

# Mathematica 11.3 Integration Test Results

Test results for the 471 problems in "4.5.7 (d trig)^m (a+b (c sec)^n)^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e + f x] (a + b \text{Sec}[e + f x]^2) dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$-\frac{(a + b) \text{ArcTanh}[\text{Cos}[e + f x]]}{f} + \frac{b \text{Sec}[e + f x]}{f}$$

Result (type 3, 84 leaves):

$$-\frac{a \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} - \frac{b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right]}{f} + \frac{a \text{Log}\left[\text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{f} + \frac{b \text{Sec}[e + f x]}{f}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e + f x]^3 (a + b \text{Sec}[e + f x]^2) dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{(a + 3b) \text{ArcTanh}[\text{Cos}[e + f x]]}{2f} - \frac{(a + b) \text{Cot}[e + f x] \text{Csc}[e + f x]}{2f} + \frac{b \text{Sec}[e + f x]}{f}$$

Result (type 3, 236 leaves):

$$-\frac{a \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{8f} - \frac{b \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{8f} - \frac{a \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right]}{2f} - \frac{3b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right]}{2f} + \frac{a \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{2f} + \frac{3b \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{2f} + \frac{a \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{8f} + \frac{b \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{8f} + \frac{b \text{Sin}\left[\frac{1}{2}(e + f x)\right]}{f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)} - \frac{b \text{Sin}\left[\frac{1}{2}(e + f x)\right]}{f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)}$$

**Problem 7: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sec}[e + f x]^2) dx$$

Optimal (type 3, 81 leaves, 5 steps):

$$\frac{3(a + 5b) \text{ArcTanh}[\text{Cos}[e + f x]]}{8f} - \frac{(3a + 7b) \text{Cot}[e + f x] \text{Csc}[e + f x]}{8f} - \frac{(a + b) \text{Cot}[e + f x] \text{Csc}[e + f x]^3}{4f} + \frac{b \text{Sec}[e + f x]}{f}$$

Result (type 3, 198 leaves):

$$\frac{1}{64f} \left( -2(3a + 7b) \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2 - (a + b) \text{Csc}\left[\frac{1}{2}(e + f x)\right]^4 + \frac{1}{-1 + \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \left( 2 \left( -3(a + 13b) + 4 \text{Cos}[e + f x] \left( 8b + 3(a + 5b) \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right] - 3(a + 5b) \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) \right) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 - (a + b) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^4 + (4(a + 2b) + (3a + 7b) \text{Cos}[e + f x]) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^4 \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right)$$

**Problem 18: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x] (a + b \text{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 52 leaves, 4 steps):

$$\frac{(a + b)^2 \text{ArcTanh}[\text{Cos}[e + f x]]}{f} + \frac{b(2a + b) \text{Sec}[e + f x]}{f} + \frac{b^2 \text{Sec}[e + f x]^3}{3f}$$

Result (type 3, 108 leaves):

$$- \left( \left( 4(b + a \text{Cos}[e + f x]^2)^2 \left( -b^2 - 3b(2a + b) \text{Cos}[e + f x]^2 + 3(a + b)^2 \text{Cos}[e + f x]^3 \left( \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right] - \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) \right) \text{Sec}[e + f x]^3 \right) / \left( 3f(a + 2b + a \text{Cos}[2(e + f x)])^2 \right) \right)$$

**Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^3 (a + b \text{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{(a+b)(a+5b) \operatorname{ArcTanh}[\cos[e+fx]]}{2f} - \frac{(3a^2+6ab+5b^2) \cot[e+fx] \csc[e+fx]}{6f} + \\
 & \frac{b(6a+5b) \sec[e+fx]}{3f} + \frac{b^2 \csc[e+fx]^2 \sec[e+fx]^3}{3f}
 \end{aligned}$$

Result (type 3, 1021 leaves):

$$\begin{aligned}
 & \frac{(-a^2-2ab-b^2) \cos[e+fx]^4 \csc\left[\frac{e}{2}+\frac{fx}{2}\right]^2 (a+b \sec[e+fx]^2)^2}{2f(a+2b+a \cos[2e+2fx])^2} - \\
 & \left( 2(a^2+6ab+5b^2) \cos[e+fx]^4 \operatorname{Log}\left[\cos\left[\frac{e}{2}+\frac{fx}{2}\right]\right] (a+b \sec[e+fx]^2)^2 \right) / \\
 & \left( f(a+2b+a \cos[2e+2fx])^2 \right) + \\
 & \left( 2(a^2+6ab+5b^2) \cos[e+fx]^4 \operatorname{Log}\left[\sin\left[\frac{e}{2}+\frac{fx}{2}\right]\right] (a+b \sec[e+fx]^2)^2 \right) / \\
 & \left( f(a+2b+a \cos[2e+2fx])^2 \right) + \frac{2b(12a+13b) \cos[e+fx]^4 \sec[e] (a+b \sec[e+fx]^2)^2}{3f(a+2b+a \cos[2e+2fx])^2} + \\
 & \frac{(a^2+2ab+b^2) \cos[e+fx]^4 \sec\left[\frac{e}{2}+\frac{fx}{2}\right]^2 (a+b \sec[e+fx]^2)^2}{2f(a+2b+a \cos[2e+2fx])^2} + \\
 & \left( 2b^2 \cos[e+fx]^4 (a+b \sec[e+fx]^2)^2 \sin\left[\frac{fx}{2}\right] \right) / \\
 & \left( 3f(a+2b+a \cos[2e+2fx])^2 \left( \cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}+\frac{fx}{2}\right] - \sin\left[\frac{e}{2}+\frac{fx}{2}\right] \right)^3 \right) + \\
 & \left( \cos[e+fx]^4 (a+b \sec[e+fx]^2)^2 \left( b^2 \cos\left[\frac{e}{2}\right] + b^2 \sin\left[\frac{e}{2}\right] \right) \right) / \\
 & \left( 3f(a+2b+a \cos[2e+2fx])^2 \left( \cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}+\frac{fx}{2}\right] - \sin\left[\frac{e}{2}+\frac{fx}{2}\right] \right)^2 \right) + \\
 & \left( 2 \cos[e+fx]^4 (a+b \sec[e+fx]^2)^2 \left( 12ab \sin\left[\frac{fx}{2}\right] + 13b^2 \sin\left[\frac{fx}{2}\right] \right) \right) / \\
 & \left( 3f(a+2b+a \cos[2e+2fx])^2 \left( \cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}+\frac{fx}{2}\right] - \sin\left[\frac{e}{2}+\frac{fx}{2}\right] \right) \right) - \\
 & \left( 2b^2 \cos[e+fx]^4 (a+b \sec[e+fx]^2)^2 \sin\left[\frac{fx}{2}\right] \right) / \\
 & \left( 3f(a+2b+a \cos[2e+2fx])^2 \left( \cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}+\frac{fx}{2}\right] + \sin\left[\frac{e}{2}+\frac{fx}{2}\right] \right)^3 \right) + \\
 & \left( \cos[e+fx]^4 (a+b \sec[e+fx]^2)^2 \left( b^2 \cos\left[\frac{e}{2}\right] - b^2 \sin\left[\frac{e}{2}\right] \right) \right) / \\
 & \left( 3f(a+2b+a \cos[2e+2fx])^2 \left( \cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}+\frac{fx}{2}\right] + \sin\left[\frac{e}{2}+\frac{fx}{2}\right] \right)^2 \right) - \\
 & \left( 2 \cos[e+fx]^4 (a+b \sec[e+fx]^2)^2 \left( 12ab \sin\left[\frac{fx}{2}\right] + 13b^2 \sin\left[\frac{fx}{2}\right] \right) \right) / \\
 & \left( 3f(a+2b+a \cos[2e+2fx])^2 \left( \cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}+\frac{fx}{2}\right] + \sin\left[\frac{e}{2}+\frac{fx}{2}\right] \right) \right)
 \end{aligned}$$

**Problem 21: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 \operatorname{Sin}[e + f x]^6 dx$$

Optimal (type 3, 148 leaves, 7 steps):

$$\frac{5}{16} (a^2 - 12 a b + 8 b^2) x - \frac{(3 a^2 - 36 a b + 8 b^2) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{16 f} +$$

$$\frac{a (a - 12 b) \operatorname{Cos}[e + f x]^3 \operatorname{Sin}[e + f x]}{24 f} - \frac{(a^2 - 12 a b + 12 b^2) \operatorname{Tan}[e + f x]}{6 f} +$$

$$\frac{a^2 \operatorname{Sin}[e + f x]^6 \operatorname{Tan}[e + f x]}{6 f} + \frac{b^2 \operatorname{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 499 leaves):

$$\frac{1}{768 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2} (b + a \operatorname{Cos}[e + f x]^2)^2 \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^3$$

$$(360 (a^2 - 12 a b + 8 b^2) f x \operatorname{Cos}[f x] + 360 (a^2 - 12 a b + 8 b^2) f x \operatorname{Cos}[2 e + f x] +$$

$$120 a^2 f x \operatorname{Cos}[2 e + 3 f x] - 1440 a b f x \operatorname{Cos}[2 e + 3 f x] + 960 b^2 f x \operatorname{Cos}[2 e + 3 f x] +$$

$$120 a^2 f x \operatorname{Cos}[4 e + 3 f x] - 1440 a b f x \operatorname{Cos}[4 e + 3 f x] + 960 b^2 f x \operatorname{Cos}[4 e + 3 f x] -$$

$$81 a^2 \operatorname{Sin}[f x] + 3444 a b \operatorname{Sin}[f x] - 3168 b^2 \operatorname{Sin}[f x] - 81 a^2 \operatorname{Sin}[2 e + f x] -$$

$$1164 a b \operatorname{Sin}[2 e + f x] + 2208 b^2 \operatorname{Sin}[2 e + f x] - 109 a^2 \operatorname{Sin}[2 e + 3 f x] + 2076 a b \operatorname{Sin}[2 e + 3 f x] -$$

$$1936 b^2 \operatorname{Sin}[2 e + 3 f x] - 109 a^2 \operatorname{Sin}[4 e + 3 f x] + 540 a b \operatorname{Sin}[4 e + 3 f x] - 144 b^2 \operatorname{Sin}[4 e + 3 f x] -$$

$$21 a^2 \operatorname{Sin}[4 e + 5 f x] + 156 a b \operatorname{Sin}[4 e + 5 f x] - 48 b^2 \operatorname{Sin}[4 e + 5 f x] - 21 a^2 \operatorname{Sin}[6 e + 5 f x] +$$

$$156 a b \operatorname{Sin}[6 e + 5 f x] - 48 b^2 \operatorname{Sin}[6 e + 5 f x] + 6 a^2 \operatorname{Sin}[6 e + 7 f x] - 12 a b \operatorname{Sin}[6 e + 7 f x] +$$

$$6 a^2 \operatorname{Sin}[8 e + 7 f x] - 12 a b \operatorname{Sin}[8 e + 7 f x] - a^2 \operatorname{Sin}[8 e + 9 f x] - a^2 \operatorname{Sin}[10 e + 9 f x])$$

**Problem 24: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b (2 a + b) \operatorname{Tan}[e + f x]}{f} + \frac{b^2 \operatorname{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 106 leaves):

$$\left( 4 (b + a \operatorname{Cos}[e + f x]^2)^2 \operatorname{Sec}[e + f x]^3 \right.$$

$$\left. (3 a^2 f x \operatorname{Cos}[e + f x]^3 + b^2 \operatorname{Sec}[e] \operatorname{Sin}[f x] + 2 b (3 a + b) \operatorname{Cos}[e + f x]^2 \operatorname{Sec}[e] \operatorname{Sin}[f x] + \right.$$

$$\left. b^2 \operatorname{Cos}[e + f x] \operatorname{Tan}[e]) \right) / \left( 3 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2 \right)$$

**Problem 25: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + f x]^2 (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$-\frac{(a+b)^2 \cot [e+f x]}{f} + \frac{2 b(a+b) \tan [e+f x]}{f} + \frac{b^2 \tan [e+f x]^3}{3 f}$$

Result (type 3, 109 leaves):

$$\left(4(b+a \cos [e+f x])^2 \sec [e+f x]^3 \left(b^2 \sec [e] \sin [f x] + \cos [e+f x]^2 \left(3(a+b)^2 \cot [e+f x] \csc [e] + b(6 a+5 b) \sec [e]\right) \sin [f x] + b^2 \cos [e+f x] \tan [e]\right)\right) / \left(3 f(a+2 b+a \cos [2(e+f x)])^2\right)$$

**Problem 27: Result more than twice size of optimal antiderivative.**

$$\int \csc [e+f x]^6 (a+b \sec [e+f x]^2)^2 dx$$

Optimal (type 3, 103 leaves, 3 steps):

$$-\frac{(a^2+6 a b+6 b^2) \cot [e+f x]}{f} - \frac{2(a+b)(a+2 b) \cot [e+f x]^3}{3 f} - \frac{(a+b)^2 \cot [e+f x]^5}{5 f} + \frac{2 b(a+2 b) \tan [e+f x]}{f} + \frac{b^2 \tan [e+f x]^3}{3 f}$$

Result (type 3, 353 leaves):

$$-\frac{1}{1920 f} \csc [e] \csc [e+f x]^5 \sec [e] \sec [e+f x]^3 \left(20 a(5 a+12 b) \sin [2 e] - 32(2 a^2+9 a b+12 b^2) \sin [2 f x] - 24 a^2 \sin [2(e+f x)] - 108 a b \sin [2(e+f x)] - 54 b^2 \sin [2(e+f x)] + 8 a^2 \sin [4(e+f x)] + 36 a b \sin [4(e+f x)] + 18 b^2 \sin [4(e+f x)] + 8 a^2 \sin [6(e+f x)] + 36 a b \sin [6(e+f x)] + 18 b^2 \sin [6(e+f x)] - 4 a^2 \sin [8(e+f x)] - 18 a b \sin [8(e+f x)] - 9 b^2 \sin [8(e+f x)] + 8 a^2 \sin [2(e+2 f x)] + 96 a b \sin [2(e+2 f x)] + 128 b^2 \sin [2(e+2 f x)] + 40 a^2 \sin [4 e+2 f x] + 8 a^2 \sin [4 e+6 f x] + 96 a b \sin [4 e+6 f x] + 128 b^2 \sin [4 e+6 f x] - 4 a^2 \sin [6 e+8 f x] - 48 a b \sin [6 e+8 f x] - 64 b^2 \sin [6 e+8 f x]\right)$$

**Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin [e+f x]^5}{a+b \sec [e+f x]^2} dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{\sqrt{b}(a+b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos [e+f x]}{\sqrt{b}}\right]}{a^{7/2} f} - \frac{(a+b)^2 \cos [e+f x]}{a^3 f} + \frac{(2 a+b) \cos [e+f x]^3}{3 a^2 f} - \frac{\cos [e+f x]^5}{5 a f}$$

Result (type 3, 425 leaves):

$$\frac{1}{1920 a^{7/2} \sqrt{b} f (a + b \operatorname{Sec}[e + f x]^2)} (a + 2 b + a \operatorname{Cos}[2 (e + f x)])$$

$$\left( \begin{aligned} & 15 (5 a^3 + 64 a^2 b + 128 a b^2 + 64 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \right. \right. \\ & \quad \left. \left. \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] + \\ & 15 (5 a^3 + 64 a^2 b + 128 a b^2 + 64 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \right. \right. \\ & \quad \left. \left. \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] - \\ & 75 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{b}}\right] - 75 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{b}}\right] - \\ & 8 \sqrt{a} \sqrt{b} \operatorname{Cos}[e + f x] \\ & (89 a^2 + 220 a b + 120 b^2 - 4 a (7 a + 5 b) \operatorname{Cos}[2 (e + f x)] + 3 a^2 \operatorname{Cos}[4 (e + f x)]) \end{aligned} \right) \operatorname{Sec}[e + f x]^2$$

**Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^3}{a + b \operatorname{Sec}[e + f x]^2} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\sqrt{b} (a + b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e + f x]}{\sqrt{b}}\right]}{a^{5/2} f} - \frac{(a + b) \operatorname{Cos}[e + f x]}{a^2 f} + \frac{\operatorname{Cos}[e + f x]^3}{3 a f}$$

Result (type 3, 376 leaves):

$$\frac{1}{48 a^{5/2} \sqrt{b} f (a + b \operatorname{Sec}[e + f x]^2)} (a + 2 b + a \operatorname{Cos}[2 (e + f x)])$$

$$\left( 3 (a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \right.$$

$$\left. \left. \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) +$$

$$3 (a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) -$$

$$3 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{b}}\right] - 3 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{b}}\right] +$$

$$4 \sqrt{a} \sqrt{b} \operatorname{Cos}[e + f x] (-5 a - 6 b + a \operatorname{Cos}[2 (e + f x)]) \Bigg) \operatorname{Sec}[e + f x]^2$$

**Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]}{a + b \operatorname{Sec}[e + f x]^2} dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e + f x]}{\sqrt{b}}\right]}{a^{3/2} f} - \frac{\operatorname{Cos}[e + f x]}{a f}$$

Result (type 3, 329 leaves):

$$\frac{1}{8 a^{3/2} \sqrt{b} f (b + a \cos [e + f x])^2} \left( (a + 4 b) \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}) \sin [e] \tan \left[ \frac{f x}{2} \right] + \cos [e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[ \frac{f x}{2} \right] \right) \right) \right] + (a + 4 b) \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}) \sin [e] \tan \left[ \frac{f x}{2} \right] + \cos [e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[ \frac{f x}{2} \right] \right) \right) \right] - a \operatorname{ArcTan} \left[ \frac{\sqrt{a} - \sqrt{a+b} \tan \left[ \frac{1}{2} (e + f x) \right]}{\sqrt{b}} \right] - a \operatorname{ArcTan} \left[ \frac{\sqrt{a} + \sqrt{a+b} \tan \left[ \frac{1}{2} (e + f x) \right]}{\sqrt{b}} \right] - 4 \sqrt{a} \sqrt{b} \cos [e + f x] \right) (a + 2 b + a \cos [2 (e + f x)])$$

**Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc [e + f x]}{a + b \sec [e + f x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{a} \cos [e + f x]}{\sqrt{b}} \right]}{\sqrt{a} (a + b) f} - \frac{\operatorname{ArcTanh} [\cos [e + f x]]}{(a + b) f}$$

Result (type 3, 239 leaves):

$$\frac{1}{(a + b) f} \left( \frac{1}{\sqrt{a}} \sqrt{b} \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}) \sin [e] \tan \left[ \frac{f x}{2} \right] + \cos [e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[ \frac{f x}{2} \right] \right) \right) \right] + \frac{1}{\sqrt{a}} \sqrt{b} \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}) \sin [e] \tan \left[ \frac{f x}{2} \right] + \cos [e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[ \frac{f x}{2} \right] \right) \right) \right] - \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (e + f x) \right] \right] + \operatorname{Log} \left[ \sin \left[ \frac{1}{2} (e + f x) \right] \right] \right)$$

**Problem 32: Result unnecessarily involves complex numbers and more than**



twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[e + f x]^3}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$\frac{\sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cos}[e + f x]}{\sqrt{b}}\right]}{(a + b)^2 f} - \frac{(a - b) \text{ArcTanh}[\text{Cos}[e + f x]]}{2 (a + b)^2 f} - \frac{\text{Cot}[e + f x] \text{Csc}[e + f x]}{2 (a + b) f}$$

Result (type 3, 371 leaves):

$$\begin{aligned} & - \frac{1}{16 (a + b)^2 f (a + b \text{Sec}[e + f x]^2)} (a + 2 b + a \text{Cos}[2 (e + f x)]) \\ & \left( -8 \sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a + b}) \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \right) \text{Sin}[e] \text{Tan}\left[\frac{f x}{2}\right] + \right. \right. \\ & \quad \left. \left. \text{Cos}[e] \left( \sqrt{a} - \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) - \\ & 8 \sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a + b}) \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \right) \text{Sin}[e] \text{Tan}\left[\frac{f x}{2}\right] + \right. \\ & \quad \left. \left. \text{Cos}[e] \left( \sqrt{a} + \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) + a \text{Csc}\left[\frac{1}{2} (e + f x)\right]^2 + \\ & b \text{Csc}\left[\frac{1}{2} (e + f x)\right]^2 + 4 a \text{Log}[\text{Cos}\left[\frac{1}{2} (e + f x)\right]] - 4 b \text{Log}[\text{Cos}\left[\frac{1}{2} (e + f x)\right]] - \\ & 4 a \text{Log}[\text{Sin}\left[\frac{1}{2} (e + f x)\right]] + 4 b \text{Log}[\text{Sin}\left[\frac{1}{2} (e + f x)\right]] - \\ & a \text{Sec}\left[\frac{1}{2} (e + f x)\right]^2 - b \text{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \Big) \text{Sec}[e + f x]^2 \end{aligned}$$

Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[e + f x]^5}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 129 leaves, 6 steps):

$$\frac{a^{3/2} \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cos}[e + f x]}{\sqrt{b}}\right]}{(a + b)^3 f} - \frac{(3 a^2 - 6 a b - b^2) \text{ArcTanh}[\text{Cos}[e + f x]]}{8 (a + b)^3 f} - \frac{(3 a - b) \text{Cot}[e + f x] \text{Csc}[e + f x]}{8 (a + b)^2 f} - \frac{\text{Cot}[e + f x] \text{Csc}[e + f x]^3}{4 (a + b) f}$$

Result (type 3, 903 leaves):

$$\begin{aligned}
 & \left( a^{3/2} \sqrt{b} \right. \\
 & \quad \text{ArcTan} \left[ \frac{1}{2\sqrt{b}} \text{Sec} \left[ \frac{fx}{2} \right] \left( 2\sqrt{a} \cos \left[ e + \frac{fx}{2} \right] - i\sqrt{a+b} \cos \left[ e - \frac{fx}{2} \right] \sqrt{\cos[2e] - i\sin[2e]} + \right. \right. \\
 & \quad \quad i\sqrt{a+b} \cos \left[ e + \frac{fx}{2} \right] \sqrt{\cos[2e] - i\sin[2e]} + \sqrt{a+b} \sqrt{\cos[2e] - i\sin[2e]} \\
 & \quad \quad \left. \left. \sin \left[ e - \frac{fx}{2} \right] - \sqrt{a+b} \sqrt{\cos[2e] - i\sin[2e]} \sin \left[ e + \frac{fx}{2} \right] \right) \right] \\
 & \quad \left. \left( a + 2b + a \cos[2e + 2fx] \right) \text{Sec} \left[ e + fx \right]^2 \right) / \left( 2(a+b)^3 f (a+b \text{Sec} \left[ e + fx \right]^2) \right) + \\
 & \left( a^{3/2} \sqrt{b} \text{ArcTan} \left[ \frac{1}{2\sqrt{b}} \text{Sec} \left[ \frac{fx}{2} \right] \left( 2\sqrt{a} \cos \left[ e + \frac{fx}{2} \right] + i\sqrt{a+b} \cos \left[ e - \frac{fx}{2} \right] \right. \right. \right. \\
 & \quad \quad \left. \left. \sqrt{\cos[2e] - i\sin[2e]} - i\sqrt{a+b} \cos \left[ e + \frac{fx}{2} \right] \sqrt{\cos[2e] - i\sin[2e]} - \sqrt{a+b} \right. \right. \\
 & \quad \quad \left. \left. \sqrt{\cos[2e] - i\sin[2e]} \sin \left[ e - \frac{fx}{2} \right] + \sqrt{a+b} \sqrt{\cos[2e] - i\sin[2e]} \sin \left[ e + \frac{fx}{2} \right] \right) \right] \\
 & \quad \left. \left( a + 2b + a \cos[2e + 2fx] \right) \text{Sec} \left[ e + fx \right]^2 \right) / \left( 2(a+b)^3 f (a+b \text{Sec} \left[ e + fx \right]^2) \right) + \\
 & \quad \frac{(-3a+b) (a+2b+a \cos[2e+2fx]) \text{Csc} \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 \text{Sec} \left[ e + fx \right]^2}{64(a+b)^2 f (a+b \text{Sec} \left[ e + fx \right]^2)} - \\
 & \quad \frac{(a+2b+a \cos[2e+2fx]) \text{Csc} \left[ \frac{e}{2} + \frac{fx}{2} \right]^4 \text{Sec} \left[ e + fx \right]^2}{128(a+b) f (a+b \text{Sec} \left[ e + fx \right]^2)} + \\
 & \quad \left( (-3a^2 + 6ab + b^2) (a+2b+a \cos[2e+2fx]) \text{Log} \left[ \cos \left[ \frac{e}{2} + \frac{fx}{2} \right] \right] \text{Sec} \left[ e + fx \right]^2 \right) / \\
 & \quad \left( 16(a+b)^3 f (a+b \text{Sec} \left[ e + fx \right]^2) \right) + \\
 & \quad \left( (3a^2 - 6ab - b^2) (a+2b+a \cos[2e+2fx]) \text{Log} \left[ \sin \left[ \frac{e}{2} + \frac{fx}{2} \right] \right] \text{Sec} \left[ e + fx \right]^2 \right) / \\
 & \quad \left( 16(a+b)^3 f (a+b \text{Sec} \left[ e + fx \right]^2) \right) + \\
 & \quad \frac{(3a-b) (a+2b+a \cos[2e+2fx]) \text{Sec} \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 \text{Sec} \left[ e + fx \right]^2}{64(a+b)^2 f (a+b \text{Sec} \left[ e + fx \right]^2)} + \\
 & \quad \frac{(a+2b+a \cos[2e+2fx]) \text{Sec} \left[ \frac{e}{2} + \frac{fx}{2} \right]^4 \text{Sec} \left[ e + fx \right]^2}{128(a+b) f (a+b \text{Sec} \left[ e + fx \right]^2)}
 \end{aligned}$$

**Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^6}{a+b \text{Sec}[e+fx]^2} dx$$

Optimal (type 3, 166 leaves, 7 steps):

$$\frac{(5a^3 + 30a^2b + 40ab^2 + 16b^3)x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a^4 f} -$$

$$\frac{(11a^2 + 18ab + 8b^2)\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{16a^3 f} +$$

$$\frac{(3a+2b)\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]}{8a^2 f} + \frac{\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]^3}{6af}$$

Result (type 3, 357 leaves):

$$\frac{1}{768a^4\sqrt{b}\sqrt{a+b}f(a+b\operatorname{Sec}[e+fx]^2)\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4}$$

$$(a+2b+a\operatorname{Cos}[2(e+fx)])\operatorname{Sec}[e+fx]^2\left(3\sqrt{b}(9a^4+136a^3b+384a^2b^2+384ab^3+128b^4)\right.$$

$$\operatorname{ArcTan}\left[\left(\operatorname{Sec}[fx](\operatorname{Cos}[2e]-i\operatorname{Sin}[2e])-(a+2b)\operatorname{Sin}[fx]+a\operatorname{Sin}[2e+fx]\right)\right]/$$

$$\left(2\sqrt{a+b}\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4\right)](\operatorname{Cos}[2e]-i\operatorname{Sin}[2e])+$$

$$\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4\left(3a^3(9a+8b)\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]+2\sqrt{b}\sqrt{a+b}\right.$$

$$\left.\left(-12a^3e+60a^3fx+360a^2bfx+480ab^2fx+192b^3fx-3a(15a^2+32ab+16b^2)\right.\right.$$

$$\left.\left.\operatorname{Sin}[2(e+fx)]+3a^2(3a+2b)\operatorname{Sin}[4(e+fx)]-a^3\operatorname{Sin}[6(e+fx)]\right)\right)\right)$$

**Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^4}{a+b\operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{(3a^2 + 12ab + 8b^2)x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a^3 f} -$$

$$\frac{(5a+4b)\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{8a^2 f} + \frac{\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]}{4af}$$

Result (type 3, 303 leaves):

$$\frac{1}{64 a^3 \sqrt{b} \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4} \\ (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2 \left( \sqrt{b} (3 a^3+34 a^2 b+64 a b^2+32 b^3) \right. \\ \left. \operatorname{ArcTan}\left[\left(\operatorname{Sec}[f x] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e])\right) (- (a+2 b) \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x])\right]\right) / \\ \left( 2 \sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4 \right) (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) + \\ \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4 \left( a^2 (3 a+2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right] + \sqrt{b} \sqrt{a+b} (-2 a^2 e + \right. \\ \left. 12 a^2 f x+48 a b f x+32 b^2 f x-8 a(a+b) \operatorname{Sin}[2(e+f x)]+a^2 \operatorname{Sin}[4(e+f x)]) \right) \left. \right) \left. \right) \left. \right) /$$

**Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+f x]^2}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 76 leaves, 5 steps):

$$\frac{(a+2 b) x}{2 a^2} - \frac{\sqrt{b} \sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{a^2 f} - \frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{2 a f}$$

Result (type 3, 245 leaves):

$$\left( (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2 \right. \\ \left( \frac{\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{\sqrt{b} \sqrt{a+b} f} - \frac{1}{a^2} \left( -4(a+2 b) x - \left( a^2+8 a b+8 b^2 \right) \right. \right. \\ \left. \left. \operatorname{ArcTan}\left[\left(\operatorname{Sec}[f x] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e])\right) (- (a+2 b) \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x])\right]\right) / \right. \\ \left. \left( 2 \sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4 \right) (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) \right) / \\ \left( \sqrt{a+b} f \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4 \right) + \frac{2 a \operatorname{Cos}[2 f x] \operatorname{Sin}[2 e]}{f} + \\ \left. \left. \left. \frac{2 a \operatorname{Cos}[2 e] \operatorname{Sin}[2 f x]}{f} \right) \right) \left. \right) / \left( 16 (a+b \operatorname{Sec}[e+f x]^2) \right)$$

**Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + b \operatorname{Sec}[e + f x]^2} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{x}{a} + \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \operatorname{Cot}[e+fx]}{\sqrt{b}}\right]}{a \sqrt{a+b} f}$$

Result (type 3, 182 leaves):

$$\left( (a + 2b + a \operatorname{Cos}[2(e + f x)]) \operatorname{Sec}[e + f x]^2 \left( \sqrt{a+b} f x \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} + \right. \right. \\ \left. \left. b \operatorname{ArcTan}\left[ (\operatorname{Sec}[f x] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a + 2b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2e + f x])) \right] \right) \right) / \\ \left( 2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \Big) / \\ \left( 2 a \sqrt{a+b} f (a + b \operatorname{Sec}[e + f x]^2) \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right)$$

**Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]^2}{a + b \operatorname{Sec}[e + f x]^2} dx$$

Optimal (type 3, 54 leaves, 3 steps):

$$-\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{3/2} f} - \frac{\operatorname{Cot}[e + f x]}{(a+b) f}$$

Result (type 3, 189 leaves):

$$\left( (a + 2b + a \operatorname{Cos}[2(e + f x)]) \operatorname{Sec}[e + f x]^2 \right. \\ \left( b \operatorname{ArcTan}\left[ (\operatorname{Sec}[f x] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a + 2b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2e + f x])) \right] \right) / \\ \left( 2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) + \\ \left. \sqrt{a+b} \operatorname{Csc}[e] \operatorname{Csc}[e + f x] \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \operatorname{Sin}[f x] \right) \Big) / \\ \left( 2 (a+b)^{3/2} f (a + b \operatorname{Sec}[e + f x]^2) \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right)$$

**Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]^4}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$-\frac{a \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{(a + b)^{5/2} f} - \frac{a \text{Cot}[e + f x]}{(a + b)^2 f} - \frac{\text{Cot}[e + f x]^3}{3 (a + b) f}$$

Result (type 3, 226 leaves):

$$\left( (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Sec}[e + f x]^2 \right. \\ \left. \left( 3 a b \text{ArcTan}\left[ (\text{Sec}[f x] (\text{Cos}[2 e] - i \text{Sin}[2 e]) (- (a + 2 b) \text{Sin}[f x] + a \text{Sin}[2 e + f x])) \right] \right) \right) / \\ \left( 2 \sqrt{a + b} \sqrt{b (\text{Cos}[e] - i \text{Sin}[e])^4} \right) (\text{Cos}[2 e] - i \text{Sin}[2 e]) + \\ \frac{1}{4} \sqrt{a + b} \text{Csc}[e] \text{Csc}[e + f x]^3 \sqrt{b (\text{Cos}[e] - i \text{Sin}[e])^4} \\ \left. \left( 6 a \text{Sin}[f x] - 3 b \text{Sin}[2 e + f x] + (-2 a + b) \text{Sin}[2 e + 3 f x] \right) \right) / \\ \left( 6 (a + b)^{5/2} f (a + b \text{Sec}[e + f x]^2) \sqrt{b (\text{Cos}[e] - i \text{Sin}[e])^4} \right)$$

**Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]^6}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 105 leaves, 4 steps):

$$-\frac{a^2 \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{(a + b)^{7/2} f} - \frac{a^2 \text{Cot}[e + f x]}{(a + b)^3 f} - \frac{(2 a + b) \text{Cot}[e + f x]^3}{3 (a + b)^2 f} - \frac{\text{Cot}[e + f x]^5}{5 (a + b) f}$$

Result (type 3, 318 leaves):

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$$\begin{aligned}
 & 480 (a+b)^{7/2} f (a+b \operatorname{Sec}[e+fx]^2) \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \\
 & (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^2 \\
 & \left( 240 a^2 b \operatorname{ArcTan}[(\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) - (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx])] \right) / \\
 & \left( 2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) + \sqrt{a+b} \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^5 \\
 & \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} (10 (8 a^2 + b^2) \operatorname{Sin}[fx] - 30 b (3 a + b) \operatorname{Sin}[2e+fx] - \\
 & 40 a^2 \operatorname{Sin}[2e+3fx] + 30 a b \operatorname{Sin}[2e+3fx] + 10 b^2 \operatorname{Sin}[2e+3fx] + \\
 & 15 a b \operatorname{Sin}[4e+3fx] + 8 a^2 \operatorname{Sin}[4e+5fx] - 9 a b \operatorname{Sin}[4e+5fx] - 2 b^2 \operatorname{Sin}[4e+5fx]) \Big)
 \end{aligned}$$

**Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^5}{(a+b \operatorname{Sec}[e+fx]^2)^2} dx$$

Optimal (type 3, 161 leaves, 6 steps):

$$\begin{aligned}
 & \frac{\sqrt{b} (a+b) (3a+7b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e+fx]}{\sqrt{b}}\right]}{2 a^{9/2} f} - \frac{(a+b) (3a+7b) \operatorname{Cos}[e+fx]}{2 a^4 f} + \\
 & \frac{(a+b) (3a+7b) \operatorname{Cos}[e+fx]^3}{6 a^3 b f} - \frac{\operatorname{Cos}[e+fx]^5}{5 a^2 f} - \frac{(a+b)^2 \operatorname{Cos}[e+fx]^5}{2 a^2 b f (b+a \operatorname{Cos}[e+fx]^2)}
 \end{aligned}$$

Result (type 3, 454 leaves):

$$\frac{1}{3840 a^{9/2} f} \left( \frac{1}{b^{3/2}} 15 (3 a^4 + 384 a^2 b^2 + 1280 a b^3 + 896 b^4) \right. \\ \text{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}) \text{Sin}[e] \text{Tan} \left[ \frac{f x}{2} \right] + \right. \right. \\ \left. \left. \text{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan} \left[ \frac{f x}{2} \right] \right) \right) \right] + \frac{1}{b^{3/2}} \\ 15 (3 a^4 + 384 a^2 b^2 + 1280 a b^3 + 896 b^4) \text{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}) \right. \right. \\ \left. \left. \text{Sin}[e] \text{Tan} \left[ \frac{f x}{2} \right] + \text{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan} \left[ \frac{f x}{2} \right] \right) \right) \right] - \\ \frac{45 a^4 \text{ArcTan} \left[ \frac{\sqrt{a} - \sqrt{a+b} \text{Tan} \left[ \frac{1}{2} (e+f x) \right]}{\sqrt{b}} \right]}{b^{3/2}} - \frac{45 a^4 \text{ArcTan} \left[ \frac{\sqrt{a} + \sqrt{a+b} \text{Tan} \left[ \frac{1}{2} (e+f x) \right]}{\sqrt{b}} \right]}{b^{3/2}} - \\ \left. \left( 16 \sqrt{a} \text{Cos}[e+f x] \right. \right. \\ \left. \left. (150 a^3 + 1436 a^2 b + 2960 a b^2 + 1680 b^3 + a (125 a^2 + 688 a b + 560 b^2) \text{Cos}[2 (e+f x)] - \right. \right. \\ \left. \left. 2 a^2 (11 a + 14 b) \text{Cos}[4 (e+f x)] + 3 a^3 \text{Cos}[6 (e+f x)] \right) \right) / (a + 2 b + a \text{Cos}[2 (e+f x)]) \left. \right)$$

**Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e+f x]^3}{(a+b \text{Sec}[e+f x]^2)^2} dx$$

Optimal (type 3, 114 leaves, 5 steps):

$$\frac{\sqrt{b} (3 a + 5 b) \text{ArcTan} \left[ \frac{\sqrt{a} \text{Cos}[e+f x]}{\sqrt{b}} \right]}{2 a^{7/2} f} - \\ \frac{(a + 2 b) \text{Cos}[e+f x]}{a^3 f} + \frac{\text{Cos}[e+f x]^3}{3 a^2 f} - \frac{b (a + b) \text{Cos}[e+f x]}{2 a^3 f (b + a \text{Cos}[e+f x]^2)}$$

Result (type 3, 403 leaves):



$$\frac{1}{384 a^{7/2} f} \left( \frac{1}{b^{3/2}} 3 (3 a^3 + 192 a b^2 + 320 b^3) \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b}) \sqrt{(\cos[e] - i \sin[e])^2} \right) \sin[e] \right. \right. \\
 \left. \left. \operatorname{Tan} \left[ \frac{f x}{2} \right] + \cos[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\cos[e] - i \sin[e])^2} \operatorname{Tan} \left[ \frac{f x}{2} \right] \right) \right] \right) + \frac{1}{b^{3/2}} \\
 3 (3 a^3 + 192 a b^2 + 320 b^3) \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b}) \sqrt{(\cos[e] - i \sin[e])^2} \right) \right. \\
 \left. \sin[e] \operatorname{Tan} \left[ \frac{f x}{2} \right] + \cos[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\cos[e] - i \sin[e])^2} \operatorname{Tan} \left[ \frac{f x}{2} \right] \right) \right] - \\
 \frac{9 a^3 \operatorname{ArcTan} \left[ \frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]}{\sqrt{b}} \right]}{b^{3/2}} - \frac{9 a^3 \operatorname{ArcTan} \left[ \frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]}{\sqrt{b}} \right]}{b^{3/2}} - \\
 \left( 32 \sqrt{a} \cos[e+f x] (9 a^2 + 56 a b + 60 b^2 + 4 a (2 a + 5 b) \cos[2(e+f x)] - a^2 \cos[4(e+f x)]) \right) / \\
 \left. (a + 2 b + a \cos[2(e+f x)]) \right)$$

**Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+f x]}{(a+b \sec[e+f x])^2} dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$\frac{3 \sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{a} \cos[e+f x]}{\sqrt{b}} \right]}{2 a^{5/2} f} - \frac{3 \cos[e+f x]}{2 a^2 f} + \frac{\cos[e+f x]^3}{2 a f (b+a \cos[e+f x]^2)}$$

Result (type 3, 393 leaves):

$$\frac{1}{64 a^{5/2} f (a + b \operatorname{Sec}[e + f x])^2} (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2$$

$$\left( \frac{1}{b^{3/2}} (a^2 + 24 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) + \frac{1}{b^{3/2}}$$

$$(a^2 + 24 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) -$$

$$\frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right]}{b^{3/2}} -$$

$$\left. \frac{16 \sqrt{a} \operatorname{Cos}[e + f x] (a + 3 b + a \operatorname{Cos}[2 (e + f x)])}{a + 2 b + a \operatorname{Cos}[2 (e + f x)]} \right) \operatorname{Sec}[e + f x]^4$$

**Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$\frac{\sqrt{b} (3 a + b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e+fx]}{\sqrt{b}}\right]}{2 a^{3/2} (a+b)^2 f} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[e + f x]]}{(a+b)^2 f} - \frac{b \operatorname{Cos}[e + f x]}{2 a (a+b) f (b + a \operatorname{Cos}[e + f x])^2}$$

Result (type 3, 384 leaves):

$$\begin{aligned}
 & \frac{1}{8 (a+b)^2 f (a+b \operatorname{Sec}[e+f x])^2} (a+2 b+a \operatorname{Cos}[2 (e+f x)]) \operatorname{Sec}[e+f x]^3 \\
 & \left( -\frac{2 b (a+b)}{a} + \frac{1}{a^{3/2}} \sqrt{b} (3 a+b) \operatorname{ArcTan}\left[ \frac{1}{\sqrt{b}} \left( \left( -\sqrt{a}-i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \right) \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sin}[e] \operatorname{Tan}\left[ \frac{f x}{2} \right] + \operatorname{Cos}[e] \left( \sqrt{a}-\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[ \frac{f x}{2} \right] \right) \right) \right] \right) \\
 & (a+2 b+a \operatorname{Cos}[2 (e+f x)]) \operatorname{Sec}[e+f x] + \frac{1}{a^{3/2}} \sqrt{b} (3 a+b) \\
 & \operatorname{ArcTan}\left[ \frac{1}{\sqrt{b}} \left( \left( -\sqrt{a}+i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[ \frac{f x}{2} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}[e] \left( \sqrt{a}+\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[ \frac{f x}{2} \right] \right) \right) \right] (a+2 b+a \operatorname{Cos}[2 (e+f x)]) \\
 & \operatorname{Sec}[e+f x] - 2 (a+2 b+a \operatorname{Cos}[2 (e+f x)]) \operatorname{Log}\left[ \operatorname{Cos}\left[ \frac{1}{2} (e+f x) \right] \right] \operatorname{Sec}[e+f x] + \\
 & 2 (a+2 b+a \operatorname{Cos}[2 (e+f x)]) \operatorname{Log}\left[ \operatorname{Sin}\left[ \frac{1}{2} (e+f x) \right] \right] \operatorname{Sec}[e+f x] \Big)
 \end{aligned}$$

**Problem 45: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+f x]^3}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$\begin{aligned}
 & \frac{(3 a-b) \sqrt{b} \operatorname{ArcTan}\left[ \frac{\sqrt{a} \operatorname{Cos}[e+f x]}{\sqrt{b}} \right]}{2 \sqrt{a} (a+b)^3 f} - \frac{(a-3 b) \operatorname{ArcTanh}[\operatorname{Cos}[e+f x]]}{2 (a+b)^3 f} + \\
 & \frac{(a-b) \operatorname{Cos}[e+f x]}{2 (a+b)^2 f (b+a \operatorname{Cos}[e+f x])^2} - \frac{\operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]}{2 (a+b) f (b+a \operatorname{Cos}[e+f x])^2}
 \end{aligned}$$

Result (type 3, 468 leaves):

$$\frac{1}{32 (a+b)^3 f (a+b \operatorname{Sec}[e+fx])^2} (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^3$$

$$\left( -8b(a+b) - \frac{1}{\sqrt{a}} 4\sqrt{b} (-3a+b) \operatorname{ArcTan}\left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2}) \right. \right. \right.$$

$$\left. \left. \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right)$$

$$(a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx] - \frac{1}{\sqrt{a}} 4\sqrt{b} (-3a+b)$$

$$\operatorname{ArcTan}\left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right) (a+2b+a \operatorname{Cos}[2(e+fx)])$$

$$\operatorname{Sec}[e+fx] - (a+b) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] -$$

$$4(a-3b) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx] +$$

$$4(a-3b) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx] +$$

$$(a+b) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]$$

**Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]^5}{(a+b \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{3\sqrt{a}(a-b)\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e+fx]}{\sqrt{b}}\right]}{2(a+b)^4 f} -$$

$$\frac{3(a^2 - 6ab + b^2) \operatorname{ArcTanh}[\operatorname{Cos}[e+fx]]}{8(a+b)^4 f} + \frac{3a(a-3b) \operatorname{Cos}[e+fx]}{8(a+b)^3 f (b+a \operatorname{Cos}[e+fx]^2)} -$$

$$\frac{(a-5b) \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]}{8(a+b)^2 f (b+a \operatorname{Cos}[e+fx]^2)} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3}{4(a+b) f (b+a \operatorname{Cos}[e+fx]^2)}$$

Result (type 3, 450 leaves):

$$\begin{aligned}
 & \frac{1}{256 (a+b)^4 f (a+b \operatorname{Sec}[e+fx])^2} (a+2b+a \operatorname{Cos}[2(e+fx)]) \\
 & \left( 96 \sqrt{a} (a-b) \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a}-i\sqrt{a+b}) \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}[e] \left( \sqrt{a}-\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right) (a+2b+a \operatorname{Cos}[2(e+fx)]) + \\
 & 96 \sqrt{a} (a-b) \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a}+i\sqrt{a+b}) \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \right. \\
 & \quad \left. \operatorname{Cos}[e] \left( \sqrt{a}+\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right) (a+2b+a \operatorname{Cos}[2(e+fx)]) - \\
 & 2(a+b) (11a^2+43ab-4b^2+4(2a^2-5ab+5b^2) \operatorname{Cos}[2(e+fx)] - \\
 & \quad 3a(a-3b) \operatorname{Cos}[4(e+fx)]) \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 - \\
 & 24(a^2-6ab+b^2) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]\right] + \\
 & 24(a^2-6ab+b^2) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx]^4
 \end{aligned}$$

**Problem 47: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^6}{(a+b \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 3, 267 leaves, 8 steps):

$$\begin{aligned}
 & \frac{(5a^3+60a^2b+120ab^2+64b^3)x}{16a^5} - \frac{\sqrt{b}(a+b)^{3/2}(3a+8b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^5 f} - \\
 & \frac{(33a^2+82ab+48b^2) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48a^3 f (a+b+b \operatorname{Tan}[e+fx]^2)} + \frac{(9a+8b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24a^2 f (a+b+b \operatorname{Tan}[e+fx]^2)} + \\
 & \frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]^3}{6af(a+b+b \operatorname{Tan}[e+fx]^2)} - \frac{b(19a^2+52ab+32b^2) \operatorname{Tan}[e+fx]}{16a^4 f (a+b+b \operatorname{Tan}[e+fx]^2)}
 \end{aligned}$$

Result (type 3, 2987 leaves):

$$\begin{aligned}
 & - \left( (a+2b+a \operatorname{Cos}[2e+2fx])^2 \operatorname{Sec}[e+fx]^4 \right. \\
 & \quad \left( 16x + \left( (-a^3+6a^2b+24ab^2+16b^3) \operatorname{ArcTan}\left[ (\operatorname{Sec}[fx] (\operatorname{Cos}[2e]-i\operatorname{Sin}[2e]) \right. \right. \right. \\
 & \quad \left. \left. \left. (- (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]) \right) \right] \right) / \left( 2\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4} \right) \right) \\
 & \quad \left( \operatorname{Cos}[2e]-i\operatorname{Sin}[2e] \right) / \left( b(a+b)^{3/2} f \sqrt{b (\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4} \right) + \\
 & \quad \left( (a^2+8ab+8b^2) ((a+2b) \operatorname{Sin}[2e]-a \operatorname{Sin}[2fx]) \right) / \\
 & \quad \left( b(a+b) f (a+2b+a \operatorname{Cos}[2(e+fx)]) (\operatorname{Cos}[e]-\operatorname{Sin}[e]) (\operatorname{Cos}[e]+\operatorname{Sin}[e]) \right) \left. \right) / \\
 & \left( 512a^2 (a+b \operatorname{Sec}[e+fx])^2 \right) + \left( 3(a+2b+a \operatorname{Cos}[2e+2fx])^2 \right)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Sec}[e + f x]^4 \\
 & \left( -64 (a + 2 b) x + \left( (a^4 - 16 a^3 b - 144 a^2 b^2 - 256 a b^3 - 128 b^4) \right. \right. \\
 & \quad \text{ArcTan}\left[ \left( \text{Sec}[f x] \left( \text{Cos}[2 e] - i \text{Sin}[2 e] \right) \left( -(a + 2 b) \text{Sin}[f x] + a \text{Sin}[2 e + f x] \right) \right) \right] / \\
 & \quad \left( 2 \sqrt{a + b} \sqrt{b \left( \text{Cos}[e] - i \text{Sin}[e] \right)^4} \right) \left( \text{Cos}[2 e] - i \text{Sin}[2 e] \right) \right) / \\
 & \quad \left( b (a + b)^{3/2} f \sqrt{b \left( \text{Cos}[e] - i \text{Sin}[e] \right)^4} + \frac{16 a \text{Cos}[2 f x] \text{Sin}[2 e]}{f} + \right. \\
 & \quad \left. \frac{16 a \text{Cos}[2 e] \text{Sin}[2 f x]}{f} - \left( (a^3 + 18 a^2 b + 48 a b^2 + 32 b^3) \left( (a + 2 b) \text{Sin}[2 e] - a \text{Sin}[2 f x] \right) \right) \right) / \\
 & \quad \left. \left( b (a + b) f (a + 2 b + a \text{Cos}[2 (e + f x)]) \left( \text{Cos}[e] - \text{Sin}[e] \right) \left( \text{Cos}[e] + \text{Sin}[e] \right) \right) \right) \right) / \\
 & \left( 4096 a^3 (a + b \text{Sec}[e + f x]^2)^2 \right) + \left( 3 (a + 2 b + a \text{Cos}[2 e + 2 f x])^2 \right. \\
 & \quad \text{Sec}[e + f x]^4 \\
 & \quad \left. \left( \frac{(a + 2 b) \text{ArcTan}\left[ \frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}} \right]}{(a + b)^{3/2}} - \frac{a \sqrt{b} \text{Sin}[2 (e + f x)]}{(a + b) (a + 2 b + a \text{Cos}[2 (e + f x)])} \right) \right) / \\
 & \quad \left( 2048 b^{3/2} f (a + b \text{Sec}[e + f x]^2)^2 \right) - \\
 & \quad \left( (a + 2 b + a \text{Cos}[2 e + 2 f x])^2 \text{Sec}[e + f x]^4 \right. \\
 & \quad \left. \left( -\frac{a \text{ArcTan}\left[ \frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}} \right]}{(a + b)^{3/2}} + \frac{\sqrt{b} (a + 2 b) \text{Sin}[2 (e + f x)]}{(a + b) (a + 2 b + a \text{Cos}[2 (e + f x)])} \right) \right) / \\
 & \quad \left( 2048 b^{3/2} f (a + b \text{Sec}[e + f x]^2)^2 \right) + \\
 & \quad \frac{1}{256 (a + b \text{Sec}[e + f x]^2)^2} \\
 & \quad (a + 2 b + a \text{Cos}[2 e + 2 f x])^2 \\
 & \quad \text{Sec}[e + f x]^4 \\
 & \quad \left( \frac{1}{a + b} (-a^5 + 30 a^4 b + 480 a^3 b^2 + 1600 a^2 b^3 + 1920 a b^4 + 768 b^5) \left( \left( \text{ArcTan}[\text{Sec}[f x] \right. \right. \right. \\
 & \quad \left. \left. \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right. \right. \\
 & \quad \left. \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Cos}[2 e] \right) \right) / \\
 & \quad \left( 8 a^4 b \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) - \left( i \text{ArcTan}[\text{Sec}[f x] \right. \\
 & \quad \left. \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left( -a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x] \right) \sin[2 e] \right) / \left( 8 a^4 b \sqrt{a+b} f \right. \\
 & \left. \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) + \frac{1}{8 a^4 b (a+b) f (a+2 b+a \cos[2 e+2 f x])} \\
 & \text{Sec}[2 e] \left( 160 a^4 b f x \cos[2 e] + 1248 a^3 b^2 f x \cos[2 e] + 3392 a^2 b^3 f x \cos[2 e] + \right. \\
 & 3840 a b^4 f x \cos[2 e] + 1536 b^5 f x \cos[2 e] + 80 a^4 b f x \cos[2 f x] + \\
 & 464 a^3 b^2 f x \cos[2 f x] + 768 a^2 b^3 f x \cos[2 f x] + 384 a b^4 f x \cos[2 f x] + \\
 & 80 a^4 b f x \cos[4 e+2 f x] + 464 a^3 b^2 f x \cos[4 e+2 f x] + 768 a^2 b^3 f x \cos[4 e+2 f x] + \\
 & 384 a b^4 f x \cos[4 e+2 f x] + a^5 \sin[2 e] + 34 a^4 b \sin[2 e] + 224 a^3 b^2 \sin[2 e] + \\
 & 576 a^2 b^3 \sin[2 e] + 640 a b^4 \sin[2 e] + 256 b^5 \sin[2 e] - a^5 \sin[2 f x] - 62 a^4 b \sin[2 f x] - \\
 & 318 a^3 b^2 \sin[2 f x] - 512 a^2 b^3 \sin[2 f x] - 256 a b^4 \sin[2 f x] - 30 a^4 b \sin[4 e+2 f x] - \\
 & 158 a^3 b^2 \sin[4 e+2 f x] - 256 a^2 b^3 \sin[4 e+2 f x] - 128 a b^4 \sin[4 e+2 f x] - \\
 & 12 a^4 b \sin[2 e+4 f x] - 36 a^3 b^2 \sin[2 e+4 f x] - 24 a^2 b^3 \sin[2 e+4 f x] - \\
 & \left. 12 a^4 b \sin[6 e+4 f x] - 36 a^3 b^2 \sin[6 e+4 f x] - 24 a^2 b^3 \sin[6 e+4 f x] + 2 a^4 b \sin[ \right. \\
 & \left. 4 e+6 f x] + 2 a^3 b^2 \sin[4 e+6 f x] + 2 a^4 b \sin[8 e+6 f x] + 2 a^3 b^2 \sin[8 e+6 f x] \right) + \\
 & \frac{1}{512 (a+b \text{Sec}[e+f x])^2} (a+2 b+a \cos[2 e+2 f x])^2 \\
 & \text{Sec}[e+f x]^4 \\
 & \left( -\frac{1}{a+b} (a^6 - 48 a^5 b - 1200 a^4 b^2 - 6400 a^3 b^3 - 13440 a^2 b^4 - 12288 a b^5 - 4096 b^6) \right. \\
 & \left( \left( \text{ArcTan}[\text{Sec}[f x] \left( \frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]} - \right. \right. \right. \\
 & \left. \left. \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) (-a \sin[f x] - 2 b \sin[f x] + \right. \\
 & \left. \left. a \sin[2 e+f x]) \right) \cos[2 e] \right) / \left( 8 a^5 b \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) - \\
 & \left( i \text{ArcTan}[\text{Sec}[f x] \left( \frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]} - \right. \right. \right. \\
 & \left. \left. \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) (-a \sin[f x] - 2 b \sin[f x] + \right. \\
 & \left. \left. a \sin[2 e+f x]) \right) \sin[2 e] \right) / \left( 8 a^5 b \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) \left. \right) - \\
 & \frac{1}{24 a^5 b (a+b) f (a+2 b+a \cos[2 e+2 f x])} \text{Sec}[2 e] \left( -960 a^5 b f x \cos[2 e] - \right. \\
 & 10944 a^4 b^2 f x \cos[2 e] - 44544 a^3 b^3 f x \cos[2 e] - 83712 a^2 b^4 f x \cos[2 e] - \\
 & 73728 a b^5 f x \cos[2 e] - 24576 b^6 f x \cos[2 e] - 480 a^5 b f x \cos[2 f x] - \\
 & 4512 a^4 b^2 f x \cos[2 f x] - 13248 a^3 b^3 f x \cos[2 f x] - 15360 a^2 b^4 f x \cos[2 f x] - \\
 & 6144 a b^5 f x \cos[2 f x] - 480 a^5 b f x \cos[4 e+2 f x] - 4512 a^4 b^2 f x \cos[4 e+2 f x] - 13248 \\
 & a^3 b^3 f x \cos[4 e+2 f x] - 15360 a^2 b^4 f x \cos[4 e+2 f x] - 6144 a b^5 f x \cos[4 e+2 f x] - \\
 & 3 a^6 \sin[2 e] - 156 a^5 b \sin[2 e] - 1500 a^4 b^2 \sin[2 e] - 5760 a^3 b^3 \sin[2 e] - \\
 & 10560 a^2 b^4 \sin[2 e] - 9216 a b^5 \sin[2 e] - 3072 b^6 \sin[2 e] + 3 a^6 \sin[2 f x] + \\
 & 366 a^5 b \sin[2 f x] + 3000 a^4 b^2 \sin[2 f x] + 8400 a^3 b^3 \sin[2 f x] + 9600 a^2 b^4 \sin[2 f x] + \\
 & \left. 3840 a b^5 \sin[2 f x] + 216 a^5 b \sin[4 e+2 f x] + 1800 a^4 b^2 \sin[4 e+2 f x] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 5040 a^3 b^3 \operatorname{Sin}[4 e + 2 f x] + 5760 a^2 b^4 \operatorname{Sin}[4 e + 2 f x] + 2304 a b^5 \operatorname{Sin}[4 e + 2 f x] + \\
 & 76 a^5 b \operatorname{Sin}[2 e + 4 f x] + 460 a^4 b^2 \operatorname{Sin}[2 e + 4 f x] + 768 a^3 b^3 \operatorname{Sin}[2 e + 4 f x] + \\
 & 384 a^2 b^4 \operatorname{Sin}[2 e + 4 f x] + 76 a^5 b \operatorname{Sin}[6 e + 4 f x] + 460 a^4 b^2 \operatorname{Sin}[6 e + 4 f x] + \\
 & 768 a^3 b^3 \operatorname{Sin}[6 e + 4 f x] + 384 a^2 b^4 \operatorname{Sin}[6 e + 4 f x] - 16 a^5 b \operatorname{Sin}[4 e + 6 f x] - \\
 & 48 a^4 b^2 \operatorname{Sin}[4 e + 6 f x] - 32 a^3 b^3 \operatorname{Sin}[4 e + 6 f x] - 16 a^5 b \operatorname{Sin}[8 e + 6 f x] - \\
 & 48 a^4 b^2 \operatorname{Sin}[8 e + 6 f x] - 32 a^3 b^3 \operatorname{Sin}[8 e + 6 f x] + 4 a^5 b \operatorname{Sin}[6 e + 8 f x] + \\
 & 4 a^4 b^2 \operatorname{Sin}[6 e + 8 f x] + 4 a^5 b \operatorname{Sin}[10 e + 8 f x] + 4 a^4 b^2 \operatorname{Sin}[10 e + 8 f x] \Big)
 \end{aligned}$$

**Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^4}{(a + b \operatorname{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 191 leaves, 7 steps):

$$\frac{3(a^2 + 8ab + 8b^2)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^4f} - \frac{(5a+6b)\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{8a^2f(a+b+b\operatorname{Tan}[e+fx]^2)} + \frac{\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]}{4af(a+b+b\operatorname{Tan}[e+fx]^2)} - \frac{3b(3a+4b)\operatorname{Tan}[e+fx]}{8a^3f(a+b+b\operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 1354 leaves):

$$\begin{aligned}
 & - \left( \left( (a + 2b + a \operatorname{Cos}[2e + 2fx])^2 \operatorname{Sec}[e + fx]^4 \right. \right. \\
 & \quad \left( 16x + \left( (-a^3 + 6a^2b + 24ab^2 + 16b^3) \operatorname{ArcTan}\left[ (\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right. \right. \right. \\
 & \quad \left. \left. \left. (- (a + 2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right) \right] / \left( 2\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) \right) \\
 & \quad \left. \left. \left. (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) \right] / \left( b (a+b)^{3/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) + \right. \\
 & \quad \left. \left. \left. \left( (a^2 + 8ab + 8b^2) \left( (a + 2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx] \right) \right) / \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. (b (a+b) f (a + 2b + a \operatorname{Cos}[2(e + fx)]) (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])) \right) \right) \right) \right) \right) \right) / \\
 & \quad \left( 256 a^2 (a + b \operatorname{Sec}[e + fx]^2)^2 \right) + \left( 3 (a + 2b + a \operatorname{Cos}[2e + 2fx])^2 \right. \\
 & \quad \left. \operatorname{Sec}[e + fx]^4 \right. \\
 & \quad \left. \left( \frac{(a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{3/2}} - \frac{a\sqrt{b}\operatorname{Sin}[2(e+fx)]}{(a+b)(a+2b+a\operatorname{Cos}[2(e+fx)])} \right) \right) / \\
 & \quad \left( 1024 b^{3/2} f (a + b \operatorname{Sec}[e + fx]^2)^2 \right) + \\
 & \quad \frac{1}{128 (a + b \operatorname{Sec}[e + fx]^2)^2} \\
 & \quad (a + 2b + a \operatorname{Cos}[2e + 2fx])^2
 \end{aligned}$$



$$\begin{aligned}
 & \text{Sec}[e + f x]^4 \\
 & \left( \frac{1}{a + b} (-a^5 + 30 a^4 b + 480 a^3 b^2 + 1600 a^2 b^3 + 1920 a b^4 + 768 b^5) \left( \left( \text{ArcTan}[\text{Sec}[f x]] \right. \right. \right. \\
 & \quad \left. \left. \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right. \right. \\
 & \quad \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Cos}[2 e] \right) / \\
 & \quad \left( 8 a^4 b \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) - \left( i \text{ArcTan}[\text{Sec}[f x]] \right. \\
 & \quad \left. \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right. \\
 & \quad \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Sin}[2 e] \Big) / \left( 8 a^4 b \sqrt{a + b} f \right. \\
 & \quad \left. \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) \Big) + \frac{1}{8 a^4 b (a + b) f (a + 2 b + a \text{Cos}[2 e + 2 f x])} \\
 & \text{Sec}[2 e] \left( 160 a^4 b f x \text{Cos}[2 e] + 1248 a^3 b^2 f x \text{Cos}[2 e] + 3392 a^2 b^3 f x \text{Cos}[2 e] + \right. \\
 & \quad 3840 a b^4 f x \text{Cos}[2 e] + 1536 b^5 f x \text{Cos}[2 e] + 80 a^4 b f x \text{Cos}[2 f x] + \\
 & \quad 464 a^3 b^2 f x \text{Cos}[2 f x] + 768 a^2 b^3 f x \text{Cos}[2 f x] + 384 a b^4 f x \text{Cos}[2 f x] + \\
 & \quad 80 a^4 b f x \text{Cos}[4 e + 2 f x] + 464 a^3 b^2 f x \text{Cos}[4 e + 2 f x] + 768 a^2 b^3 f x \text{Cos}[4 e + 2 f x] + \\
 & \quad 384 a b^4 f x \text{Cos}[4 e + 2 f x] + a^5 \text{Sin}[2 e] + 34 a^4 b \text{Sin}[2 e] + 224 a^3 b^2 \text{Sin}[2 e] + \\
 & \quad 576 a^2 b^3 \text{Sin}[2 e] + 640 a b^4 \text{Sin}[2 e] + 256 b^5 \text{Sin}[2 e] - a^5 \text{Sin}[2 f x] - 62 a^4 b \text{Sin}[2 f x] - \\
 & \quad 318 a^3 b^2 \text{Sin}[2 f x] - 512 a^2 b^3 \text{Sin}[2 f x] - 256 a b^4 \text{Sin}[2 f x] - 30 a^4 b \text{Sin}[4 e + 2 f x] - \\
 & \quad 158 a^3 b^2 \text{Sin}[4 e + 2 f x] - 256 a^2 b^3 \text{Sin}[4 e + 2 f x] - 128 a b^4 \text{Sin}[4 e + 2 f x] - \\
 & \quad 12 a^4 b \text{Sin}[2 e + 4 f x] - 36 a^3 b^2 \text{Sin}[2 e + 4 f x] - 24 a^2 b^3 \text{Sin}[2 e + 4 f x] - \\
 & \quad 12 a^4 b \text{Sin}[6 e + 4 f x] - 36 a^3 b^2 \text{Sin}[6 e + 4 f x] - 24 a^2 b^3 \text{Sin}[6 e + 4 f x] + 2 a^4 b \\
 & \quad \left. \text{Sin}[4 e + 6 f x] + 2 a^3 b^2 \text{Sin}[4 e + 6 f x] + 2 a^4 b \text{Sin}[8 e + 6 f x] + 2 a^3 b^2 \text{Sin}[8 e + 6 f x] \right) \Big)
 \end{aligned}$$

**Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e + f x]^2}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$\begin{aligned}
 & \frac{(a + 4 b) x}{2 a^3} - \frac{\sqrt{b} (3 a + 4 b) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{2 a^3 \sqrt{a + b} f} - \\
 & \frac{\text{Cos}[e + f x] \text{Sin}[e + f x]}{2 a f (a + b + b \text{Tan}[e + f x]^2)} - \frac{b \text{Tan}[e + f x]}{a^2 f (a + b + b \text{Tan}[e + f x]^2)}
 \end{aligned}$$

Result (type 3, 825 leaves):

$$\begin{aligned}
 & - \left( \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \right. \right. \\
 & \quad \left. \left( 16x + \left( (-a^3 + 6a^2b + 24ab^2 + 16b^3) \operatorname{ArcTan} \left[ \left( \frac{\sec[fx] (\cos[2e] - i \sin[2e])}{(- (a + 2b) \sin[fx] + a \sin[2e + fx])} \right) \right] \right. \right. \right. \\
 & \quad \left. \left. \left( 2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \right) \right) \left. \right. \\
 & \quad \left. \left( \cos[2e] - i \sin[2e] \right) \right) \left. \right. \left. \left( b (a+b)^{3/2} f \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \right. \\
 & \quad \left. \left( (a^2 + 8ab + 8b^2) \left( (a + 2b) \sin[2e] - a \sin[2fx] \right) \right) \right. \\
 & \quad \left. \left( b (a+b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right) \right) \right) \left. \right. \left. \right) \left. \right. \\
 & \quad \left( 128a^2 (a + b \sec[e + fx]^2)^2 \right) - \left( (a + 2b + a \cos[2e + 2fx])^2 \right. \\
 & \quad \left. \sec[e + fx]^4 \right. \\
 & \quad \left( -64(a + 2b)x + \left( (a^4 - 16a^3b - 144a^2b^2 - 256ab^3 - 128b^4) \right. \right. \\
 & \quad \left. \left. \operatorname{ArcTan} \left[ \left( \frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{\left( 2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (\cos[2e] - i \sin[2e])} \right) \right] \right. \right. \\
 & \quad \left. \left. \left( b (a+b)^{3/2} f \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \frac{16a \cos[2fx] \sin[2e]}{f} + \right. \right. \\
 & \quad \left. \left. \frac{16a \cos[2e] \sin[2fx]}{f} - \left( (a^3 + 18a^2b + 48ab^2 + 32b^3) \left( (a + 2b) \sin[2e] - a \sin[2fx] \right) \right) \right) \right. \\
 & \quad \left. \left( b (a+b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right) \right) \right) \left. \right. \left. \right) \left. \right. \\
 & \quad \left( 256a^3 (a + b \sec[e + fx]^2)^2 \right) + \left( (a + 2b + a \cos[2e + 2fx])^2 \right. \\
 & \quad \left. \sec[e + fx]^4 \right. \\
 & \quad \left( \frac{(a + 2b) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} - \frac{a \sqrt{b} \sin[2(e + fx)]}{(a+b) (a + 2b + a \cos[2(e + fx)])} \right) \right) \left. \right. \left. \right) \left. \right. \\
 & \quad \left( 128b^{3/2} f (a + b \sec[e + fx]^2)^2 \right) + \\
 & \quad \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \right. \\
 & \quad \left( - \frac{a \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} + \frac{\sqrt{b} (a + 2b) \sin[2(e + fx)]}{(a+b) (a + 2b + a \cos[2(e + fx)])} \right) \right) \left. \right. \left. \right) \left. \right. \\
 & \quad \left( 256b^{3/2} f (a + b \sec[e + fx]^2)^2 \right)
 \end{aligned}$$

**Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{3/2} f} - \frac{b \operatorname{Tan}[e + f x]}{2a (a+b) f (a+b + b \operatorname{Tan}[e + f x]^2)}$$

Result (type 3, 240 leaves):

$$\left( (a + 2b + a \operatorname{Cos}[2(e + f x)]) \operatorname{Sec}[e + f x]^4 \right. \\ \left. \left( 2x (a + 2b + a \operatorname{Cos}[2(e + f x)]) + (b (3a + 2b) \operatorname{ArcTan}[(\operatorname{Sec}[f x] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right. \right. \\ \left. \left. (- (a + 2b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2e + f x])]) / (2\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}) \right] \right) \\ \left. (a + 2b + a \operatorname{Cos}[2(e + f x)]) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) / \\ \left( (a+b)^{3/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} + \right. \\ \left. \frac{b ((a + 2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{(a+b) f (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} \right) \Big) / (8a^2 (a + b \operatorname{Sec}[e + f x]^2)^2)$$

**Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]^2}{(a + b \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$-\frac{3\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a+b}}\right]}{2(a+b)^{5/2} f} - \frac{3 \operatorname{Cot}[e + f x]}{2(a+b)^2 f} + \frac{\operatorname{Cot}[e + f x]}{2(a+b) f (a+b + b \operatorname{Tan}[e + f x]^2)}$$

Result (type 3, 242 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^4 \right. \\ \left. \left( \left( 3b \operatorname{ArcTan}\left[ \left( \sec[fx] (\cos[2e] - i \sin[2e]) \right) \left( -(a + 2b) \sin[fx] + a \sin[2e + fx] \right) \right] \right) / \right. \right. \\ \left. \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (a + 2b + a \cos[2(e + fx)]) \right. \\ \left. (\cos[2e] - i \sin[2e]) \right) / \left( \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \\ 2 (a + 2b + a \cos[2(e + fx)]) \operatorname{Csc}[e] \operatorname{Csc}[e + fx] \sin[fx] + \\ \left. \frac{b ((a + 2b) \sin[2e] - a \sin[2fx])}{a (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \left. \right) / \\ \left( 8 (a + b)^2 f (a + b \sec[e + fx]^2)^2 \right)$$

**Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + fx]^4}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$-\frac{(3a - 2b) \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + fx]}{\sqrt{a+b}}\right]}{2 (a + b)^{7/2} f} - \\ \frac{(a - b) \operatorname{Cot}[e + fx]}{(a + b)^3 f} - \frac{\operatorname{Cot}[e + fx]^3}{3 (a + b)^2 f} - \frac{ab \operatorname{Tan}[e + fx]}{2 (a + b)^3 f (a + b + b \operatorname{Tan}[e + fx]^2)}$$

Result (type 3, 637 leaves):

$$\begin{aligned}
 & - \frac{(a + 2b + a \cos[2e + 2fx])^2 \cot[e] \csc[e + fx]^2 \sec[e + fx]^4}{12(a + b)^2 f (a + b \sec[e + fx]^2)^2} + \\
 & \left( (3a - 2b) (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left( \left( b \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \operatorname{Sec}[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right] \cos[2e] \right) \right) \right) / \\
 & \quad \left( 8\sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i b \operatorname{ArcTan} \left[ \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \operatorname{Sec}[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right] \sin[2e] \right) \right) \right) / \\
 & \quad \left( 8\sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \left. \right) / \left( (a + b)^3 (a + b \sec[e + fx]^2)^2 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^2 \csc[e] \csc[e + fx]^3 \sec[e + fx]^4 \right. \\
 & \quad \left. \sin[fx] \right) / \\
 & \left( 12(a + b)^2 f (a + b \sec[e + fx]^2)^2 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^2 \csc[e] \csc[e + fx] \right. \\
 & \quad \left. \sec[e + fx]^4 (a \sin[fx] - 2b \sin[fx]) \right) / \\
 & \left( 6(a + b)^3 f (a + b \sec[e + fx]^2)^2 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^4 \right. \\
 & \quad \left. (a b \sin[2e] + 2b^2 \sin[2e] - a b \sin[2fx]) \right) / \\
 & \left( 8(a + b)^3 f (a + b \sec[e + fx]^2)^2 (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right)
 \end{aligned}$$

**Problem 53: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e + fx]^6}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{a(3a - 4b) \sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{2(a + b)^{9/2} f} - \frac{(5a^2 - 10ab - b^2) \cot[e + fx]}{5(a + b)^4 f} - \frac{(10a + 3b) \cot[e + fx]^3}{15(a + b)^3 f} \\
 & - \frac{\cot[e + fx]^5}{5(a + b) f (a + b \tan[e + fx]^2)} - \frac{b(5a^2 + 2b^2) \tan[e + fx]}{10(a + b)^4 f (a + b \tan[e + fx]^2)}
 \end{aligned}$$

Result (type 3, 777 leaves):

$$\frac{1}{7680 (a+b)^4 f (a+b \operatorname{Sec}[e+fx])^2} (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^4 \left( \left( 960 a (3a-4b) b \operatorname{ArcTan} \left[ \frac{(\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]))}{\left( 2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right)} \right] (a+2b+a \operatorname{Cos}[2(e+fx)]) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) / \left( \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) - \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^5 \operatorname{Sec}[2e] \left( 10 a (16 a^2 + 34 a b + 123 b^2) \operatorname{Sin}[fx] - a (16 a^2 - 223 a b + 1336 b^2) \operatorname{Sin}[3fx] + 240 a^3 \operatorname{Sin}[2e-fx] + 640 a^2 b \operatorname{Sin}[2e-fx] - 1460 a b^2 \operatorname{Sin}[2e-fx] + 240 b^3 \operatorname{Sin}[2e-fx] - 240 a^3 \operatorname{Sin}[2e+fx] - 715 a^2 b \operatorname{Sin}[2e+fx] + 860 a b^2 \operatorname{Sin}[2e+fx] - 240 b^3 \operatorname{Sin}[2e+fx] + 160 a^3 \operatorname{Sin}[4e+fx] + 415 a^2 b \operatorname{Sin}[4e+fx] + 1830 a b^2 \operatorname{Sin}[4e+fx] + 165 a^2 b \operatorname{Sin}[2e+3fx] - 30 a b^2 \operatorname{Sin}[2e+3fx] + 120 b^3 \operatorname{Sin}[2e+3fx] - 16 a^3 \operatorname{Sin}[4e+3fx] + 208 a^2 b \operatorname{Sin}[4e+3fx] - 1036 a b^2 \operatorname{Sin}[4e+3fx] + 180 a^2 b \operatorname{Sin}[6e+3fx] - 330 a b^2 \operatorname{Sin}[6e+3fx] + 120 b^3 \operatorname{Sin}[6e+3fx] + 48 a^3 \operatorname{Sin}[2e+5fx] - 268 a^2 b \operatorname{Sin}[2e+5fx] + 290 a b^2 \operatorname{Sin}[2e+5fx] - 24 b^3 \operatorname{Sin}[2e+5fx] + 48 a^3 \operatorname{Sin}[6e+5fx] - 223 a^2 b \operatorname{Sin}[6e+5fx] + 230 a b^2 \operatorname{Sin}[6e+5fx] - 24 b^3 \operatorname{Sin}[6e+5fx] - 45 a^2 b \operatorname{Sin}[8e+5fx] + 60 a b^2 \operatorname{Sin}[8e+5fx] - 16 a^3 \operatorname{Sin}[4e+7fx] + 83 a^2 b \operatorname{Sin}[4e+7fx] - 6 a b^2 \operatorname{Sin}[4e+7fx] - 15 a^2 b \operatorname{Sin}[6e+7fx] - 16 a^3 \operatorname{Sin}[8e+7fx] + 68 a^2 b \operatorname{Sin}[8e+7fx] - 6 a b^2 \operatorname{Sin}[8e+7fx] \right) \right)$$

**Problem 54: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^5}{(a+b \operatorname{Sec}[e+fx])^3} dx$$

Optimal (type 3, 214 leaves, 6 steps):

$$\frac{\sqrt{b} (15 a^2 + 70 a b + 63 b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \operatorname{Cos}[e+fx]}{\sqrt{b}} \right]}{8 a^{11/2} f} - \frac{(3 a^2 + 14 a b + 13 b^2) \operatorname{Cos}[e+fx]}{2 a^5 f} + \frac{(a+3b) (3a+5b) \operatorname{Cos}[e+fx]^3}{12 a^4 b f} - \frac{\operatorname{Cos}[e+fx]^5}{5 a^3 f} - \frac{(a+b)^2 \operatorname{Cos}[e+fx]^7}{4 a^2 b f (b+a \operatorname{Cos}[e+fx])^2} - \frac{b (a+b) (3a+11b) \operatorname{Cos}[e+fx]}{8 a^5 f (b+a \operatorname{Cos}[e+fx])^2}$$

Result (type 3, 1641 leaves):

$$\frac{1}{491520 a^{11/2} b^{5/2} f (a+b \operatorname{Sec}[e+fx])^3} (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^6 \left( -900 a^{11/2} b^{3/2} \operatorname{Cos}[e+fx] - 109000 a^{9/2} b^{5/2} \operatorname{Cos}[e+fx] - 936000 a^{7/2} b^{7/2} \operatorname{Cos}[e+fx] - 2803072 a^{5/2} b^{9/2} \operatorname{Cos}[e+fx] - 3763200 a^{3/2} b^{11/2} \operatorname{Cos}[e+fx] - \right)$$

$$\begin{aligned}
 & 1935360 \sqrt{a} b^{13/2} \cos [e+f x]-900 a^{11/2} b^{3/2} \cos [e+f x] \cos [2(e+f x)]+ \\
 & 900 a^{9/2} b^{3/2} \cos [e+f x](a+2 b+a \cos [2(e+f x)])+24000 a^{7/2} b^{5/2} \cos [e+f x] \\
 & (a+2 b+a \cos [2(e+f x)])+43200 a^{5/2} b^{7/2} \cos [e+f x](a+2 b+a \cos [2(e+f x)])+ \\
 & 225 a^5 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}-\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 115200 a^2 b^3 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}-\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 537600 a b^4 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}-\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 483840 b^5 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}-\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 225 a^5 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}+\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 115200 a^2 b^3 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}+\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 537600 a b^4 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}+\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 483840 b^5 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right]+ \right.\right. \\
 & \left.\left.\cos [e]\left(\sqrt{a}+\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right](a+2 b+a \cos [2(e+f x)])^2- \\
 & 225 a^5 \operatorname{ArcTan}\left[\frac{\sqrt{a}-\sqrt{a+b} \tan \left[\frac{1}{2}(e+f x)\right]}{\sqrt{b}}\right](a+2 b+a \cos [2(e+f x)])^2- \\
 & 225 a^5 \operatorname{ArcTan}\left[\frac{\sqrt{a}+\sqrt{a+b} \tan \left[\frac{1}{2}(e+f x)\right]}{\sqrt{b}}\right](a+2 b+a \cos [2(e+f x)])^2+ \\
 & 19200 a^{5/2} b^{5/2} \cos [e] \cos [f x](a+2 b+a \cos [2(e+f x)])^2- \\
 & 20352 a^{9/2} b^{5/2} \cos [e+f x] \cos [4(e+f x)]- \\
 & 115712 a^{7/2} b^{7/2} \cos [e+f x] \cos [4(e+f x)]- \\
 & 129024 a^{5/2} b^{9/2} \cos [e+f x] \cos [4(e+f x)]+2048 a^{9/2} b^{5/2} \cos [e+f x] \cos [6(e+f x)]+
 \end{aligned}$$

$$\begin{aligned}
 & 4608 a^{7/2} b^{7/2} \cos [e+f x] \cos [6(e+f x)] - 384 a^{9/2} b^{5/2} \cos [e+f x] \cos [8(e+f x)] - \\
 & 19200 a^{5/2} b^{5/2} (a+2 b+a \cos [2(e+f x)])^2 \sin [e] \sin [f x] - \\
 & 32496 a^{9/2} b^{5/2} \csc [e+f x] \sin [4(e+f x)] - 252080 a^{7/2} b^{7/2} \csc [e+f x] \sin [4(e+f x)] - \\
 & 577024 a^{5/2} b^{9/2} \csc [e+f x] \sin [4(e+f x)] - 403200 a^{3/2} b^{11/2} \csc [e+f x] \sin [4(e+f x)]
 \end{aligned}$$

**Problem 55: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin [e+f x]^3}{(a+b \sec [e+f x]^2)^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\begin{aligned}
 & \frac{5 \sqrt{b} (3 a+7 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos [e+f x]}{\sqrt{b}}\right]}{8 a^{9/2} f} - \frac{(a+3 b) \cos [e+f x]}{a^4 f} + \\
 & \frac{\cos [e+f x]^3}{3 a^3 f} + \frac{b^2 (a+b) \cos [e+f x]}{4 a^4 f (b+a \cos [e+f x]^2)^2} - \frac{b (9 a+13 b) \cos [e+f x]}{8 a^4 f (b+a \cos [e+f x]^2)}
 \end{aligned}$$

Result (type 3, 1392 leaves):

$$\begin{aligned}
 & 3 \left( -\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{a}-\sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{b}}\right]}{\sqrt{a}} - \frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{a}+\sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{b}}\right]}{\sqrt{a}} - \right. \\
 & \left. \frac{2 \sqrt{b} \cos [e+f x] (3 a+10 b+3 a \cos [2(e+f x)])}{(a+2 b+a \cos [2(e+f x)])^2} \right) (a+2 b+a \cos [2 e+2 f x])^3 \\
 & \sec [e+f x]^6 \left/ \left( 8192 b^{5/2} f (a+b \sec [e+f x]^2)^3 \right) + \frac{1}{2048 a^{3/2} b^{5/2} f (a+b \sec [e+f x]^2)^3} \right. \\
 & \left( (3 a-4 b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \right. \\
 & \left. \left. \left. \cos [e]\left(\sqrt{a}-\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right]\right)\right)\right] \right) + \right. \\
 & \left. (3 a-4 b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i \sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2}\right) \sin [e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \right. \\
 & \left. \left. \left. \cos [e]\left(\sqrt{a}+\sqrt{a+b} \sqrt{(\cos [e]-i \sin [e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right]\right)\right)\right] \right) + \right. \\
 & \left. \left( 2 \sqrt{a} \sqrt{b} \cos [e+f x] (3 a^2+6 a b+8 b^2+a (3 a-4 b) \cos [2(e+f x)]) \right) \right/ \\
 & \left. (a+2 b+a \cos [2(e+f x)])^2 \right) (a+2 b+a \cos [2 e+2 f x])^3 \sec [e+f x]^6 -
 \end{aligned}$$



$$\begin{aligned}
 & \frac{1}{49152 a^{9/2} b^{5/2} f (a+b \operatorname{Sec}[e+f x])^3} \left( -3 (3 a^4 - 40 a^3 b + 720 a^2 b^2 + 6720 a b^3 + 8960 b^4) \right. \\
 & \quad \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) - \\
 & \quad 3 (3 a^4 - 40 a^3 b + 720 a^2 b^2 + 6720 a b^3 + 8960 b^4) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \\
 & \quad \left. \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) - \\
 & \quad \left( 2 \sqrt{a} \sqrt{b} \operatorname{Cos}[e+f x] (9 a^5 - 90 a^4 b - 10144 a^3 b^2 - 48672 a^2 b^3 - 85120 a b^4 - \right. \\
 & \quad \left. 53760 b^5 + a (9 a^4 - 120 a^3 b - 12432 a^2 b^2 - 47936 a b^3 - 44800 b^4) \operatorname{Cos}[2(e+f x)] - \right. \\
 & \quad \left. 128 a^2 b^2 (15 a + 28 b) \operatorname{Cos}[4(e+f x)] + 128 a^3 b^2 \operatorname{Cos}[6(e+f x)] \right) \Big/ \\
 & \quad \left. (a+2 b+a \operatorname{Cos}[2(e+f x)])^2 \right) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 - \\
 & \frac{1}{16384 a^{7/2} f (a+b \operatorname{Sec}[e+f x])^2} \Big)^3 (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \\
 & \quad \operatorname{Sec}[e+f x]^6 \\
 & \quad \left( \frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \right. \\
 & \quad \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) + \frac{1}{b^{5/2}} \right. \\
 & \quad \left. 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) \right] - \\
 & \quad 512 \sqrt{a} \operatorname{Cos}[e] \operatorname{Cos}[f x] + \frac{8 \sqrt{a} (a^3 + 24 a^2 b + 80 a b^2 + 64 b^3) \operatorname{Cos}[e+f x]}{b (a+2 b+a \operatorname{Cos}[2(e+f x)])^2} + \\
 & \quad \left. \frac{2 \sqrt{a} (3 a^3 - 24 a^2 b - 400 a b^2 - 576 b^3) \operatorname{Cos}[e+f x]}{b^2 (a+2 b+a \operatorname{Cos}[2(e+f x)])} + 512 \sqrt{a} \operatorname{Sin}[e] \operatorname{Sin}[f x] \right)
 \end{aligned}$$

**Problem 56: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+f x]}{(a+b \operatorname{Sec}[e+f x])^3} dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$\frac{15 \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right]}{8 a^{7/2} f} - \frac{15 \cos[e+fx]}{8 a^3 f} + \frac{\cos[e+fx]^5}{4 a f (b+a \cos[e+fx]^2)^2} + \frac{5 \cos[e+fx]^3}{8 a^2 f (b+a \cos[e+fx]^2)}$$

Result (type 3, 656 leaves):

$$\frac{1}{4096 a^{7/2} b^{5/2} f (a+b \sec[e+fx]^2)^3} (a+2b+a \cos[2(e+fx)])^3 \sec[e+fx]^6 \left( 15 (a^3+64b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a}-i\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2}) \right) \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left( \sqrt{a}-\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right] + 15 (a^3+64b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a}+i\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2}) \right) \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left( \sqrt{a}+\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right] + \frac{1}{(a+2b+a \cos[2(e+fx)])^2} \sqrt{a} \left( 24 a^4 \sqrt{b} \cos[e+fx] - 24 a^3 b^{3/2} \cos[e+fx] - 144 a^2 b^{5/2} \cos[e+fx] + 512 b^{9/2} \cos[e+fx] - 72 a^3 b^{3/2} \cos[e+fx] \cos[2(e+fx)] - 24 a^3 \sqrt{b} \cos[e+fx] (a+2b+a \cos[2(e+fx)]) + 72 a^2 b^{3/2} \cos[e+fx] (a+2b+a \cos[2(e+fx)]) - 1152 b^{7/2} \cos[e+fx] (a+2b+a \cos[2(e+fx)]) \right) - 15 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a}-\sqrt{a+b} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right] (a+2b+a \cos[2(e+fx)])^2 - 15 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a}+\sqrt{a+b} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right] (a+2b+a \cos[2(e+fx)])^2 - 512 b^{5/2} \cos[e] \cos[fx] (a+2b+a \cos[2(e+fx)])^2 + 512 b^{5/2} (a+2b+a \cos[2(e+fx)])^2 \sin[e] \sin[fx] + 6 a^4 \sqrt{b} \operatorname{Csc}[e+fx] \sin[4(e+fx)] \right)$$

**Problem 57: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]}{(a+b \sec[e+fx]^2)^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right] - \frac{\operatorname{ArcTanh}[\cos[e+fx]]}{(a+b)^3 f}}{8 a^{5/2} (a+b)^3 f} - \frac{b \cos[e+fx]^3}{4 a (a+b) f (b+a \cos[e+fx]^2)^2} - \frac{b (7 a+3 b) \cos[e+fx]}{8 a^2 (a+b)^2 f (b+a \cos[e+fx]^2)}$$

Result (type 3, 447 leaves):

$$\frac{1}{64 (a+b)^3 f (a+b \sec[e+fx]^2)^3} (a+2b+a \cos[2(e+fx)]) \sec[e+fx]^5$$

$$\left( \frac{8 b^2 (a+b)^2}{a^2} - \frac{2 b (a+b) (9 a+5 b) (a+2 b+a \cos[2(e+fx)])}{a^2} + \frac{1}{a^{5/2}} \right.$$

$$\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i \sqrt{a+b} \sqrt{(\cos[e] - i \sin[e])^2}) \right. \right.$$

$$\left. \left. \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\cos[e] - i \sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right) \right]$$

$$(a+2b+a \cos[2(e+fx)])^2 \sec[e+fx] + \frac{1}{a^{5/2}} \sqrt{b} (15 a^2 + 10 a b + 3 b^2)$$

$$\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i \sqrt{a+b} \sqrt{(\cos[e] - i \sin[e])^2}) \sin[e] \tan\left[\frac{fx}{2}\right] + \right. \right.$$

$$\left. \left. \cos[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\cos[e] - i \sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right) \right] (a+2b+a \cos[2(e+fx)])^2$$

$$\sec[e+fx] - 8 (a+2b+a \cos[2(e+fx)])^2 \log\left[\cos\left[\frac{1}{2}(e+fx)\right]\right] \sec[e+fx] +$$

$$\left. 8 (a+2b+a \cos[2(e+fx)])^2 \log\left[\sin\left[\frac{1}{2}(e+fx)\right]\right] \sec[e+fx] \right)$$

**Problem 58: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e+fx]^3}{(a+b \sec[e+fx]^2)^3} dx$$

Optimal (type 3, 213 leaves, 7 steps):

$$\frac{\sqrt{b} (15 a^2 - 10 a b - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right] - \frac{\operatorname{ArcTanh}[\cos[e+fx]]}{2 (a+b)^4 f}}{8 a^{3/2} (a+b)^4 f} + \frac{(2 a-b) b \cos[e+fx]}{4 a (a+b)^2 f (b+a \cos[e+fx]^2)^2} + \frac{(4 a^2 - 9 a b - b^2) \cos[e+fx]}{8 a (a+b)^3 f (b+a \cos[e+fx]^2)} - \frac{\cos[e+fx] \cot[e+fx]^2}{2 (a+b) f (b+a \cos[e+fx]^2)^2}$$

Result (type 3, 532 leaves):

$$\frac{1}{64 (a+b)^4 f (a+b \operatorname{Sec}[e+fx])^3} (a+2b+a \operatorname{Cos}[2(e+fx)])$$

$$\operatorname{Sec}[e+fx]^5 \left( \frac{8b^2(a+b)^2}{a} - \frac{2b(a+b)(9a+b)(a+2b+a \operatorname{Cos}[2(e+fx)])}{a} - \frac{1}{a^{3/2}} \right.$$

$$\sqrt{b}(-15a^2+10ab+b^2) \operatorname{ArcTan}\left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a}-i\sqrt{a+b}) \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \right) \right.$$

$$\left. \left. \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \operatorname{Cos}[e] \left( \sqrt{a}-\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right]$$

$$(a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Sec}[e+fx] - \frac{1}{a^{3/2}} \sqrt{b}(-15a^2+10ab+b^2)$$

$$\operatorname{ArcTan}\left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a}+i\sqrt{a+b}) \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \right.$$

$$\left. \left. \operatorname{Cos}[e] \left( \sqrt{a}+\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right] (a+2b+a \operatorname{Cos}[2(e+fx)])^2$$

$$\operatorname{Sec}[e+fx] - (a+b)(a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] -$$

$$4(a-5b)(a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx] +$$

$$4(a-5b)(a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx] +$$

$$(a+b)(a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \Big)$$

**Problem 59: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]^5}{(a+b \operatorname{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 257 leaves, 8 steps):

$$\frac{3\sqrt{b}(5a^2-10ab+b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e+fx]}{\sqrt{b}}\right]}{8\sqrt{a}(a+b)^5 f} - \frac{3(a^2-10ab+5b^2) \operatorname{ArcTanh}[\operatorname{Cos}[e+fx]]}{8(a+b)^5 f} +$$

$$\frac{(a^2-9ab+2b^2) \operatorname{Cos}[e+fx]}{8(a+b)^3 f (b+a \operatorname{Cos}[e+fx]^2)^2} + \frac{3(a^2-6ab+b^2) \operatorname{Cos}[e+fx]}{8(a+b)^4 f (b+a \operatorname{Cos}[e+fx]^2)} -$$

$$\frac{(a-7b) \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]}{8(a+b)^2 f (b+a \operatorname{Cos}[e+fx]^2)^2} - \frac{\operatorname{Cot}[e+fx]^3 \operatorname{Csc}[e+fx]}{4(a+b) f (b+a \operatorname{Cos}[e+fx]^2)^2}$$

Result (type 3, 549 leaves):

$$\begin{aligned}
 & \frac{1}{1024 (a+b)^5 f (a+b \operatorname{Sec}[e+fx]^2)^3} (a+2b+a \operatorname{Cos}[2(e+fx)]) \\
 & \left( \frac{1}{\sqrt{a}} 48 \sqrt{b} (5a^2 - 10ab + b^2) \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2}) \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sin}[e] \operatorname{Tan} \left[ \frac{fx}{2} \right] + \operatorname{Cos}[e] \left( \sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2} \operatorname{Tan} \left[ \frac{fx}{2} \right] \right) \right) \right] \right) \\
 & (a+2b+a \operatorname{Cos}[2(e+fx)])^2 + \frac{1}{\sqrt{a}} 48 \sqrt{b} (5a^2 - 10ab + b^2) \\
 & \operatorname{ArcTan} \left[ \frac{1}{\sqrt{b}} \left( (-\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan} \left[ \frac{fx}{2} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}[e] \left( \sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2} \operatorname{Tan} \left[ \frac{fx}{2} \right] \right) \right) \right] (a+2b+a \operatorname{Cos}[2(e+fx)])^2 - \\
 & 2(a+b) (30a^3 + 112a^2b + 182ab^2 - 140b^3 + (35a^3 + 78a^2b - 93ab^2 + 224b^3) \operatorname{Cos}[2(e+fx)] + \\
 & \quad 2(a^3 - 8a^2b + 53ab^2 - 10b^3) \operatorname{Cos}[4(e+fx)] - 3a^3 \operatorname{Cos}[6(e+fx)] + \\
 & \quad 18a^2b \operatorname{Cos}[6(e+fx)] - 3ab^2 \operatorname{Cos}[6(e+fx)]) \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 - \\
 & 48(a^2 - 10ab + 5b^2) (a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}(e+fx) \right] \right] + \\
 & 48(a^2 - 10ab + 5b^2) (a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2}(e+fx) \right] \right] \operatorname{Sec}[e+fx]^6
 \end{aligned}$$

**Problem 60: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^6}{(a+b \operatorname{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 314 leaves, 9 steps):

$$\begin{aligned}
 & \frac{5(a+2b)(a^2+16ab+16b^2)x}{16a^6} - \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8a^6f} \\
 & \frac{(33a^2+110ab+80b^2)\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{48a^3f(a+b+b\operatorname{Tan}[e+fx]^2)^2} + \\
 & \frac{(9a+10b)\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]}{24a^2f(a+b+b\operatorname{Tan}[e+fx]^2)^2} + \frac{\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]^3}{6af(a+b+b\operatorname{Tan}[e+fx]^2)^2} - \\
 & \frac{5b(9a^2+32ab+24b^2)\operatorname{Tan}[e+fx]}{48a^4f(a+b+b\operatorname{Tan}[e+fx]^2)^2} - \frac{5b(5a^2+20ab+16b^2)\operatorname{Tan}[e+fx]}{16a^5f(a+b+b\operatorname{Tan}[e+fx]^2)}
 \end{aligned}$$

Result (type 3, 2057 leaves):

$$\begin{aligned}
 & \left( 5(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \left( \frac{(3a^2+8ab+8b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}} \right]}{(a+b)^{5/2}} - \right. \right. \\
 & \quad \left. \left. (a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\operatorname{Cos}[2(e+fx)])\operatorname{Sin}[2(e+fx)]) \right) \right) /
 \end{aligned}$$

$$\left( (a+b)^2 (a+2b+a \cos[2(e+fx)])^2 \right) \Bigg/ \left( 65536 b^{5/2} f (a+b \sec[e+fx]^2)^3 \right) +$$

$$\frac{1}{2048 (a+b \sec[e+fx]^2)^3} (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6$$

$$\left( \frac{32 (7a^3 + 54a^2b + 120ab^2 + 80b^3) x}{a^6} - \right.$$

$$\frac{1}{(a+b)^2} (-3a^8 + 64a^7b - 2240a^6b^2 - 53760a^5b^3 - 313600a^4b^4 - 802816a^3b^5 - 1032192a^2b^6 -$$

$$655360ab^7 - 163840b^8) \left( \left( \operatorname{ArcTan}[\sec[fx]] \left( \frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e]-ib\sin[4e]}} - \right. \right. \right.$$

$$\left. \left. \frac{ib\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e]-ib\sin[4e]}} \right) (-a\sin[fx] - 2b\sin[fx] + \right.$$

$$\left. \left. a\sin[2e+fx]) \right) \cos[2e] \right) \Bigg/ \left( 64a^6b^2\sqrt{a+b}f\sqrt{b\cos[4e]-ib\sin[4e]} \right) -$$

$$\left( ib \operatorname{ArcTan}[\sec[fx]] \left( \frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e]-ib\sin[4e]}} - \right. \right.$$

$$\left. \left. \frac{ib\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e]-ib\sin[4e]}} \right) (-a\sin[fx] - 2b\sin[fx] + \right.$$

$$\left. \left. a\sin[2e+fx]) \right) \sin[2e] \right) \Bigg/ \left( 64a^6b^2\sqrt{a+b}f\sqrt{b\cos[4e]-ib\sin[4e]} \right) \Bigg) -$$

$$\frac{1}{16a^6b(a+b)f(a+2b+a\cos[2e+2fx])^2} \sec[2e] (a^7\sin[2e] + 74a^6b\sin[2e] +$$

$$984a^5b^2\sin[2e] + 5264a^4b^3\sin[2e] + 14080a^3b^4\sin[2e] +$$

$$19968a^2b^5\sin[2e] + 14336a^6b\sin[2e] + 4096b^7\sin[2e] - a^7\sin[2fx] -$$

$$72a^6b\sin[2fx] - 840a^5b^2\sin[2fx] - 3584a^4b^3\sin[2fx] -$$

$$6912a^3b^4\sin[2fx] - 6144a^2b^5\sin[2fx] - 2048ab^6\sin[2fx]) -$$

$$\frac{1}{64a^6b^2(a+b)^2f(a+2b+a\cos[2e+2fx])} \sec[2e]$$

$$(3a^8\sin[2e] - 64a^7b\sin[2e] - 4480a^6b^2\sin[2e] - 45696a^5b^3\sin[2e] -$$

$$196928a^4b^4\sin[2e] - 438272a^3b^5\sin[2e] - 528384a^2b^6\sin[2e] -$$

$$327680ab^7\sin[2e] - 81920b^8\sin[2e] - 3a^8\sin[2fx] + 66a^7b\sin[2fx] +$$

$$4056a^6b^2\sin[2fx] + 33936a^5b^3\sin[2fx] + 111360a^4b^4\sin[2fx] +$$

$$173568a^3b^5\sin[2fx] + 129024a^2b^6\sin[2fx] + 36864ab^7\sin[2fx]) -$$

$$(7a^2 + 32ab + 32b^2) \left( -\frac{6ib\cos[2e+2fx]}{a^5f} + \frac{6\sin[2e+2fx]}{a^5f} \right) -$$

$$(7a^2 + 32ab + 32b^2) \left( \frac{6ib\cos[2e+2fx]}{a^5f} + \frac{6\sin[2e+2fx]}{a^5f} \right) -$$

$$(a+2b) \left( -\frac{6ib\cos[4e+4fx]}{a^4f} - \frac{6\sin[4e+4fx]}{a^4f} \right) -$$

$$(a+2b) \left( \frac{6ib\cos[4e+4fx]}{a^4f} - \frac{6\sin[4e+4fx]}{a^4f} \right) -$$

$$\begin{aligned}
 & \left. \frac{4 \operatorname{Sin}[6 e + 6 f x]}{3 a^3 f} \right) - \\
 & \left( 15 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \right. \\
 & \quad \left( - \left( \left( 6 a^2 \operatorname{ArcTan} \left[ \left( \operatorname{Sec}[f x] \left( \operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e] \right) \left( - (a + 2 b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x] \right) \right] \right) \right) \right) \right) / \\
 & \quad \left( 2 \sqrt{a + b} \sqrt{b \left( \operatorname{Cos}[e] - i \operatorname{Sin}[e] \right)^4} \right) \right) \\
 & \quad \left( \operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e] \right) \left) / \left( \sqrt{a + b} \sqrt{b \left( \operatorname{Cos}[e] - i \operatorname{Sin}[e] \right)^4} \right) \right) + \\
 & \quad \left( a \operatorname{Sec}[2 e] \left( \left( -9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4 \right) \operatorname{Sin}[2 f x] + a \left( -3 a^3 + 2 a^2 b + 24 a \right. \right. \right. \\
 & \quad \left. \left. \left. b^2 + 16 b^3 \right) \operatorname{Sin}[2 (e + 2 f x)] \right) + \left( 3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4 \right) \operatorname{Sin}[4 e + 2 f x] \right) + \\
 & \quad \left( 9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5 \right) \operatorname{Tan}[2 e] \left) / \right. \\
 & \quad \left. \left( a^2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2 \right) \right) \left) \right) / \\
 & \left( 262144 b^2 (a + b)^2 f (a + b \operatorname{Sec}[e + f x]^2)^3 \right) + \\
 & \quad \frac{1}{65536 a^4 (a + b \operatorname{Sec}[e + f x]^2)^3} \\
 & \quad \frac{3}{(a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3} \\
 & \quad \operatorname{Sec}[e + f x]^6 \\
 & \quad \left( -1536 (a + 2 b) x - \right. \\
 & \quad \left( 3 (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \\
 & \quad \left. \operatorname{ArcTan} \left[ \left( \operatorname{Sec}[f x] \left( \operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e] \right) \left( - (a + 2 b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x] \right) \right) \right] \right) / \\
 & \quad \left( 2 \sqrt{a + b} \sqrt{b \left( \operatorname{Cos}[e] - i \operatorname{Sin}[e] \right)^4} \right) \right) \\
 & \quad \left( \operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e] \right) \left) / \left( b^2 (a + b)^{5/2} f \sqrt{b \left( \operatorname{Cos}[e] - i \operatorname{Sin}[e] \right)^4} \right) \right) + \\
 & \quad \left( 4 (a^4 + 32 a^3 b + 160 a^2 b^2 + 256 a b^3 + 128 b^4) \operatorname{Sec}[2 e] \left( (a + 2 b) \operatorname{Sin}[2 e] - a \operatorname{Sin}[2 f x] \right) \right) / \\
 & \quad \left( b (a + b) f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2 \right) + \frac{256 a \operatorname{Sin}[2 (e + f x)]}{f} + \\
 & \quad \left( a \left( -3 a^5 + 26 a^4 b + 736 a^3 b^2 + 2624 a^2 b^3 + 3200 a b^4 + 1280 b^5 \right) \operatorname{Sec}[2 e] \operatorname{Sin}[2 f x] + \right. \\
 & \quad \left. \left( 3 a^6 - 24 a^5 b - 920 a^4 b^2 - 4864 a^3 b^3 - 10112 a^2 b^4 - 9216 a b^5 - 3072 b^6 \right) \operatorname{Tan}[2 e] \right) / \\
 & \quad \left. \left( b^2 (a + b)^2 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \right) \right) \left) \right)
 \end{aligned}$$

**Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^4}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 238 leaves, 8 steps):

$$\frac{3 (a^2 + 12 a b + 16 b^2) x}{8 a^5} - \frac{3 \sqrt{b} (5 a^2 + 20 a b + 16 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{8 a^5 \sqrt{a+b} f} - \frac{(5 a + 8 b) \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{8 a^2 f (a+b+b \operatorname{Tan}[e+f x]^2)^2} + \frac{\operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]}{4 a f (a+b+b \operatorname{Tan}[e+f x]^2)^2} - \frac{b (7 a + 12 b) \operatorname{Tan}[e+f x]}{8 a^3 f (a+b+b \operatorname{Tan}[e+f x]^2)^2} - \frac{3 b (a+2 b) \operatorname{Tan}[e+f x]}{2 a^4 f (a+b+b \operatorname{Tan}[e+f x]^2)}$$

Result (type 3, 3109 leaves):

$$\left( 3 (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left( \frac{(3 a^2+8 a b+8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{(a \sqrt{b} (3 a^2+16 a b+16 b^2+3 a (a+2 b) \operatorname{Cos}[2 (e+f x)]) \operatorname{Sin}[2 (e+f x)])}{((a+b)^2 (a+2 b+a \operatorname{Cos}[2 (e+f x)])^2)} \right) \right) / \left( 16384 b^{5/2} f (a+b \operatorname{Sec}[e+f x]^2)^3 \right) + \left( (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left( -\frac{3 a (a+2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{(\sqrt{b} (3 a^3+14 a^2 b+24 a b^2+16 b^3+a (3 a^2+4 a b+4 b^2) \operatorname{Cos}[2 (e+f x)]) \operatorname{Sin}[2 (e+f x)])}{((a+b)^2 (a+2 b+a \operatorname{Cos}[2 (e+f x)])^2)} \right) \right) / \left( 16384 b^{5/2} f (a+b \operatorname{Sec}[e+f x]^2)^3 \right) - \frac{1}{512 (a+b \operatorname{Sec}[e+f x]^2)^3} 3 (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left( \frac{1}{(a+b)^2} (3 a^5-10 a^4 b+80 a^3 b^2+480 a^2 b^3+640 a b^4+256 b^5) \left( \left( \operatorname{ArcTan}[\operatorname{Sec}[f x]] \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} \right) - (a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \operatorname{Cos}[2 e] \right) \right) / \left( 64 a^3 b^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) - \left( i \operatorname{ArcTan}[\operatorname{Sec}[f x]] \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} \right) - (a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \operatorname{Sin}[2 e] \right) \right) / \left( 64 a^3 b^2 \sqrt{a+b} f \right)$$



$$\begin{aligned}
 & \left. \left. \left. \left. \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) + \frac{1}{128 a^3 b^2 (a+b)^2 f (a+2b+a \cos[2e+2fx])^2} \right. \\
 & \text{Sec}[2e] \left( 768 a^4 b^2 f x \cos[2e] + 3584 a^3 b^3 f x \cos[2e] + 6912 a^2 b^4 f x \cos[2e] + \right. \\
 & 6144 a b^5 f x \cos[2e] + 2048 b^6 f x \cos[2e] + 512 a^4 b^2 f x \cos[2fx] + \\
 & 2048 a^3 b^3 f x \cos[2fx] + 2560 a^2 b^4 f x \cos[2fx] + 1024 a b^5 f x \cos[2fx] + \\
 & 512 a^4 b^2 f x \cos[4e+2fx] + 2048 a^3 b^3 f x \cos[4e+2fx] + 2560 a^2 b^4 f x \cos[4e+2fx] + \\
 & 1024 a b^5 f x \cos[4e+2fx] + 128 a^4 b^2 f x \cos[2e+4fx] + 256 a^3 b^3 f x \cos[2e+4fx] + \\
 & 128 a^2 b^4 f x \cos[2e+4fx] + 128 a^4 b^2 f x \cos[6e+4fx] + 256 a^3 b^3 f x \cos[6e+4fx] + \\
 & 128 a^2 b^4 f x \cos[6e+4fx] - 9 a^6 \sin[2e] + 12 a^5 b \sin[2e] + 684 a^4 b^2 \sin[2e] + \\
 & 2880 a^3 b^3 \sin[2e] + 5280 a^2 b^4 \sin[2e] + 4608 a b^5 \sin[2e] + 1536 b^6 \sin[2e] + \\
 & 9 a^6 \sin[2fx] - 14 a^5 b \sin[2fx] - 608 a^4 b^2 \sin[2fx] - 2112 a^3 b^3 \sin[2fx] - \\
 & 2560 a^2 b^4 \sin[2fx] - 1024 a b^5 \sin[2fx] - 3 a^6 \sin[4e+2fx] + 10 a^5 b \sin[4e+2fx] + \\
 & 304 a^4 b^2 \sin[4e+2fx] + 1056 a^3 b^3 \sin[4e+2fx] + 1280 a^2 b^4 \sin[4e+2fx] + \\
 & 512 a b^5 \sin[4e+2fx] + 3 a^6 \sin[2e+4fx] - 12 a^5 b \sin[2e+4fx] - \\
 & \left. \left. \left. \left. 204 a^4 b^2 \sin[2e+4fx] - 384 a^3 b^3 \sin[2e+4fx] - 192 a^2 b^4 \sin[2e+4fx] \right) \right) \right) + \\
 & \frac{1}{512 (a+b \text{Sec}[e+fx])^3} (a+2b+a \cos[2e+2fx])^3 \\
 & \text{Sec}[e+fx]^6 \\
 & \left( \frac{12 (7 a^2 + 32 a b + 32 b^2) x}{a^5} + \right. \\
 & \frac{1}{(a+b)^2} (a^7 - 14 a^6 b + 336 a^5 b^2 + 5600 a^4 b^3 + 22400 a^3 b^4 + 37632 a^2 b^5 + 28672 a b^6 + 8192 b^7) \\
 & \left( \left( 3 \text{ArcTan}[\text{Sec}[fx] \left( \frac{\cos[2e]}{2 \sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \right. \right. \right. \\
 & \left. \left. \left. \frac{i \sin[2e]}{2 \sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right) (-a \sin[fx] - 2 b \sin[fx] + \right. \\
 & \left. \left. a \sin[2e+fx]) \right) \cos[2e] \right) / \left( 64 a^5 b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \\
 & \left( 3 i \text{ArcTan}[\text{Sec}[fx] \left( \frac{\cos[2e]}{2 \sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \right. \right. \right. \\
 & \left. \left. \left. \frac{i \sin[2e]}{2 \sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right) (-a \sin[fx] - 2 b \sin[fx] + \right. \\
 & \left. \left. a \sin[2e+fx]) \right) \sin[2e] \right) / \left( 64 a^5 b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \Big) + \\
 & \left( \text{Sec}[2e] (-a^6 \sin[2e] - 52 a^5 b \sin[2e] - 500 a^4 b^2 \sin[2e] - 1920 a^3 b^3 \sin[2e] - 3520 \right. \\
 & a^2 b^4 \sin[2e] - 3072 a b^5 \sin[2e] - 1024 b^6 \sin[2e] + a^6 \sin[2fx] + 50 a^5 b \sin[2fx] + \\
 & 400 a^4 b^2 \sin[2fx] + 1120 a^3 b^3 \sin[2fx] + 1280 a^2 b^4 \sin[2fx] + 512 a b^5 \sin[2fx]) \Big) / \\
 & \left( 16 a^5 b (a+b) f (a+2b+a \cos[2e+2fx])^2 \right) + \\
 & \frac{1}{64 a^5 b^2 (a+b)^2 f (a+2b+a \cos[2e+2fx])} \\
 & \text{Sec}[2e] (-3 a^7 \sin[2e] + 42 a^6 b \sin[2e] + 2192 a^5 b^2 \sin[2e] + 16480 a^4 b^3 \sin[2e] +
 \end{aligned}$$

$$\begin{aligned}
& 51200 a^3 b^4 \sin[2e] + 77824 a^2 b^5 \sin[2e] + 57344 a b^6 \sin[2e] + 16384 b^7 \sin[2e] + \\
& 3 a^7 \sin[2fx] - 44 a^6 b \sin[2fx] - 1900 a^5 b^2 \sin[2fx] - 10880 a^4 b^3 \sin[2fx] - \\
& 23360 a^3 b^4 \sin[2fx] - 21504 a^2 b^5 \sin[2fx] - 7168 a b^6 \sin[2fx] + \\
& (a+2b) \left( -\frac{12 i \cos[2e+2fx]}{a^4 f} - \frac{12 \sin[2e+2fx]}{a^4 f} \right) + \\
& (a+2b) \left( \frac{12 i \cos[2e+2fx]}{a^4 f} - \frac{12 \sin[2e+2fx]}{a^4 f} \right) + \\
& \left. \frac{2 \sin[4e+4fx]}{a^3 f} \right) - \\
& \left( (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6 \right. \\
& \left. \left( -\left( \left( 6 a^2 \operatorname{ArcTan}[(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])]) \right] \right) \right) \right) / \\
& \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \\
& \left( \cos[2e] - i \sin[2e] \right) \left) / \left( \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \\
& (a \sec[2e] \left( (-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \sin[2fx] + a (-3 a^3 + 2 a^2 b + 24 a \right. \\
& \left. b^2 + 16 b^3) \sin[2(e+2fx)] + (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \sin[4e+2fx] \right) + \\
& \left. (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \tan[2e] \right) / \\
& \left. \left( a^2 (a+2b+a \cos[2(e+fx)])^2 \right) \right) \left) \right) / \\
& \left( 8192 b^2 (a+b)^2 f (a+b \sec[e+fx]^2)^3 \right) + \frac{1}{16384 a^4 (a+b \sec[e+fx]^2)^3} \\
& (a+2b+a \cos[2e+2fx])^3 \\
& \sec[e+fx]^6 \\
& \left( -1536 (a+2b) x - \right. \\
& \left. \left( 3 (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right) \right. \\
& \left. \operatorname{ArcTan}[(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])]) \right] / \\
& \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \\
& \left( \cos[2e] - i \sin[2e] \right) \left) / \left( b^2 (a+b)^{5/2} f \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \\
& \left( 4 (a^4 + 32 a^3 b + 160 a^2 b^2 + 256 a b^3 + 128 b^4) \sec[2e] \left( (a+2b) \sin[2e] - a \sin[2fx] \right) \right) / \\
& \left( b (a+b) f (a+2b+a \cos[2(e+fx)])^2 \right) + \frac{256 a \sin[2(e+fx)]}{f} + \\
& \left( a (-3 a^5 + 26 a^4 b + 736 a^3 b^2 + 2624 a^2 b^3 + 3200 a b^4 + 1280 b^5) \sec[2e] \sin[2fx] + \right. \\
& \left. (3 a^6 - 24 a^5 b - 920 a^4 b^2 - 4864 a^3 b^3 - 10112 a^2 b^4 - 9216 a b^5 - 3072 b^6) \tan[2e] \right) / \\
& \left. \left( b^2 (a+b)^2 f (a+2b+a \cos[2(e+fx)]) \right) \right) \left) \right)
\end{aligned}$$

Problem 62: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sin}[e + f x]^2}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 184 leaves, 7 steps):

$$\frac{(a + 6 b) x}{2 a^4} - \frac{\sqrt{b} (15 a^2 + 40 a b + 24 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 a^4 (a + b)^{3/2} f} - \frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{2 a f (a + b + b \operatorname{Tan}[e + f x]^2)^2} - \frac{3 b \operatorname{Tan}[e + f x]}{4 a^2 f (a + b + b \operatorname{Tan}[e + f x]^2)^2} - \frac{b (11 a + 12 b) \operatorname{Tan}[e + f x]}{8 a^3 (a + b) f (a + b + b \operatorname{Tan}[e + f x]^2)}$$

Result (type 3, 2515 leaves):

$$\left( 5 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \left( \frac{(3 a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}}\right]}{(a + b)^{5/2}} - \frac{(a \sqrt{b} (3 a^2 + 16 a b + 16 b^2 + 3 a (a + 2 b) \operatorname{Cos}[2 (e + f x)]) \operatorname{Sin}[2 (e + f x)])}{((a + b)^2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2)} \right) \right) / (8192 b^{5/2} f (a + b \operatorname{Sec}[e + f x]^2)^3) + \left( (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \left( - \frac{3 a (a + 2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}}\right]}{(a + b)^{5/2}} + \frac{(\sqrt{b} (3 a^3 + 14 a^2 b + 24 a b^2 + 16 b^3 + a (3 a^2 + 4 a b + 4 b^2) \operatorname{Cos}[2 (e + f x)]) \operatorname{Sin}[2 (e + f x)])}{((a + b)^2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2)} \right) \right) / (2048 b^{5/2} f (a + b \operatorname{Sec}[e + f x]^2)^3) + \frac{1}{128 (a + b \operatorname{Sec}[e + f x]^2)^3} (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \left( \frac{24 (a + 2 b) x}{a^4} - \frac{1}{(a + b)^2} (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) - \left( \left( 3 \operatorname{ArcTan}[\operatorname{Sec}[f x]] \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \operatorname{Cos}[2 e] \right) \right) / (64 a^4 b^2 \sqrt{a + b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}) \right) + \left( 3 i \operatorname{ArcTan}[\operatorname{Sec}[f x]] \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right)$$

$$\begin{aligned}
 & \left. \frac{\frac{\text{I Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]}}}{\text{Sin}[2 e + f x]}}{\left( -a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \right)} \right) \left( 64 a^4 b^2 \sqrt{a+b} f \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]} \right) \Bigg) - \\
 & \left( \text{Sec}[2 e] \left( a^5 \text{Sin}[2 e] + 34 a^4 b \text{Sin}[2 e] + 224 a^3 b^2 \text{Sin}[2 e] + 576 a^2 b^3 \text{Sin}[2 e] + \right. \right. \\
 & \quad \left. \left. 640 a b^4 \text{Sin}[2 e] + 256 b^5 \text{Sin}[2 e] - a^5 \text{Sin}[2 f x] - 32 a^4 b \text{Sin}[2 f x] - \right. \right. \\
 & \quad \left. \left. 160 a^3 b^2 \text{Sin}[2 f x] - 256 a^2 b^3 \text{Sin}[2 f x] - 128 a b^4 \text{Sin}[2 f x] \right) \right) / \\
 & \left( 16 a^4 b (a+b) f (a+2 b+a \text{Cos}[2 e+2 f x])^2 \right) - \\
 & \left( \text{Sec}[2 e] \left( 3 a^6 \text{Sin}[2 e] - 24 a^5 b \text{Sin}[2 e] - 920 a^4 b^2 \text{Sin}[2 e] - 4864 a^3 b^3 \text{Sin}[2 e] - \right. \right. \\
 & \quad \left. \left. 10112 a^2 b^4 \text{Sin}[2 e] - 9216 a b^5 \text{Sin}[2 e] - 3072 b^6 \text{Sin}[2 e] - \right. \right. \\
 & \quad \left. \left. 3 a^6 \text{Sin}[2 f x] + 26 a^5 b \text{Sin}[2 f x] + 736 a^4 b^2 \text{Sin}[2 f x] + \right. \right. \\
 & \quad \left. \left. 2624 a^3 b^3 \text{Sin}[2 f x] + 3200 a^2 b^4 \text{Sin}[2 f x] + 1280 a b^5 \text{Sin}[2 f x] \right) \right) / \\
 & \left( 64 a^4 b^2 (a+b)^2 f (a+2 b+a \text{Cos}[2 e+2 f x]) \right) - \frac{4 \text{Sin}[2 e+2 f x]}{a^3 f} \Bigg) + \\
 & \frac{1}{32 (a+b \text{Sec}[e+f x]^2)^3} (a+2 b+a \text{Cos}[2 e+2 f x])^3 \\
 & \text{Sec}[e+f x]^6 \\
 & \left( -\frac{1}{(a+b)^2} (3 a^5 - 10 a^4 b + 80 a^3 b^2 + 480 a^2 b^3 + 640 a b^4 + 256 b^5) \right. \\
 & \quad \left( \left( \text{ArcTan}[\text{Sec}[f x] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]}} - \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{\text{I Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]}} \right) \right) (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + \right. \right. \\
 & \quad \left. \left. a \text{Sin}[2 e+f x]) \right) \text{Cos}[2 e] \right) / \left( 64 a^3 b^2 \sqrt{a+b} f \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]} \right) - \\
 & \quad \left( \text{I ArcTan}[\text{Sec}[f x] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]}} - \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{\text{I Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]}} \right) \right) (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + \right. \right. \\
 & \quad \left. \left. a \text{Sin}[2 e+f x]) \right) \text{Sin}[2 e] \right) / \left( 64 a^3 b^2 \sqrt{a+b} f \sqrt{b \text{Cos}[4 e] - \text{I b Sin}[4 e]} \right) \Bigg) - \\
 & \frac{1}{128 a^3 b^2 (a+b)^2 f (a+2 b+a \text{Cos}[2 e+2 f x])^2} \text{Sec}[2 e] (768 a^4 b^2 f x \text{Cos}[2 e] + \\
 & \quad 3584 a^3 b^3 f x \text{Cos}[2 e] + 6912 a^2 b^4 f x \text{Cos}[2 e] + 6144 a b^5 f x \text{Cos}[2 e] + \\
 & \quad 2048 b^6 f x \text{Cos}[2 e] + 512 a^4 b^2 f x \text{Cos}[2 f x] + 2048 a^3 b^3 f x \text{Cos}[2 f x] + \\
 & \quad 2560 a^2 b^4 f x \text{Cos}[2 f x] + 1024 a b^5 f x \text{Cos}[2 f x] + 512 a^4 b^2 f x \text{Cos}[4 e+2 f x] + \\
 & \quad 2048 a^3 b^3 f x \text{Cos}[4 e+2 f x] + 2560 a^2 b^4 f x \text{Cos}[4 e+2 f x] + 1024 a b^5 f x \text{Cos}[4 e+2 f x] + \\
 & \quad 128 a^4 b^2 f x \text{Cos}[2 e+4 f x] + 256 a^3 b^3 f x \text{Cos}[2 e+4 f x] + 128 a^2 b^4 f x \text{Cos}[2 e+4 f x] + \\
 & \quad 128 a^4 b^2 f x \text{Cos}[6 e+4 f x] + 256 a^3 b^3 f x \text{Cos}[6 e+4 f x] + 128 a^2 b^4 f x \text{Cos}[6 e+4 f x] - \\
 & \quad 9 a^6 \text{Sin}[2 e] + 12 a^5 b \text{Sin}[2 e] + 684 a^4 b^2 \text{Sin}[2 e] + 2880 a^3 b^3 \text{Sin}[2 e] + \\
 & \quad 5280 a^2 b^4 \text{Sin}[2 e] + 4608 a b^5 \text{Sin}[2 e] + 1536 b^6 \text{Sin}[2 e] + 9 a^6 \text{Sin}[2 f x] - \\
 & \quad 14 a^5 b \text{Sin}[2 f x] - 608 a^4 b^2 \text{Sin}[2 f x] - 2112 a^3 b^3 \text{Sin}[2 f x] - 2560 a^2 b^4 \text{Sin}[2 f x] -
 \end{aligned}$$

$$\begin{aligned}
 & 1024 a b^5 \sin[2 f x] - 3 a^6 \sin[4 e + 2 f x] + 10 a^5 b \sin[4 e + 2 f x] + \\
 & 304 a^4 b^2 \sin[4 e + 2 f x] + 1056 a^3 b^3 \sin[4 e + 2 f x] + 1280 a^2 b^4 \sin[4 e + 2 f x] + \\
 & 512 a b^5 \sin[4 e + 2 f x] + 3 a^6 \sin[2 e + 4 f x] - 12 a^5 b \sin[2 e + 4 f x] - \\
 & 204 a^4 b^2 \sin[2 e + 4 f x] - 384 a^3 b^3 \sin[2 e + 4 f x] - 192 a^2 b^4 \sin[2 e + 4 f x] \Big) - \\
 & \left( (a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \right. \\
 & \left. - \left( \left( 6 a^2 \operatorname{ArcTan}[\sec[f x] (\cos[2 e] - i \sin[2 e]) (- (a + 2 b) \sin[f x] + a \sin[2 e + f x])] \right) / \right. \right. \\
 & \left. \left( 2 \sqrt{a + b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \right) \\
 & \left. (\cos[2 e] - i \sin[2 e]) \right) / \left( \sqrt{a + b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \\
 & (a \sec[2 e] \left( (-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \sin[2 f x] + a (-3 a^3 + 2 a^2 b + 24 a \right. \\
 & \left. b^2 + 16 b^3) \sin[2 (e + 2 f x)] + (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \sin[4 e + 2 f x] \right) + \\
 & \left. (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \tan[2 e] \right) / \\
 & \left. \left( a^2 (a + 2 b + a \cos[2 (e + f x)])^2 \right) \right) / \left( 4096 b^2 (a + b)^2 f (a + b \sec[e + f x]^2)^3 \right)
 \end{aligned}$$

**Problem 63: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\begin{aligned}
 & \frac{x}{a^3} - \frac{\sqrt{b} (15 a^2 + 20 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{5/2} f} - \\
 & \frac{b \tan[e + f x]}{4 a (a + b) f (a + b + b \tan[e + f x]^2)^2} - \frac{b (7 a + 4 b) \tan[e + f x]}{8 a^2 (a + b)^2 f (a + b + b \tan[e + f x]^2)}
 \end{aligned}$$

Result (type 3, 627 leaves):

$$\begin{aligned}
& x \frac{(a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{8a^3 (a + b \sec[e + fx]^2)^3} + \\
& \left( (15a^2 + 20ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \left( b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b}} - \frac{i \sin[2e]}{2\sqrt{a+b}} \right] \right. \right. \right. \\
& \left. \left. \left. \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
& \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \cos[2e] \right) \right) \right) / \\
& \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b}} - \frac{i \sin[2e]}{2\sqrt{a+b}} \right] \right. \\
& \left. \left. \left. \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
& \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) \right) \right) / \\
& \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \left. \right) / \left( (a+b)^2 (a+b \sec[e + fx]^2)^3 \right) + \\
& \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6 (9a^2 b \sin[2e] + 28a b^2 \sin[2e] + \right. \\
& \left. 16b^3 \sin[2e] - 9a^2 b \sin[2fx] - 6a b^2 \sin[2fx]) \right) / \\
& \left( 64a^3 (a+b)^2 f (a+b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) \right. \\
& \left. (\cos[e] + \sin[e]) \right) + \\
& \left( (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^6 (-a b^2 \sin[2e] - 2b^3 \sin[2e] + a b^2 \sin[2fx]) \right) / \\
& \left( 16a^3 (a+b) f (a+b \sec[e + fx]^2)^3 \right. \\
& \left. (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right)
\end{aligned}$$

**Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e + fx]^2}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$\begin{aligned}
& -\frac{15\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{8(a+b)^{7/2} f} - \frac{15 \cot[e + fx]}{8(a+b)^3 f} + \\
& \frac{\cot[e + fx]}{4(a+b) f (a+b + b \tan[e + fx]^2)^2} + \frac{5 \cot[e + fx]}{8(a+b)^2 f (a+b + b \tan[e + fx]^2)}
\end{aligned}$$

Result (type 3, 987 leaves):

$$\begin{aligned}
 & \left( (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \left( 15b \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{ib\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a\sin[fx] - 2b\sin[fx] + a\sin[2e + fx]) \cos[2e] \right) \right] \right) \right) / \\
 & \quad \left( 64\sqrt{a+b} f \sqrt{b\cos[4e] - ib\sin[4e]} \right) - \left( 15ib \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{ib\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a\sin[fx] - 2b\sin[fx] + a\sin[2e + fx]) \sin[2e] \right) \right] \right) \right) / \\
 & \quad \left( 64\sqrt{a+b} f \sqrt{b\cos[4e] - ib\sin[4e]} \right) \left. \right) \left. \right) / \\
 & \left( (a+b)^3 (a+b \sec[e+fx]^2)^3 \right) + \frac{1}{512 a^2 (a+b)^3 f (a+b \sec[e+fx]^2)^3} \\
 & (a + 2b + a \cos[2e + 2fx]) \\
 & \operatorname{Csc}[ \\
 & \quad e] \operatorname{Csc}[ \\
 & \quad e + fx] \sec[ \\
 & \quad 2e] \sec[e + fx]^6 \\
 & (-32 a^4 \sin[fx] - 64 a^3 b \sin[fx] + 22 a^2 b^2 \sin[fx] + 80 a b^3 \sin[fx] + \\
 & \quad 16 b^4 \sin[fx] + 32 a^4 \sin[3fx] + 46 a^3 b \sin[3fx] - 54 a^2 b^2 \sin[3fx] - \\
 & \quad 8 a b^3 \sin[3fx] - 48 a^4 \sin[2e - fx] - 128 a^3 b \sin[2e - fx] - \\
 & \quad 106 a^2 b^2 \sin[2e - fx] + 80 a b^3 \sin[2e - fx] + 16 b^4 \sin[2e - fx] + \\
 & \quad 48 a^4 \sin[2e + fx] + 146 a^3 b \sin[2e + fx] + 182 a^2 b^2 \sin[2e + fx] + \\
 & \quad 80 a b^3 \sin[2e + fx] + 16 b^4 \sin[2e + fx] - 32 a^4 \sin[4e + fx] - \\
 & \quad 82 a^3 b \sin[4e + fx] - 54 a^2 b^2 \sin[4e + fx] - 80 a b^3 \sin[4e + fx] - \\
 & \quad 16 b^4 \sin[4e + fx] - 8 a^4 \sin[2e + 3fx] + 18 a^3 b \sin[2e + 3fx] + \\
 & \quad 54 a^2 b^2 \sin[2e + 3fx] + 8 a b^3 \sin[2e + 3fx] + 32 a^4 \sin[4e + 3fx] + \\
 & \quad 73 a^3 b \sin[4e + 3fx] + 24 a^2 b^2 \sin[4e + 3fx] + 8 a b^3 \sin[4e + 3fx] - \\
 & \quad 8 a^4 \sin[6e + 3fx] - 9 a^3 b \sin[6e + 3fx] - 24 a^2 b^2 \sin[6e + 3fx] - \\
 & \quad 8 a b^3 \sin[6e + 3fx] + 8 a^4 \sin[2e + 5fx] - 9 a^3 b \sin[2e + 5fx] - 2 a^2 b^2 \sin[2e + 5fx] + \\
 & \quad 9 a^3 b \sin[4e + 5fx] + 2 a^2 b^2 \sin[4e + 5fx] + 8 a^4 \sin[6e + 5fx])
 \end{aligned}$$

**Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + fx]^4}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 164 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{5 (3 a - 4 b) \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{8 (a+b)^{9/2} f} - \frac{(a-2 b) \operatorname{Cot}[e+f x]}{(a+b)^4 f} - \frac{\operatorname{Cot}[e+f x]^3}{3 (a+b)^3 f} - \\
 & \frac{a b \operatorname{Tan}[e+f x]}{4 (a+b)^3 f (a+b+b \operatorname{Tan}[e+f x]^2)^2} - \frac{(7 a-4 b) b \operatorname{Tan}[e+f x]}{8 (a+b)^4 f (a+b+b \operatorname{Tan}[e+f x]^2)}
 \end{aligned}$$

Result (type 3, 1234 leaves):



$$\begin{aligned}
 & \left( (3a - 4b) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \left( 5b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right] \right. \right. \right. \\
 & \quad \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \cos[2e] \right) \right) / \right. \\
 & \quad \left( 64 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( 5i b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right] \right. \\
 & \quad \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) \right) / \right. \\
 & \quad \left. \left. \left. \left( 64 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) \right) / \right. \\
 & \quad \left( (a+b)^4 (a+b \sec[e+fx]^2)^3 \right) + \frac{1}{6144 a (a+b)^4 f (a+b \sec[e+fx]^2)^3} \\
 & (a + 2b + a \cos[2e + 2fx]) \\
 & \operatorname{Csc}[ \\
 & \quad e] \operatorname{Csc}[e + fx]^3 \operatorname{Sec}[ \\
 & \quad 2e] \operatorname{Sec}[e + fx]^6 \\
 & (-176 a^4 \sin[fx] - 488 a^3 b \sin[fx] - 252 a^2 b^2 \sin[fx] - 504 a b^3 \sin[fx] - \\
 & \quad 144 b^4 \sin[fx] + 96 a^4 \sin[3fx] + 71 a^3 b \sin[3fx] - \\
 & \quad 344 a^2 b^2 \sin[3fx] + 1208 a b^3 \sin[3fx] - 48 b^4 \sin[3fx] - \\
 & \quad 224 a^4 \sin[2e - fx] - 576 a^3 b \sin[2e - fx] - 124 a^2 b^2 \sin[2e - fx] + \\
 & \quad 2184 a b^3 \sin[2e - fx] - 144 b^4 \sin[2e - fx] + 224 a^4 \sin[2e + fx] + \\
 & \quad 657 a^3 b \sin[2e + fx] + 538 a^2 b^2 \sin[2e + fx] - 984 a b^3 \sin[2e + fx] - \\
 & \quad 144 b^4 \sin[2e + fx] - 176 a^4 \sin[4e + fx] - 569 a^3 b \sin[4e + fx] - \\
 & \quad 666 a^2 b^2 \sin[4e + fx] - 1704 a b^3 \sin[4e + fx] + 144 b^4 \sin[4e + fx] - \\
 & \quad 48 a^4 \sin[2e + 3fx] - 111 a^3 b \sin[2e + 3fx] - 360 a^2 b^2 \sin[2e + 3fx] - \\
 & \quad 312 a b^3 \sin[2e + 3fx] + 48 b^4 \sin[2e + 3fx] + 96 a^4 \sin[4e + 3fx] + \\
 & \quad 152 a^3 b \sin[4e + 3fx] - 146 a^2 b^2 \sin[4e + 3fx] + 728 a b^3 \sin[4e + 3fx] + \\
 & \quad 48 b^4 \sin[4e + 3fx] - 48 a^4 \sin[6e + 3fx] - 192 a^3 b \sin[6e + 3fx] - \\
 & \quad 558 a^2 b^2 \sin[6e + 3fx] + 168 a b^3 \sin[6e + 3fx] - 48 b^4 \sin[6e + 3fx] - \\
 & \quad 16 a^4 \sin[2e + 5fx] + 598 a^2 b^2 \sin[2e + 5fx] - 48 a b^3 \sin[2e + 5fx] - \\
 & \quad 72 a^3 b \sin[4e + 5fx] - 150 a^2 b^2 \sin[4e + 5fx] + 48 a b^3 \sin[4e + 5fx] - \\
 & \quad 16 a^4 \sin[6e + 5fx] - 27 a^3 b \sin[6e + 5fx] + 388 a^2 b^2 \sin[6e + 5fx] - \\
 & \quad 45 a^3 b \sin[8e + 5fx] + 60 a^2 b^2 \sin[8e + 5fx] - 16 a^4 \sin[4e + 7fx] + \\
 & \quad 83 a^3 b \sin[4e + 7fx] - 6 a^2 b^2 \sin[4e + 7fx] - 27 a^3 b \sin[6e + 7fx] + \\
 & \quad 6 a^2 b^2 \sin[6e + 7fx] - 16 a^4 \sin[8e + 7fx] + 56 a^3 b \sin[8e + 7fx])
 \end{aligned}$$

Problem 66: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\begin{aligned} & - \frac{\sqrt{b} (15 a^2 - 40 a b + 8 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 (a + b)^{11/2} f} - \frac{(5 a^2 - 20 a b + 2 b^2) \text{Cot}[e + f x]}{5 (a + b)^5 f} \\ & - \frac{(10 a + b) \text{Cot}[e + f x]^3}{15 (a + b)^4 f} - \frac{\text{Cot}[e + f x]^5}{5 (a + b) f (a + b + b \text{Tan}[e + f x]^2)^2} \\ & - \frac{b (5 a^2 + 4 b^2) \text{Tan}[e + f x]}{20 (a + b)^4 f (a + b + b \text{Tan}[e + f x]^2)^2} - \frac{b (35 a^2 - 40 a b + 24 b^2) \text{Tan}[e + f x]}{40 (a + b)^5 f (a + b + b \text{Tan}[e + f x]^2)} \end{aligned}$$

Result (type 3, 908 leaves):

$$\begin{aligned}
 & \left( (-4 a \cos [e] + 11 b \cos [e]) (a + 2 b + a \cos [2 e + 2 f x])^3 \csc [e] \csc [e + f x]^2 \sec [e + f x]^6 \right) / \\
 & \left( 120 (a + b)^4 f (a + b \sec [e + f x]^2)^3 \right) - \\
 & \frac{(a + 2 b + a \cos [2 e + 2 f x])^3 \cot [e] \csc [e + f x]^4 \sec [e + f x]^6}{40 (a + b)^3 f (a + b \sec [e + f x]^2)^3} + \\
 & \left( (15 a^2 - 40 a b + 8 b^2) (a + 2 b + a \cos [2 e + 2 f x])^3 \sec [e + f x]^6 \left( \left( b \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \frac{\cos [2 e]}{2 \sqrt{a + b} \sqrt{b \cos [4 e] - i b \sin [4 e]}} - \frac{i \sin [2 e]}{2 \sqrt{a + b} \sqrt{b \cos [4 e] - i b \sin [4 e]}} \right] \right) \right) \right) / \\
 & \left( -a \sin [f x] - 2 b \sin [f x] + a \sin [2 e + f x] \right) \cos [2 e] \left. \right) / \\
 & \left( 64 \sqrt{a + b} f \sqrt{b \cos [4 e] - i b \sin [4 e]} \right) - \left( i b \operatorname{ArcTan} \left[ \right. \right. \\
 & \left. \left. \left. \left. \frac{\cos [2 e]}{2 \sqrt{a + b} \sqrt{b \cos [4 e] - i b \sin [4 e]}} - \frac{i \sin [2 e]}{2 \sqrt{a + b} \sqrt{b \cos [4 e] - i b \sin [4 e]}} \right] \right) \right) \right) / \\
 & \left( -a \sin [f x] - 2 b \sin [f x] + a \sin [2 e + f x] \right) \sin [2 e] \left. \right) / \\
 & \left( 64 \sqrt{a + b} f \sqrt{b \cos [4 e] - i b \sin [4 e]} \right) \left. \right) / \left( (a + b)^5 (a + b \sec [e + f x]^2)^3 \right) + \\
 & \left( (a + 2 b + a \cos [2 e + 2 f x])^3 \csc [e] \csc [e + f x]^5 \sec [e + f x]^6 \sin [f x] \right) / \\
 & \left( 40 (a + b)^3 f (a + b \sec [e + f x]^2)^3 \right) + \\
 & \left( (a + 2 b + a \cos [2 e + 2 f x])^3 \csc [e] \csc [e + f x]^3 \right. \\
 & \left. \sec [e + f x]^6 (4 a \sin [f x] - 11 b \sin [f x]) \right) / \\
 & \left( 120 (a + b)^4 f (a + b \sec [e + f x]^2)^3 \right) + \\
 & \left( (a + 2 b + a \cos [2 e + 2 f x])^3 \csc [e] \csc [e + f x] \sec [e + f x]^6 \right. \\
 & \left. (8 a^2 \sin [f x] - 59 a b \sin [f x] + 23 b^2 \sin [f x]) \right) / \\
 & \left( 120 (a + b)^5 f (a + b \sec [e + f x]^2)^3 \right) + \\
 & \left( (a + 2 b + a \cos [2 e + 2 f x]) \sec [2 e] \sec [e + f x]^6 \right. \\
 & \left. (-a b^2 \sin [2 e] - 2 b^3 \sin [2 e] + a b^2 \sin [2 f x]) \right) / \\
 & \left( 16 (a + b)^4 f (a + b \sec [e + f x]^2)^3 \right) + \\
 & \left( (a + 2 b + a \cos [2 e + 2 f x])^2 \sec [2 e] \sec [e + f x]^6 \right. \\
 & \left. (9 a^2 b \sin [2 e] + 16 a b^2 \sin [2 e] - 8 b^3 \sin [2 e] - 9 a^2 b \sin [2 f x] + 6 a b^2 \sin [2 f x]) \right) / \\
 & \left( 64 (a + b)^5 f (a + b \sec [e + f x]^2)^3 \right)
 \end{aligned}$$

**Problem 67: Unable to integrate problem.**

$$\int \sqrt{a + b \sec [e + f x]^2} \sin [e + f x]^5 dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f} +$$

$$\frac{2(5a+b) \operatorname{Cos}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{15a^2 f} - \frac{\operatorname{Cos}[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{5a f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^5 dx$$

**Problem 68: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^3 dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} -$$

$$\frac{\operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f} + \frac{\operatorname{Cos}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{3a f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^3 dx$$

**Problem 69: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx] dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f}$$

Result (type 8, 25 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx] dx$$

**Problem 71: Unable to integrate problem.**

$$\int \operatorname{Csc}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 124 leaves, 7 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{(a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{2\sqrt{a+b} f} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{2f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Csc}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

**Problem 72: Unable to integrate problem.**

$$\int \operatorname{Csc}[e+fx]^5 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 183 leaves, 8 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{(3a^2 + 12ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{8(a+b)^{3/2} f} - \frac{(3a+4b) \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{8(a+b) f} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{4f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Csc}[e+fx]^5 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

**Problem 73: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^6 dx$$

Optimal (type 3, 240 leaves, 9 steps):

$$\frac{(5a^3 - 15a^2b - 5ab^2 - b^3) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right] + \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16a^{5/2}f} + \frac{(a-b)(5a+b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16a^2f} - \frac{(5a-b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24af} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{6f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^6 dx$$

**Problem 74: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^4 dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$\frac{(3a^2 - 6ab - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right] + \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8a^{3/2}f} + \frac{(3a-b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8af} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^4 dx$$

**Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^2 dx$$

Optimal (type 3, 123 leaves, 7 steps):

$$\frac{(a-b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2\sqrt{a} f} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 432 leaves):

$$\frac{1}{4\sqrt{2} f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]}} e^{-i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2}$$

$$\operatorname{Cos}[e+fx] \left( i(-1+e^{2i(e+fx)}) + \left( 2e^{2i(e+fx)} \left( 2afx-2bfx - \right. \right. \right.$$

$$i(a-b) \operatorname{Log}\left[ a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] +$$

$$i(a-b) \operatorname{Log}\left[ a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] -$$

$$4\sqrt{a}\sqrt{b} \operatorname{Log}\left[ \left( \left( -\sqrt{b}(-1+e^{2i(e+fx)}) + i\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) f \right) / \right.$$

$$\left. \left. \left. \left( 2b(1+e^{2i(e+fx)}) \right) \right) \right] \right) \left. \right) \sqrt{a+b \operatorname{Sec}[e+fx]^2}$$

**Problem 76: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f}$$

Result (type 8, 18 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

**Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 68 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f}$$

Result (type 3, 285 leaves):

$$- \left( \left( \left( \left( 1 + e^{2i(e+fx)} \right) \sqrt{4b + a e^{-2i(e+fx)} \left( 1 + e^{2i(e+fx)} \right)^2} \right. \right. \right. \\ \left. \left. \left( i \sqrt{4b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} + \sqrt{b} \left( -1 + e^{2i(e+fx)} \right) \right. \right. \right. \\ \left. \left. \left. \operatorname{Log}\left[ \frac{1}{1 + e^{2i(e+fx)}} \left( -4 \sqrt{b} \left( -1 + e^{2i(e+fx)} \right) f + 4 i \sqrt{4b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} f \right) \right] \right) \right) \right) \\ \left. \left. \left. \left. \sqrt{a + b \operatorname{Sec}[e+fx]^2} \right) / \left( \sqrt{2} \left( -1 + e^{2i(e+fx)} \right) \sqrt{4b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right. \right. \right. \right. \\ \left. \left. \left. \left. f \sqrt{a + 2b + a \operatorname{Cos}[2(e+fx)]} \right) \right) \right) \right)$$

**Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f} - \frac{\operatorname{Cot}[e+fx]^3 (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{3(a+b)f}$$

Result (type 3, 309 leaves):



$$\left( \sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx] \right. \\ \left. - \left( i \left( 2a (1-4e^{2i(e+fx)} + e^{4i(e+fx)}) + b (3-10e^{2i(e+fx)} + 3e^{4i(e+fx)}) \right) \right) / \right. \\ \left. \left( (a+b) (-1+e^{2i(e+fx)})^3 \right) - \frac{3\sqrt{b} \operatorname{Log} \left[ \frac{-4\sqrt{b} (-1+e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} f}{1+e^{2i(e+fx)}} \right]}{\sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2}} \right) \\ \left. \sqrt{a+b \operatorname{Sec}[e+fx]^2} \right) / \left( 3f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \right)$$

**Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^6 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{f} - \frac{\operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f} - \\ \frac{2(5a+4b) \operatorname{Cot}[e+fx]^3 (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{15(a+b)^2 f} - \frac{\operatorname{Cot}[e+fx]^5 (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{5(a+b) f}$$

Result (type 3, 422 leaves):

$$\frac{1}{15 f \sqrt{a + 2 b + a \operatorname{Cos}[2 e + 2 f x]}} \sqrt{2} e^{i(e+f x)} \sqrt{4 b + a e^{-2 i(e+f x)} (1 + e^{2 i(e+f x)})^2} \operatorname{Cos}[e + f x]$$

$$\left( - \left( i \left( 8 a^2 (1 - 6 e^{2 i(e+f x)} + 16 e^{4 i(e+f x)} - 6 e^{6 i(e+f x)} + e^{8 i(e+f x)}) + b^2 (15 - 80 e^{2 i(e+f x)} + 178 e^{4 i(e+f x)} - 80 e^{6 i(e+f x)} + 15 e^{8 i(e+f x)}) + a b (25 - 136 e^{2 i(e+f x)} + 318 e^{4 i(e+f x)} - 136 e^{6 i(e+f x)} + 25 e^{8 i(e+f x)}) \right) / \left( (a + b)^2 (-1 + e^{2 i(e+f x)})^5 \right) \right) - \frac{15 \sqrt{b} \operatorname{Log} \left[ \frac{-4 \sqrt{b} (-1 + e^{2 i(e+f x)}) f + 4 i \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2} f}{1 + e^{2 i(e+f x)}} \right]}{\sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}} \right) \sqrt{a + b \operatorname{Sec}[e + f x]^2}$$

**Problem 80: Unable to integrate problem.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Sin}[e + f x]^5 dx$$

Optimal (type 3, 196 leaves, 7 steps):

$$\frac{(3 a - 4 b) \sqrt{b} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \operatorname{Sec}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} \right]}{2 f} + \frac{(3 a - 4 b) b \operatorname{Sec}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2}}{2 a f} - \frac{(3 a - 4 b) \operatorname{Cos}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2}}{3 a f} + \frac{2 \operatorname{Cos}[e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^{5/2}}{3 a f} - \frac{\operatorname{Cos}[e + f x]^5 (a + b \operatorname{Sec}[e + f x]^2)^{5/2}}{5 a f}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Sin}[e + f x]^5 dx$$

**Problem 81: Unable to integrate problem.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Sin}[e + f x]^3 dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{(3 a - 2 b) \sqrt{b} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \operatorname{Sec}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} \right]}{2 f} + \frac{(3 a - 2 b) b \operatorname{Sec}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2}}{2 a f} - \frac{(3 a - 2 b) \operatorname{Cos}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2}}{3 a f} + \frac{\operatorname{Cos}[e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^{5/2}}{3 a f}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \sin[e + f x]^3 dx$$

**Problem 82: Unable to integrate problem.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \sin[e + f x] dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\frac{3 a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} + \frac{3 b \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{2 f} - \frac{\cos[e+f x] (a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{f}$$

Result (type 8, 25 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \sin[e + f x] dx$$

**Problem 83: Unable to integrate problem.**

$$\int \operatorname{Csc}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 122 leaves, 7 steps):

$$\frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} - \frac{(a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{f} + \frac{b \operatorname{Sec}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2}}{2 f}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Csc}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

**Problem 84: Unable to integrate problem.**

$$\int \operatorname{Csc}[e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$\frac{\sqrt{b} (3a + 4b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{2f} - \frac{\sqrt{a+b} (a+4b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{2f} +$$

$$\frac{b \operatorname{Sec}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{2f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Csc}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Problem 85: Unable to integrate problem.

$$\int \operatorname{Csc}[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 218 leaves, 9 steps):

$$\frac{3\sqrt{b} (a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{2f} -$$

$$\frac{3(a^2 + 8ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{8\sqrt{a+b}f} + \frac{3(a+4b) \operatorname{Sec}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{8f} -$$

$$\frac{3(a+2b) \operatorname{Csc}[e+fx]^2 \operatorname{Sec}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{8f} -$$

$$\frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{4f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Csc}[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Problem 86: Unable to integrate problem.

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^6 dx$$

Optimal (type 3, 298 leaves, 10 steps):

$$\begin{aligned}
 & \frac{(5a^3 - 45a^2b + 15ab^2 + b^3) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16a^{3/2}f} + \frac{(3a - 5b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} - \\
 & \frac{(5a^2 - 26ab + b^2) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16af} + \frac{1}{48af} \\
 & \frac{(5a^2 - 40ab + 3b^2) \operatorname{Sin}[e+fx]^2 \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24f} + \\
 & \frac{(5a - 3b) \operatorname{Sin}[e+fx]^4 \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24f} - \\
 & \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^5 (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{6f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^6 dx$$

**Problem 87: Unable to integrate problem.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^4 dx$$

Optimal (type 3, 217 leaves, 9 steps):

$$\begin{aligned}
 & \frac{3(a^2 - 6ab + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8\sqrt{a}f} + \\
 & \frac{3(a-b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} - \frac{3(a-3b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} + \\
 & \frac{3(a-b) \operatorname{Sin}[e+fx]^2 \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} - \\
 & \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^3 (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^4 dx$$

**Problem 88: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^2 dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$\frac{\sqrt{a} (a - 3b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{(3a - b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} +$$

$$\frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{2f}$$

Result (type 3, 493 leaves):

$$\frac{1}{2\sqrt{2} f (a + 2b + a \operatorname{Cos}[2e + 2fx])^{3/2}} e^{-i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2}$$

$$\operatorname{Cos}[e+fx]^3 \left( \frac{i(-1 + e^{2i(e+fx)}) (-4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)}{(1 + e^{2i(e+fx)})^2} + \right.$$

$$\left. \left( 2e^{2i(e+fx)} \left( 2\sqrt{a} (a - 3b) fx - i\sqrt{a} (a - 3b) \right. \right. \right.$$

$$\left. \left. \left. \operatorname{Log}\left[ a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right] + i\sqrt{a} (a - 3b) \right. \right. \right.$$

$$\left. \left. \left. \operatorname{Log}\left[ a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right] + \right. \right.$$

$$\left. \left. \left. 2\sqrt{b} (-3a + b) \operatorname{Log}\left[ \left( \left( \sqrt{b} (-1 + e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) f \right) \right. \right. \right.$$

$$\left. \left. \left. \left( b(-3a + b) (1 + e^{2i(e+fx)}) \right) \right) \right] \right] \right] /$$

$$\left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) (a + b \operatorname{Sec}[e+fx]^2)^{3/2}$$

**Problem 89: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 118 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} +$$

$$\frac{\sqrt{b} (3a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves):

$$\begin{aligned}
 & \frac{1}{f (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}} \sqrt{2} e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} \\
 & \operatorname{Cos}[e + f x]^3 \left( -\frac{i b (-1 + e^{2 i (e+f x)})}{(1 + e^{2 i (e+f x)})^2} + \frac{1}{\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}} \right. \\
 & \left. \left( 2 a^{3/2} f x - i a^{3/2} \operatorname{Log}[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] + \right. \right. \\
 & \left. \left. i a^{3/2} \operatorname{Log}[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] - \right. \right. \\
 & \left. \left. 3 a \sqrt{b} \operatorname{Log}\left[ \left( -2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f \right) / \right. \right. \right. \\
 & \left. \left. \left. (b (3 a + b) (1 + e^{2 i (e+f x)})) \right] \right] - \right. \\
 & \left. \left. b^{3/2} \operatorname{Log}\left[ \left( -2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f \right) / \right. \right. \right. \\
 & \left. \left. \left. (b (3 a + b) (1 + e^{2 i (e+f x)})) \right] \right] \right) \right) (a + b \operatorname{Sec}[e + f x]^2)^{3/2}
 \end{aligned}$$

**Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + f x]^2 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$\begin{aligned}
 & \frac{3 \sqrt{b} (a + b) \operatorname{ArcTanh}\left[ \frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}} \right]}{2 f} + \\
 & \frac{3 b \operatorname{Tan}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{2 f} - \frac{\operatorname{Cot}[e + f x] (a + b \operatorname{Tan}[e + f x]^2)^{3/2}}{f}
 \end{aligned}$$

Result (type 3, 310 leaves):

$$\left( \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right. \\ \left. \cos[e+fx]^3 \left( -\frac{i \left( 2a (1 + e^{2i(e+fx)})^2 + b (3 + 2 e^{2i(e+fx)} + 3 e^{4i(e+fx)}) \right)}{(-1 + e^{2i(e+fx)}) (1 + e^{2i(e+fx)})^2} \right) \right. \\ \left. \left( 3 \sqrt{b} (a+b) \operatorname{Log} \left[ \frac{1}{1 + e^{2i(e+fx)}} \left( -4 \sqrt{b} (-1 + e^{2i(e+fx)}) f + \right. \right. \right. \right. \\ \left. \left. \left. 4 i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) \right] \right) \left/ \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right) \right) \\ \left. (a + b \operatorname{Sec}[e+fx]^2)^{3/2} \right) \left/ (f (a + 2b + a \operatorname{Cos}[2e + 2fx])^{3/2}) \right)$$

**Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^4 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 172 leaves, 6 steps):

$$\frac{\sqrt{b} (3a + 5b) \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{2f} + \frac{b (3a + 5b) \operatorname{Tan}[e+fx] \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{2(a+b)f} \\ - \frac{(3a + 5b) \operatorname{Cot}[e+fx] (a + b + b \operatorname{Tan}[e+fx]^2)^{3/2}}{3(a+b)f} - \frac{\operatorname{Cot}[e+fx]^3 (a + b + b \operatorname{Tan}[e+fx]^2)^{5/2}}{3(a+b)f}$$

Result (type 3, 369 leaves):



$$\begin{aligned}
 & \left( \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right. \\
 & \left. \cos[e+fx]^3 \left( - \left( i \left( 4a (1 + e^{2i(e+fx)})^2 (1 - 4e^{2i(e+fx)} + e^{4i(e+fx)}) + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. b (15 - 20e^{2i(e+fx)} - 22e^{4i(e+fx)} - 20e^{6i(e+fx)} + 15e^{8i(e+fx)}) \right) \right) \right) / \\
 & \left( (-1 + e^{2i(e+fx)})^3 (1 + e^{2i(e+fx)})^2 \right) - \left( 3\sqrt{b} (3a + 5b) \operatorname{Log} \left[ \frac{1}{1 + e^{2i(e+fx)}} \right] \right. \\
 & \left. \left( -4\sqrt{b} (-1 + e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) \right) / \\
 & \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \left( a + b \operatorname{Sec}[e+fx]^2 \right)^{3/2} / \\
 & (3f (a + 2b + a \cos[2e + 2fx])^{3/2})
 \end{aligned}$$

**Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^6 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$\begin{aligned}
 & \frac{\sqrt{b} (3a + 7b) \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{2f} + \frac{b (3a + 7b) \operatorname{Tan}[e+fx] \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{2(a+b)f} - \\
 & \frac{(3a + 7b) \operatorname{Cot}[e+fx] (a + b \operatorname{Tan}[e+fx]^2)^{3/2}}{3(a+b)f} - \\
 & \frac{2 \operatorname{Cot}[e+fx]^3 (a + b \operatorname{Tan}[e+fx]^2)^{5/2}}{3(a+b)f} - \frac{\operatorname{Cot}[e+fx]^5 (a + b \operatorname{Tan}[e+fx]^2)^{5/2}}{5(a+b)f}
 \end{aligned}$$

Result (type 3, 512 leaves):

$$\frac{1}{15 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^{3/2}} \sqrt{2} e^{i (e + f x)}$$

$$\sqrt{4 b + a e^{-2 i (e + f x)} (1 + e^{2 i (e + f x)})^2} \operatorname{Cos}[e + f x]^3 \left( -\frac{1}{(a + b) (-1 + e^{2 i (e + f x)})^5 (1 + e^{2 i (e + f x)})^2} \right.$$

$$i (16 a^2 (1 + e^{2 i (e + f x)})^2 (1 - 6 e^{2 i (e + f x)} + 16 e^{4 i (e + f x)} - 6 e^{6 i (e + f x)} + e^{8 i (e + f x)}) +$$

$$b^2 (105 - 350 e^{2 i (e + f x)} + 231 e^{4 i (e + f x)} + 412 e^{6 i (e + f x)} + 231 e^{8 i (e + f x)} - 350 e^{10 i (e + f x)} +$$

$$105 e^{12 i (e + f x)}) + a b (115 - 402 e^{2 i (e + f x)} + 317 e^{4 i (e + f x)} + 708 e^{6 i (e + f x)} +$$

$$317 e^{8 i (e + f x)} - 402 e^{10 i (e + f x)} + 115 e^{12 i (e + f x)}) \left. \right) - \left( 15 \sqrt{b} (3 a + 7 b) \right.$$

$$\left. \operatorname{Log} \left[ \frac{1}{1 + e^{2 i (e + f x)}} \left( -4 \sqrt{b} (-1 + e^{2 i (e + f x)}) f + 4 i \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} f \right) \right] \right) /$$

$$\left( \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} \right) \left( a + b \operatorname{Sec}[e + f x]^2 \right)^{3/2}$$

**Problem 96: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}} \right]}{\sqrt{a+b} f}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Csc}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

**Problem 97: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]^3}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$\frac{a \operatorname{ArcTanh} \left[ \frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}} \right]}{2 (a+b)^{3/2} f} - \frac{\operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2}}{2 (a+b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Csc}[e + f x]^3}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

**Problem 98: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]^5}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\begin{aligned} & - \frac{3 a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{8 (a+b)^{5/2} f} - \frac{(5 a+2 b) \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{8 (a+b)^2 f} \\ & - \frac{\operatorname{Cot}[e+f x]^3 \operatorname{Csc}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{4 (a+b) f} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Csc}[e + f x]^5}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

**Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^6}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 193 leaves, 7 steps):

$$\begin{aligned} & \frac{5 (a+b)^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{16 a^{7/2} f} - \frac{1}{48 a^3 f} \\ & + \frac{(33 a^2 + 40 a b + 15 b^2) \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] \sqrt{a+b \operatorname{Tan}[e+f x]^2}}{24 a^2 f} + \\ & + \frac{(9 a + 5 b) \operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x] \sqrt{a+b \operatorname{Tan}[e+f x]^2}}{6 a f} \\ & + \frac{\operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]^3 \sqrt{a+b \operatorname{Tan}[e+f x]^2}}{6 a f} \end{aligned}$$

Result (type 3, 2258 leaves):

$$\begin{aligned} & \frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+2 b+a \operatorname{Cos}[2 (e+f x)]}}\right] \sqrt{a+2 b+a \operatorname{Cos}[2 (e+f x)]} \operatorname{Sec}[e+f x]}{64 \sqrt{2} \sqrt{a} f \sqrt{a+b \operatorname{Sec}[e+f x]^2}} \end{aligned}$$

$$\frac{1}{1536 \sqrt{2} a^{7/2} f \sqrt{a+b \operatorname{Sec}[e+f x]^2}} e^{-i(13e+5fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2}$$

$$\sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \left( (1+e^{14ie}) \left( i \sqrt{a} (-60b^2 e^{4ifx} (-e^{12ie} + e^{2ifx}) + \right. \right.$$

$$10ab e^{2ifx} (-e^{10ie} - 6e^{4ifx} + 6e^{2i(6e+fx)} + e^{2i(e+3fx)}) +$$

$$a^2 (2e^{8ie} - 11e^{6ifx} + 11e^{4i(3e+fx)} - 5e^{2i(5e+fx)} + 5e^{2i(e+4fx)} - 2e^{2i(2e+5fx)}) \Big) -$$

$$\left( 6(a^3 + 12a^2b + 30ab^2 + 20b^3) e^{6ifx} \left( -i \operatorname{Log}[e^{-2ie} (a+2b+a e^{2i(e+fx)}) + \right. \right.$$

$$\left. \left. \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] + e^{14ie} \left( 2fx + i \operatorname{Log}[e^{-2ie} (a + \right. \right.$$

$$\left. \left. a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right) \Big) \Big) \Big) /$$

$$\left( \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) + (-1+e^{14ie}) \left( -i \sqrt{a} \right.$$

$$(60b^2 e^{4ifx} (e^{12ie} + e^{2ifx}) - 10ab e^{2ifx} (e^{10ie} - 6e^{4ifx} - 6e^{2i(6e+fx)} + e^{2i(e+3fx)}) +$$

$$a^2 (2e^{8ie} + 11e^{6ifx} + 11e^{4i(3e+fx)} - 5e^{2i(5e+fx)} - 5e^{2i(e+4fx)} + 2e^{2i(2e+5fx)}) \Big) +$$

$$\left( 6(a^3 + 12a^2b + 30ab^2 + 20b^3) e^{6ifx} \left( i \operatorname{Log}[e^{-2ie} (a+2b+a e^{2i(e+fx)}) + \right. \right.$$

$$\left. \left. \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] + e^{14ie} \left( 2fx + i \operatorname{Log}[e^{-2ie} (a + \right. \right.$$

$$\left. \left. a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right) \Big) \Big) \Big) /$$

$$\left( \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \Big) \Big) \Big) \operatorname{Sec}[e+f x] +$$

$$\frac{1}{128 \sqrt{2} a^{3/2} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} f \sqrt{a+b \operatorname{Sec}[e+f x]^2}}$$

9

$$e^{-i(e+fx)}$$

$$\sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2}$$

$$\sqrt{a+2b+a \operatorname{Cos}[2e+2fx]}$$

$$\left( -\sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} + \right.$$

$$\left. \sqrt{a} e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} - \right.$$

$$\begin{aligned}
 & 2 i a e^{2 i (e+f x)} f x - \\
 & 4 i b e^{2 i (e+f x)} f x - \\
 & (a+2 b) e^{2 i (e+f x)} \operatorname{Log}\left[e^{-2 i e}\left(a+2 b+a e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right]+ \\
 & (a+2 b) e^{2 i (e+f x)} \\
 & \operatorname{Log}\left[e^{-2 i e}\left(a+a e^{2 i (e+f x)}+2 b e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right] \\
 & \operatorname{Sec}[e+f x]+\frac{1}{256 \sqrt{2} a^{5 / 2} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f \sqrt{a+b \operatorname{Sec}[e+f x]^2}} \\
 5 & \\
 & e^{-3 i (e+f x)} \\
 & \sqrt{4 b+a e^{-2 i (e+f x)}\left(1+e^{2 i (e+f x)}\right)^2} \\
 & \sqrt{a+2 b+a \operatorname{Cos}[2 e+2 f x]} \\
 & \left(i a^{3 / 2} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}-\right. \\
 & 3 i a^{3 / 2} e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}- \\
 & 6 i \sqrt{a} b e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}+ \\
 & 3 i a^{3 / 2} e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}+ \\
 & 6 i \sqrt{a} b e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}- \\
 & i a^{3 / 2} e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}+4 a^2 e^{4 i (e+f x)} f x+ \\
 & 24 a b e^{4 i (e+f x)} f x+24 b^2 e^{4 i (e+f x)} f x-2 i\left(a^2+6 a b+6 b^2\right) e^{4 i (e+f x)} \\
 & \operatorname{Log}\left[e^{-2 i e}\left(a+2 b+a e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right]+ \\
 & 2 i\left(a^2+6 a b+6 b^2\right) e^{4 i (e+f x)} \operatorname{Log}\left[ \right. \\
 & \left. e^{-2 i e}\left(a+a e^{2 i (e+f x)}+2 b e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right] \operatorname{Sec}[e+f x]
 \end{aligned}$$

Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin[e + f x]^4}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\frac{3 (a + b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}}\right]}{8 a^{5/2} f} - \frac{(5 a + 3 b) \cos[e + f x] \sin[e + f x] \sqrt{a + b \tan[e + f x]^2}}{8 a^2 f} + \frac{\cos[e + f x]^3 \sin[e + f x] \sqrt{a + b \tan[e + f x]^2}}{4 a f}$$

Result (type 3, 1286 leaves):

$$\begin{aligned}
 & \frac{\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{a}\sin[e+fx]}{\sqrt{a+2b+a\cos[2e+2fx]}}\right]\sqrt{a+2b+a\cos[2e+2fx]}\sec[e+fx]}{8\sqrt{2}\sqrt{a}f\sqrt{a+b\sec[e+fx]^2}} + \\
 & \frac{1}{32\sqrt{2}a^{3/2}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f\sqrt{a+b\sec[e+fx]^2}} \\
 & \frac{3i e^{-i(e+fx)}\sqrt{4b+a e^{-2i(e+fx)}(1+e^{2i(e+fx)})^2}\sqrt{a+2b+a\cos[2e+2fx]}}{\left(-\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}+\sqrt{a}e^{2i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}-\right. \\
 & \quad \left.2ia e^{2i(e+fx)}fx-4ib e^{2i(e+fx)}fx-(a+2b)e^{2i(e+fx)}\text{Log}\left[e^{-2ie}\left(a+2b+a e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]+(a+2b)e^{2i(e+fx)}\right. \\
 & \quad \left.\text{Log}\left[e^{-2ie}\left(a+a e^{2i(e+fx)}+2b e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right)} \\
 & \frac{1}{\text{Sec}[e+fx]} + \frac{1}{64\sqrt{2}a^{5/2}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f\sqrt{a+b\sec[e+fx]^2}} \\
 & \frac{e^{-3i(e+fx)}\sqrt{4b+a e^{-2i(e+fx)}(1+e^{2i(e+fx)})^2}\sqrt{a+2b+a\cos[2e+2fx]}}{\left(i a^{3/2}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}-3i a^{3/2}e^{2i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}-\right. \\
 & \quad \left.6i\sqrt{a}b e^{2i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}+3i a^{3/2}e^{4i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}-\right. \\
 & \quad \left.\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}+6i\sqrt{a}b e^{4i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}-\right. \\
 & \quad \left.ia^{3/2}e^{6i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}+4a^2e^{4i(e+fx)}fx+\right. \\
 & \quad \left.24ab e^{4i(e+fx)}fx+24b^2e^{4i(e+fx)}fx-2i(a^2+6ab+6b^2)e^{4i(e+fx)}\right. \\
 & \quad \left.\text{Log}\left[e^{-2ie}\left(a+2b+a e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]+2i(a^2+6ab+6b^2)e^{4i(e+fx)}\right. \\
 & \quad \left.\text{Log}\left[e^{-2ie}\left(a+a e^{2i(e+fx)}+2b e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right)\text{Sec}[e+fx]
 \end{aligned}$$

**Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^2}{\sqrt{a+b \sec[e+fx]^2}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{2 a^{3/2} f} - \frac{\cos[e+fx] \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{2 a f}$$

Result (type 3, 558 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} \sin[e+fx]}{\sqrt{a+2 b+a \cos[2 e+2 f x]}}\right] \sqrt{a+2 b+a \cos[2 e+2 f x]} \sec[e+fx]}{4 \sqrt{2} \sqrt{a} f \sqrt{a+b \sec[e+fx]^2}} +$$

$$\frac{1}{8 \sqrt{2} a^{3/2} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} f \sqrt{a+b \sec[e+fx]^2}}$$

$$i e^{-i(e+fx)} \sqrt{4 b + a e^{-2 i(e+fx)} (1 + e^{2 i(e+fx)})^2} \sqrt{a+2 b+a \cos[2 e+2 f x]}$$

$$\left( -\sqrt{a} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} + \sqrt{a} e^{2 i(e+fx)} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} - \right.$$

$$2 i a e^{2 i(e+fx)} f x - 4 i b e^{2 i(e+fx)} f x - (a+2 b) e^{2 i(e+fx)} \operatorname{Log}\left[e^{-2 i e} \right.$$

$$\left. \left( a+2 b+a e^{2 i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} \right) \right] + (a+2 b) e^{2 i(e+fx)} \operatorname{Log}\left[ \right.$$

$$\left. e^{-2 i e} \left( a+a e^{2 i(e+fx)} + 2 b e^{2 i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} \right) \right] \right) \sec[e+fx]$$

**Problem 102: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a+b \sec[e+fx]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{\sqrt{a} f}$$

Result (type 8, 18 leaves):



$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Problem 109: Unable to integrate problem.

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{(a+b)^{3/2} f} - \frac{b \operatorname{Sec}[e+fx]}{a(a+b) f \sqrt{a+b \operatorname{Sec}[e+fx]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Problem 110: Unable to integrate problem.

$$\int \frac{\operatorname{Csc}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 126 leaves, 6 steps):

$$-\frac{(a-2b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{2(a+b)^{5/2} f} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]}{2(a+b) f \sqrt{a+b \operatorname{Sec}[e+fx]^2}} - \frac{3b \operatorname{Sec}[e+fx]}{2(a+b)^2 f \sqrt{a+b \operatorname{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Csc}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Problem 111: Unable to integrate problem.

$$\int \frac{\operatorname{Csc}[e + f x]^5}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 177 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 a (a - 4 b) \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{8 (a+b)^{7/2} f} - \frac{5 a \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]}{8 (a+b)^2 f \sqrt{a+b \operatorname{Sec}[e+f x]^2}} - \\
& \frac{\operatorname{Cot}[e+f x]^3 \operatorname{Csc}[e+f x]}{4 (a+b) f \sqrt{a+b \operatorname{Sec}[e+f x]^2}} - \frac{(13 a - 2 b) b \operatorname{Sec}[e+f x]}{8 (a+b)^3 f \sqrt{a+b \operatorname{Sec}[e+f x]^2}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Csc}[e+f x]^5}{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}} dx$$

**Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}} dx$$

Optimal (type 3, 242 leaves, 8 steps):

$$\begin{aligned}
& \frac{5 (a+b)^2 (a+7 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{16 a^{9/2} f} - \\
& \frac{(a+b) (33 a+35 b) \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{48 a^3 f \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}} + \frac{(9 a+7 b) \operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]}{24 a^2 f \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}} + \\
& \frac{\operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]^3}{6 a f \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}} - \frac{b (81 a^2+190 a b+105 b^2) \operatorname{Tan}[e+f x]}{48 a^4 f \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}}
\end{aligned}$$

Result (type 3, 3051 leaves):

$$\begin{aligned}
& \left( 3 e^{-3 i (e+f x)} \sqrt{4 b+a e^{-2 i (e+f x)} \left(1+e^{2 i (e+f x)}\right)^2 (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3} \right)^{3/2} \\
& \left( i a^{7/2} + i a^{5/2} b - 5 i a^{7/2} e^{2 i (e+f x)} - 15 i a^{5/2} b e^{2 i (e+f x)} - 10 i a^{3/2} b^2 e^{2 i (e+f x)} - \right. \\
& 13 i a^{7/2} e^{4 i (e+f x)} - 104 i a^{5/2} b e^{4 i (e+f x)} - 210 i a^{3/2} b^2 e^{4 i (e+f x)} - 120 i \sqrt{a} b^3 e^{4 i (e+f x)} + \\
& 13 i a^{7/2} e^{6 i (e+f x)} + 104 i a^{5/2} b e^{6 i (e+f x)} + 210 i a^{3/2} b^2 e^{6 i (e+f x)} + 120 i \sqrt{a} b^3 e^{6 i (e+f x)} + \\
& 5 i a^{7/2} e^{8 i (e+f x)} + 15 i a^{5/2} b e^{8 i (e+f x)} + 10 i a^{3/2} b^2 e^{8 i (e+f x)} - i a^{7/2} e^{10 i (e+f x)} - \\
& i a^{5/2} b e^{10 i (e+f x)} + 24 a^3 e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a \left(1+e^{2 i (e+f x)}\right)^2} f x + \\
& 144 a^2 b e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a \left(1+e^{2 i (e+f x)}\right)^2} f x + \\
& 240 a b^2 e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a \left(1+e^{2 i (e+f x)}\right)^2} f x +
\end{aligned}$$

$$\begin{aligned}
 & 120 b^3 e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f x - \\
 & 12 i\left(a^3+6 a^2 b+10 a b^2+5 b^3\right) e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} \\
 & \operatorname{Log}\left[e^{-2 i e}\left(a+2 b+a e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right]+ \\
 & 12 i\left(a^3+6 a^2 b+10 a b^2+5 b^3\right) e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} \\
 & \operatorname{Log}\left[e^{-2 i e}\left(a+a e^{2 i (e+f x)}+2 b e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right] \\
 & \operatorname{Sec}[e+f x]^3\left/\left(512 \sqrt{2} a^{7 / 2}(a+b)\left(4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2\right)\right.\right. \\
 & \left.\left.f\left(a+b \operatorname{Sec}[e+f x]^2\right)^{3 / 2}\right)+\right. \\
 & \left(i e^{-5 i (e+f x)} \sqrt{4 b+a e^{-2 i (e+f x)}\left(1+e^{2 i (e+f x)}\right)^2}\left(a+2 b+a \operatorname{Cos}[2 e+2 f x]\right)^{3 / 2}\right. \\
 & \left.-2 a^{9 / 2}-2 a^{7 / 2} b+7 a^{9 / 2} e^{2 i (e+f x)}+21 a^{7 / 2} b e^{2 i (e+f x)}+14 a^{5 / 2} b^2 e^{2 i (e+f x)}-\right. \\
 & 27 a^{9 / 2} e^{4 i (e+f x)}-167 a^{7 / 2} b e^{4 i (e+f x)}-280 a^{5 / 2} b^2 e^{4 i (e+f x)}-140 a^{3 / 2} b^3 e^{4 i (e+f x)}- \\
 & 63 a^{9 / 2} e^{6 i (e+f x)}-790 a^{7 / 2} b e^{6 i (e+f x)}-2830 a^{5 / 2} b^2 e^{6 i (e+f x)}-3780 a^{3 / 2} b^3 e^{6 i (e+f x)}- \\
 & 1680 \sqrt{a} b^4 e^{6 i (e+f x)}+63 a^{9 / 2} e^{8 i (e+f x)}+790 a^{7 / 2} b e^{8 i (e+f x)}+2830 a^{5 / 2} b^2 e^{8 i (e+f x)}+ \\
 & 3780 a^{3 / 2} b^3 e^{8 i (e+f x)}+1680 \sqrt{a} b^4 e^{8 i (e+f x)}+27 a^{9 / 2} e^{10 i (e+f x)}+ \\
 & 167 a^{7 / 2} b e^{10 i (e+f x)}+280 a^{5 / 2} b^2 e^{10 i (e+f x)}+140 a^{3 / 2} b^3 e^{10 i (e+f x)}- \\
 & 7 a^{9 / 2} e^{12 i (e+f x)}-21 a^{7 / 2} b e^{12 i (e+f x)}-14 a^{5 / 2} b^2 e^{12 i (e+f x)}+2 a^{9 / 2} e^{14 i (e+f x)}+ \\
 & \left.2 a^{7 / 2} b e^{14 i (e+f x)}-120 i a^4 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f x -\right. \\
 & 1200 i a^3 b e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f x - \\
 & 3600 i a^2 b^2 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f x - \\
 & 4200 i a b^3 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f x - \\
 & 1680 i b^4 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} f x - \\
 & 60\left(a^4+10 a^3 b+30 a^2 b^2+35 a b^3+14 b^4\right) e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2} \\
 & \operatorname{Log}\left[e^{-2 i e}\left(a+2 b+a e^{2 i (e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i (e+f x)}+a\left(1+e^{2 i (e+f x)}\right)^2}\right)\right]+
 \end{aligned}$$

$$\begin{aligned}
& 60 (a^4 + 10 a^3 b + 30 a^2 b^2 + 35 a b^3 + 14 b^4) e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \\
& \operatorname{Log}\left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] \\
& \operatorname{Sec}[e + f x]^3 \Big/ \left(1536 \sqrt{2} a^{9/2} (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)\right. \\
& \left. f (a + b \operatorname{Sec}[e + f x]^2)^{3/2}\right) - \\
& \left(e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}\right. \\
& \left(-3 i a^{3/2} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} - 4 i \sqrt{a} b \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \right. \\
& \left. 3 i a^{3/2} e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \right. \\
& \left. 4 i \sqrt{a} b e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + 4 a^2 f x + 4 a b f x + \right. \\
& \left. 8 a^2 e^{2 i (e+f x)} f x + 24 a b e^{2 i (e+f x)} f x + 16 b^2 e^{2 i (e+f x)} f x + 4 a^2 e^{4 i (e+f x)} f x + \right. \\
& \left. 4 a b e^{4 i (e+f x)} f x - 2 i (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)\right. \\
& \left. \operatorname{Log}\left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] + \right. \\
& \left. 2 i (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)\right. \\
& \left. \operatorname{Log}\left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] \right) \\
& \operatorname{Sec}[e + f x]^3 \Big/ \left(64 \sqrt{2} a^{3/2} (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)\right)^{3/2} \\
& f (a + b \operatorname{Sec}[e + f x]^2)^{3/2} + \\
& \frac{3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2} \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{256 (a + b) f \sqrt{a + 2 b + a \operatorname{Cos}[2 (e + f x)]} (a + b \operatorname{Sec}[e + f x]^2)^{3/2}}
\end{aligned}$$

**Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^4}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{3(a+b)(a+5b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8a^{7/2}f} - \frac{5(a+b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{8a^2f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} +$$

$$\frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{4af \sqrt{a+b \operatorname{Tan}[e+fx]^2}} - \frac{b(13a+15b) \operatorname{Tan}[e+fx]}{8a^3f \sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 3, 2543 leaves):

$$\left( i e^{-i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \operatorname{Cos}[2e+2fx])^{3/2} \right.$$

$$\left( -2a^{5/2} - 2a^{3/2}b - 7a^{5/2}e^{2i(e+fx)} - 30a^{3/2}b e^{2i(e+fx)} - 24\sqrt{a} b^2 e^{2i(e+fx)} + \right.$$

$$\left. 7a^{5/2}e^{4i(e+fx)} + 30a^{3/2}b e^{4i(e+fx)} + 24\sqrt{a} b^2 e^{4i(e+fx)} + 2a^{5/2}e^{6i(e+fx)} + \right.$$

$$\left. 2a^{3/2}b e^{6i(e+fx)} - 12i a^2 e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} f x - \right.$$

$$\left. 36i a b e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} f x - \right.$$

$$\left. 24i b^2 e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} f x - \right.$$

$$\left. 6(a^2+3ab+2b^2) e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right.$$

$$\left. \operatorname{Log}\left[ e^{-2i} e \left( a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \right] + \right.$$

$$\left. 6(a^2+3ab+2b^2) e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right.$$

$$\left. \operatorname{Log}\left[ e^{-2i} e \left( a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \right] \right]$$

$$\operatorname{Sec}[e+fx]^3 \left/ \left( 128\sqrt{2} a^{5/2} (a+b) \left( 4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2 \right) \right) \right.$$

$$\left. f (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) +$$

$$\left( e^{-3i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \operatorname{Cos}[2e+2fx])^{3/2} \right.$$

$$\left( i a^{7/2} + i a^{5/2}b - 5i a^{7/2}e^{2i(e+fx)} - 15i a^{5/2}b e^{2i(e+fx)} - 10i a^{3/2}b^2 e^{2i(e+fx)} - \right.$$

$$\left. 13i a^{7/2}e^{4i(e+fx)} - 104i a^{5/2}b e^{4i(e+fx)} - 210i a^{3/2}b^2 e^{4i(e+fx)} - 120i \sqrt{a} b^3 e^{4i(e+fx)} + \right.$$

$$\left. 13i a^{7/2}e^{6i(e+fx)} + 104i a^{5/2}b e^{6i(e+fx)} + 210i a^{3/2}b^2 e^{6i(e+fx)} + 120i \sqrt{a} b^3 e^{6i(e+fx)} + \right.$$

$$\left. 5i a^{7/2}e^{8i(e+fx)} + 15i a^{5/2}b e^{8i(e+fx)} + 10i a^{3/2}b^2 e^{8i(e+fx)} - i a^{7/2}e^{10i(e+fx)} - \right.$$

$$\begin{aligned}
& i a^{5/2} b e^{10 i (e+f x)} + 24 a^3 e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x + \\
& 144 a^2 b e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x + \\
& 240 a b^2 e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x + \\
& 120 b^3 e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x - \\
& 12 i (a^3 + 6 a^2 b + 10 a b^2 + 5 b^3) e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \\
& \operatorname{Log}\left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] + \\
& 12 i (a^3 + 6 a^2 b + 10 a b^2 + 5 b^3) e^{4 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \\
& \operatorname{Log}\left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] \Big] \\
& \operatorname{Sec}[e + f x]^3 \Big/ \left(128 \sqrt{2} a^{7/2} (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)\right) \\
& f (a + b \operatorname{Sec}[e + f x]^2)^{3/2} - \\
& \left(3 e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}\right. \\
& \left(-3 i a^{3/2} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} - 4 i \sqrt{a} b \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \right. \\
& 3 i a^{3/2} e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \\
& 4 i \sqrt{a} b e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + 4 a^2 f x + 4 a b f x + \\
& 8 a^2 e^{2 i (e+f x)} f x + 24 a b e^{2 i (e+f x)} f x + 16 b^2 e^{2 i (e+f x)} f x + 4 a^2 e^{4 i (e+f x)} f x + \\
& 4 a b e^{4 i (e+f x)} f x - 2 i (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2) \\
& \left. \operatorname{Log}\left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] + \right. \\
& \left. 2 i (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)\right. \\
& \left. \operatorname{Log}\left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] \Big] \right) \\
& \operatorname{Sec}[e + f x]^3 \Big/ \left(128 \sqrt{2} a^{3/2} (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)^{3/2}\right)
\end{aligned}$$

$$\frac{f (a + b \operatorname{Sec}[e + f x]^2)^{3/2} + 3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2} \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{128 (a + b) f \sqrt{a + 2 b + a \operatorname{Cos}[2 (e + f x)]} (a + b \operatorname{Sec}[e + f x]^2)^{3/2}}$$

**Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^2}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 121 leaves, 6 steps):

$$\frac{(a + 3 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{2 a^{5/2} f} - \frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{2 a f \sqrt{a + b \operatorname{Tan}[e + f x]^2}} - \frac{3 b \operatorname{Tan}[e + f x]}{2 a^2 f \sqrt{a + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 3, 1522 leaves):

$$\begin{aligned} & \left( i e^{-i (e + f x)} \sqrt{4 b + a e^{-2 i (e + f x)}} (1 + e^{2 i (e + f x)})^2 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2} \right. \\ & \left( -2 a^{5/2} - 2 a^{3/2} b - 7 a^{5/2} e^{2 i (e + f x)} - 30 a^{3/2} b e^{2 i (e + f x)} - 24 \sqrt{a} b^2 e^{2 i (e + f x)} + \right. \\ & \left. 7 a^{5/2} e^{4 i (e + f x)} + 30 a^{3/2} b e^{4 i (e + f x)} + 24 \sqrt{a} b^2 e^{4 i (e + f x)} + 2 a^{5/2} e^{6 i (e + f x)} + \right. \\ & \left. 2 a^{3/2} b e^{6 i (e + f x)} - 12 i a^2 e^{2 i (e + f x)} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} f x - \right. \\ & \left. 36 i a b e^{2 i (e + f x)} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} f x - \right. \\ & \left. 24 i b^2 e^{2 i (e + f x)} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} f x - \right. \\ & \left. 6 (a^2 + 3 a b + 2 b^2) e^{2 i (e + f x)} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} \right. \\ & \left. \operatorname{Log}\left[e^{-2 i e} \left( a + 2 b + a e^{2 i (e + f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} \right) \right] + \right. \\ & \left. 6 (a^2 + 3 a b + 2 b^2) e^{2 i (e + f x)} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} \right. \\ & \left. \operatorname{Log}\left[e^{-2 i e} \left( a + a e^{2 i (e + f x)} + 2 b e^{2 i (e + f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} \right) \right] \right) \\ & \operatorname{Sec}[e + f x]^3 \Big/ \left( 32 \sqrt{2} a^{5/2} (a + b) (4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2) \right) \\ & f (a + b \operatorname{Sec}[e + f x]^2)^{3/2} - \end{aligned}$$

$$\begin{aligned}
 & \left( e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 (a+2b+a \cos[2e+2fx])^{3/2} \right. \\
 & \left( -3i a^{3/2} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} - 4i \sqrt{a} b \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} + \right. \\
 & 3i a^{3/2} e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} + \\
 & 4i \sqrt{a} b e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} + 4a^2 f x + 4ab f x + \\
 & 8a^2 e^{2i(e+fx)} f x + 24ab e^{2i(e+fx)} f x + 16b^2 e^{2i(e+fx)} f x + 4a^2 e^{4i(e+fx)} f x + \\
 & 4ab e^{4i(e+fx)} f x - 2i(a+b) (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2) \\
 & \left. \left. \log \left[ e^{-2ie} \left( a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \right. \right. \\
 & 2i(a+b) (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2) \\
 & \left. \left. \log \left[ e^{-2ie} \left( a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] \right] \right) \\
 & \sec[e+fx]^3 \Big/ \left( 32\sqrt{2} a^{3/2} (a+b) (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \right. \\
 & \left. f (a+b \sec[e+fx]^2)^{3/2} \right) + \\
 & \frac{(a+2b+a \cos[2e+2fx])^{3/2} \sec[e+fx]^2 \tan[e+fx]}{16(a+b) f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b \sec[e+fx]^2)^{3/2}}
 \end{aligned}$$

**Problem 115: Unable to integrate problem.**

$$\int \frac{1}{(a+b \sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\text{ArcTan} \left[ \frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}} \right]}{a^{3/2} f} - \frac{b \tan[e+fx]}{a(a+b) f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{(a+b \sec[e+fx]^2)^{3/2}} dx$$



### Problem 122: Unable to integrate problem.

$$\int \frac{\text{Csc}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b} \text{Sec}[e+fx]}{\sqrt{a+b \text{Sec}[e+fx]^2}}\right]}{(a+b)^{5/2} f} - \frac{b \text{Sec}[e+fx]}{3 a (a+b) f (a+b \text{Sec}[e+fx]^2)^{3/2}} - \frac{b (5 a+2 b) \text{Sec}[e+fx]}{3 a^2 (a+b)^2 f \sqrt{a+b \text{Sec}[e+fx]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Csc}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

### Problem 123: Unable to integrate problem.

$$\int \frac{\text{Csc}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$-\frac{(a-4b) \text{ArcTanh}\left[\frac{\sqrt{a+b} \text{Sec}[e+fx]}{\sqrt{a+b \text{Sec}[e+fx]^2}}\right]}{2 (a+b)^{7/2} f} - \frac{\text{Cot}[e+fx] \text{Csc}[e+fx]}{2 (a+b) f (a+b \text{Sec}[e+fx]^2)^{3/2}} - \frac{5 b \text{Sec}[e+fx]}{6 (a+b)^2 f (a+b \text{Sec}[e+fx]^2)^{3/2}} - \frac{(13 a-2 b) b \text{Sec}[e+fx]}{6 a (a+b)^3 f \sqrt{a+b \text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Csc}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

### Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 234 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(3 a^2 - 24 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{8 (a+b)^{9/2} f} - \\
 & \frac{(5 a - 2 b) \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]}{8 (a+b)^2 f (a+b \operatorname{Sec}[e+f x]^2)^{3/2}} - \frac{\operatorname{Cot}[e+f x]^3 \operatorname{Csc}[e+f x]}{4 (a+b) f (a+b \operatorname{Sec}[e+f x]^2)^{3/2}} - \\
 & \frac{(23 a - 12 b) b \operatorname{Sec}[e+f x]}{24 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^{3/2}} - \frac{5 (11 a - 10 b) b \operatorname{Sec}[e+f x]}{24 (a+b)^4 f \sqrt{a+b \operatorname{Sec}[e+f x]^2}}
 \end{aligned}$$

Result (type 6, 1709 leaves):

$$\begin{aligned}
 & - \left( \left( 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cot}[e+f x]^4 \operatorname{Csc}[e+f x]^3 \right) / \right. \\
 & \left( 20 \sqrt{2} f (a+b \operatorname{Sec}[e+f x]^2)^{5/2} (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \right. \\
 & \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] - \right. \\
 & 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] + \\
 & \left. \left. 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Sin}[e+f x]^2 \right) \right. \\
 & \left. - \left( \left( 7 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Cos}[e+f x]^4 \operatorname{Cot}[e+f x] \right) / \left( 4 \sqrt{2} (a+b-a \operatorname{Sin}[e+f x]^2)^{7/2} \right. \right. \right. \\
 & \left. \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] - \right. \right. \\
 & 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] + \\
 & \left. \left. 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Sin}[e+f x]^2 \right) \right) \right) + \\
 & \left( 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cos}[e+f x]^2 \right. \\
 & \left. \operatorname{Cot}[e+f x] \right) / \left( 10 \sqrt{2} (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \right. \\
 & \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] - \right. \\
 & 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] + \\
 & \left. \left. 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Sin}[e+f x]^2 \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cot}[e+f x]^3 \right) / \\
 & \left( 10 \sqrt{2} (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \right. \\
 & \left. \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] - \right. \right. \\
 & \quad 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] + \\
 & \quad \left. \left. 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Sin}[e+f x]^2 \right) \right) - \\
 & \left( 7 a \operatorname{Cos}[e+f x]^2 \operatorname{Cot}[e+f x]^2 \left( -\frac{1}{7 a} 25 (a+b) f \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \right. \\
 & \quad \left. \frac{20}{7} f \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \right. \\
 & \quad \left. \left. \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right) \right) / \left( 20 \sqrt{2} f (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \right) \\
 & \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] - \right. \\
 & \quad 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] + \\
 & \quad \left. \left. 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Sin}[e+f x]^2 \right) \right) + \\
 & \left( 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cos}[e+f x]^2 \right. \\
 & \quad \left. \operatorname{Cot}[e+f x]^2 \left( 5 (a+b) \left( -\frac{1}{9 a} 49 (a+b) f \operatorname{AppellF1}\left[\frac{9}{2}, -2, \frac{9}{2}, \frac{11}{2}, \operatorname{Csc}[e+f x]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{28}{9} f \operatorname{AppellF1}\left[\frac{9}{2}, -1, \frac{7}{2}, \right. \right. \\
 & \quad \left. \left. \frac{11}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right) - \right. \\
 & \quad \left. 4 a \left( -\frac{1}{9 a} 35 (a+b) f \operatorname{AppellF1}\left[\frac{9}{2}, -1, \frac{7}{2}, \frac{11}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \right. \right. \\
 & \quad \left. \left. \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{14}{9} f \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right. \right. \\
 & \quad \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{9}{2}, \frac{11}{2}, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \right) + 14 a f \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \\
 & \quad \left. \left. -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] + \right.
 \end{aligned}$$

$$\begin{aligned}
& 7 a \left( -\frac{1}{7 a} 25 (a+b) f \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \right. \\
& \quad \left. \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{20}{7} f \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \right. \right. \\
& \quad \quad \left. \left. \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right) \operatorname{Sin}[e+f x]^2 \Big) / \\
& \left( 20 \sqrt{2} f (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \right. \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] - 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] + 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \right. \right. \\
& \quad \left. \left. \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a}\right] \operatorname{Sin}[e+f x]^2 \right)^2 \Big) \Big) \Big) \Big) \Big) \Big) \Big)
\end{aligned}$$

Problem 125: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Sin}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x]^2)^{5/2}} dx$$

Optimal (type 3, 288 leaves, 9 steps):

$$\begin{aligned}
& \frac{5 (a+b) (a^2+14 a b+21 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{16 a^{11/2} f} - \\
& \frac{(a+b) (11 a+21 b) \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{16 a^3 f (a+b \operatorname{Tan}[e+f x]^2)^{3/2}} + \\
& \frac{3 (a+b) \operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]}{8 a^2 f (a+b \operatorname{Tan}[e+f x]^2)^{3/2}} + \frac{\operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]^3}{6 a f (a+b \operatorname{Tan}[e+f x]^2)^{3/2}} - \\
& \frac{7 b (a+b) (7 a+15 b) \operatorname{Tan}[e+f x]}{48 a^4 f (a+b \operatorname{Tan}[e+f x]^2)^{3/2}} - \frac{b (113 a^2+420 a b+315 b^2) \operatorname{Tan}[e+f x]}{48 a^5 f \sqrt{a+b \operatorname{Tan}[e+f x]^2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 126: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sin}[e+f x]^4}{(a+b \operatorname{Sec}[e+f x]^2)^{5/2}} dx$$

Optimal (type 3, 227 leaves, 8 steps):

$$\frac{(3 a^2 + 30 a b + 35 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{8 a^{9/2} f} - \frac{(5 a + 7 b) \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{8 a^2 f (a+b+b \operatorname{Tan}[e+f x]^2)^{3/2}} + \frac{\operatorname{Cos}[e+f x]^3 \operatorname{Sin}[e+f x]}{4 a f (a+b+b \operatorname{Tan}[e+f x]^2)^{3/2}} - \frac{b(23 a+35 b) \operatorname{Tan}[e+f x]}{24 a^3 f (a+b+b \operatorname{Tan}[e+f x]^2)^{3/2}} - \frac{5 b(11 a+21 b) \operatorname{Tan}[e+f x]}{24 a^4 f \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}}$$

Result (type 3, 6006 leaves):

$$\frac{1}{256 (a+b \operatorname{Sec}[e+f x]^2)^{5/2}} \left( (a+2 b+a \operatorname{Cos}[2 e+2 f x])^{5/2} \left( -\frac{1}{24 \sqrt{2} a^4 b^2 (a+b)^2 \left(4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2\right)^2 f} \right. \right. \\ \left. \left. i e^{-i(17 e+3 f x)}\left(-1+e^{18 i e}\right) \sqrt{4 b+a e^{-2 i(e+f x)}\left(1+e^{2 i(e+f x)}\right)^2} \right. \right. \\ \left. \left. \begin{aligned} &(-13440 b^7 e^{4 i e+6 i f x}\left(e^{16 i e}+e^{2 i f x}\right)+a^7 e^{4 i f x}\left(1+e^{18 i e}\right)\left(1+e^{2 i(e+f x)}\right)^3 - \\ &6 a^6 b e^{4 i f x}\left(1+e^{18 i e}\right)\left(3+8 e^{2 i(e+f x)}+8 e^{4 i(e+f x)}+3 e^{6 i(e+f x)}\right)-2240 a b^6 e^{2 i(e+2 f x)} \\ &\left(2 e^{16 i e}+3 e^{2 i f x}+2 e^{4 i e+6 i f x}+3 e^{4 i(5 e+f x)}+21 e^{2 i(9 e+f x)}+21 e^{2 i(e+2 f x)}\right)- \\ &24 a^2 b^5 e^{2 i f x}\left(7 e^{16 i e}+35 e^{2 i f x}+2710 e^{4 i e+6 i f x}+840 e^{22 i e+6 i f x}+560 e^{6 i e+8 i f x}+ \\ &35 e^{8 i(3 e+f x)}+2710 e^{4 i(5 e+f x)}+560 e^{2 i(9 e+f x)}+840 e^{2 i(e+2 f x)}+7 e^{2 i(4 e+5 f x)}\right)- \\ &4 a^4 b^3\left(-6 e^{14 i e}+430 e^{4 i f x}+3987 e^{20 i e+6 i f x}+2610 e^{22 i e+8 i f x}+84 e^{2 i(8 e+f x)}+ \\ &3987 e^{4 i(e+2 f x)}+1684 e^{2 i(9 e+2 f x)}+2610 e^{2 i(e+3 f x)}+84 e^{4 i(2 e+3 f x)}+ \\ &1684 e^{2 i(3 e+5 f x)}+430 e^{2 i(12 e+5 f x)}-6 e^{2 i(5 e+7 f x)}\right)-6 a^5 b^2\left(-2 e^{14 i e}+80 e^{4 i f x}+ \\ &407 e^{20 i e+6 i f x}+318 e^{22 i e+8 i f x}+14 e^{2 i(8 e+f x)}+407 e^{4 i(e+2 f x)}+191 e^{2 i(9 e+2 f x)}+ \\ &318 e^{2 i(e+3 f x)}+14 e^{4 i(2 e+3 f x)}+191 e^{2 i(3 e+5 f x)}+80 e^{2 i(12 e+5 f x)}-2 e^{2 i(5 e+7 f x)}\right)- \\ &12 a^3 b^4\left(-e^{14 i e}+175 e^{4 i f x}+3750 e^{20 i e+6 i f x}+1835 e^{22 i e+8 i f x}+ \\ &35 e^{2 i(8 e+f x)}+3750 e^{4 i(e+2 f x)}+1214 e^{2 i(9 e+2 f x)}+1835 e^{2 i(e+3 f x)}+ \\ &35 e^{4 i(2 e+3 f x)}+1214 e^{2 i(3 e+5 f x)}+175 e^{2 i(12 e+5 f x)}-e^{2 i(5 e+7 f x)}\right) \end{aligned} \right. \\ \left. \left. \frac{1}{24 \sqrt{2} a^4 b^2 (a+b)^2 \left(4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2\right)^2 f} \right. \right. \\ \left. \left. \left(1+e^{18 i e}\right) \sqrt{4 b+a e^{-2 i(e+f x)}\left(1+e^{2 i(e+f x)}\right)^2} \right. \right. \\ \left. \left. \begin{aligned} &\left(13440 b^7 e^{4 i e+6 i f x}\left(-e^{16 i e}+e^{2 i f x}\right)+a^7 e^{4 i f x}\left(-1+e^{18 i e}\right)\left(1+e^{2 i(e+f x)}\right)^3 - \\ &6 a^6 b e^{4 i f x}\left(-1+e^{18 i e}\right)\left(3+8 e^{2 i(e+f x)}+8 e^{4 i(e+f x)}+3 e^{6 i(e+f x)}\right)+2240 a b^6 e^{2 i(e+2 f x)} \\ &\left(-2 e^{16 i e}+3 e^{2 i f x}+2 e^{4 i e+6 i f x}-3 e^{4 i(5 e+f x)}-21 e^{2 i(9 e+f x)}+21 e^{2 i(e+2 f x)}\right)+ \\ &24 a^2 b^5 e^{2 i f x}\left(-7 e^{16 i e}+35 e^{2 i f x}+2710 e^{4 i e+6 i f x}-840 e^{22 i e+6 i f x}+560 e^{6 i e+8 i f x}- \\ &35 e^{8 i(3 e+f x)}-2710 e^{4 i(5 e+f x)}-560 e^{2 i(9 e+f x)}+840 e^{2 i(e+2 f x)}+7 e^{2 i(4 e+5 f x)}\right)- \\ &12 a^3 b^4\left(-e^{14 i e}-175 e^{4 i f x}+3750 e^{20 i e+6 i f x}+1835 e^{22 i e+8 i f x}+35 e^{2 i(8 e+f x)}- \\ &3750 e^{4 i(e+2 f x)}+1214 e^{2 i(9 e+2 f x)}-1835 e^{2 i(e+3 f x)}-35 e^{4 i(2 e+3 f x)}- \\ &1214 e^{2 i(3 e+5 f x)}+175 e^{2 i(12 e+5 f x)}+e^{2 i(5 e+7 f x)}\right)-6 a^5 b^2\left(-2 e^{14 i e}-80 e^{4 i f x}+ \\ &407 e^{20 i e+6 i f x}+318 e^{22 i e+8 i f x}+14 e^{2 i(8 e+f x)}-407 e^{4 i(e+2 f x)}+191 e^{2 i(9 e+2 f x)}- \\ &318 e^{2 i(e+3 f x)}-14 e^{4 i(2 e+3 f x)}-191 e^{2 i(3 e+5 f x)}+80 e^{2 i(12 e+5 f x)}+2 e^{2 i(5 e+7 f x)}\right) \end{aligned} \right. \\ \left. \left. \right. \right) \end{aligned}$$

$$\begin{aligned}
 & 4 a^4 b^3 \left( -6 e^{14 i e} - 430 e^{4 i f x} + 3987 e^{20 i e+6 i f x} + 2610 e^{22 i e+8 i f x} + \right. \\
 & \quad 84 e^{2 i (8 e+f x)} - 3987 e^{4 i (e+2 f x)} + 1684 e^{2 i (9 e+2 f x)} - 2610 e^{2 i (e+3 f x)} - \\
 & \quad \left. 84 e^{4 i (2 e+3 f x)} - 1684 e^{2 i (3 e+5 f x)} + 430 e^{2 i (12 e+5 f x)} + 6 e^{2 i (5 e+7 f x)} \right) + \\
 & \left( 5 \left( 3 a^2 + 14 a b + 14 b^2 \right) e^{i (-17 e+f x)} \left( 1 + e^{18 i e} \right) \sqrt{4 b + a e^{-2 i (e+f x)} \left( 1 + e^{2 i (e+f x)} \right)^2} \right. \\
 & \quad \left. \left( -i \operatorname{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \right. \right. \\
 & \quad \left. \left. e^{18 i e} \left( 2 f x + i \operatorname{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] \right) \right] \right) \right) / \\
 & \left( \sqrt{2} a^{9/2} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} f \right) - \left( 5 \left( 3 a^2 + 14 a b + 14 b^2 \right) \right. \\
 & \quad \left. e^{i (-17 e+f x)} \left( -1 + e^{18 i e} \right) \sqrt{4 b + a e^{-2 i (e+f x)} \left( 1 + e^{2 i (e+f x)} \right)^2} \right. \\
 & \quad \left. \left( i \operatorname{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \right. \right. \\
 & \quad \left. \left. e^{18 i e} \left( 2 f x + i \operatorname{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] \right) \right] \right) \right) / \\
 & \left( \sqrt{2} a^{9/2} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} f \right) \operatorname{Sec}[e + f x]^5 - \\
 & \left( i e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} \left( 1 + e^{2 i (e+f x)} \right)^2} \right. \\
 & \quad \left. (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{5/2} \right. \\
 & \quad \left. - 25 a^{7/2} - \right. \\
 & \quad 58 a^{5/2} b - 32 a^{3/2} b^2 - \\
 & \quad 15 a^{7/2} e^{2 i (e+f x)} - \\
 & \quad 108 a^{5/2} b e^{2 i (e+f x)} - \\
 & \quad 192 a^{3/2} b^2 e^{2 i (e+f x)} - \\
 & \quad 96 \sqrt{a} b^3 e^{2 i (e+f x)} + \\
 & \quad 15 a^{7/2} e^{4 i (e+f x)} + \\
 & \quad 108 a^{5/2} b e^{4 i (e+f x)} + \\
 & \quad 192 a^{3/2} b^2 e^{4 i (e+f x)} + \\
 & \quad \left. 96 \sqrt{a} b^3 e^{4 i (e+f x)} + 25 a^{7/2} e^{6 i (e+f x)} + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 58 a^{5/2} b e^{6 i (e+f x)} + 32 a^{3/2} b^2 e^{6 i (e+f x)} - \\
 & 24 i a^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 48 i a b \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 24 i b^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 12 a^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] - \\
 & 24 a b \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] - \\
 & 12 b^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 12 a^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 24 a b \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 12 b^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] \Big] \\
 & \text{Sec} [e + f x]^5 \Big/ \left( 384 \sqrt{2} a^{5/2} (a + b)^2 \right. \\
 & \quad \left. \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^2 \right. \\
 & \quad \left. f \right. \\
 & \quad \left. (a + b \text{Sec} [e + f x]^2)^{5/2} \right) - \\
 & \left( i e^{-i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} \left( 1 + e^{2 i (e+f x)} \right)^2} \right. \\
 & \quad \left. (a + 2 b + a \text{Cos} [2 e + 2 f x])^{5/2} \right. \\
 & \quad \left. \left( -12 a^{9/2} - 24 a^{7/2} b - 12 a^{5/2} b^2 - 113 a^{9/2} e^{2 i (e+f x)} - \right. \right. \\
 & \quad 532 a^{7/2} b e^{2 i (e+f x)} - 740 a^{5/2} b^2 e^{2 i (e+f x)} - \\
 & \quad 320 a^{3/2} b^3 e^{2 i (e+f x)} - 87 a^{9/2} e^{4 i (e+f x)} - 690 a^{7/2} b e^{4 i (e+f x)} - \\
 & \quad \left. \left. 2040 a^{5/2} b^2 e^{4 i (e+f x)} - 2400 a^{3/2} b^3 e^{4 i (e+f x)} - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 960 \sqrt{a} b^4 e^{4i(e+fx)} + 87 a^{9/2} e^{6i(e+fx)} + 690 a^{7/2} b e^{6i(e+fx)} + \\
 & 2040 a^{5/2} b^2 e^{6i(e+fx)} + 2400 a^{3/2} b^3 e^{6i(e+fx)} + \\
 & 960 \sqrt{a} b^4 e^{6i(e+fx)} + 113 a^{9/2} e^{8i(e+fx)} + 532 a^{7/2} b e^{8i(e+fx)} + \\
 & 740 a^{5/2} b^2 e^{8i(e+fx)} + 320 a^{3/2} b^3 e^{8i(e+fx)} + \\
 & 12 a^{9/2} e^{10i(e+fx)} + 24 a^{7/2} b e^{10i(e+fx)} + 12 a^{5/2} b^2 e^{10i(e+fx)} - \\
 & 120 i a^3 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} f x - \\
 & 480 i a^2 b e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} f x - \\
 & 600 i a b^2 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} f x - \\
 & 240 i b^3 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} f x - \\
 & 60 a^3 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] - \\
 & 240 a^2 b e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] - \\
 & 300 a b^2 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] - \\
 & 120 b^3 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] + \\
 & 60 a^3 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] + \\
 & 240 a^2 b e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] + \\
 & 300 a b^2 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right] + \\
 & 120 b^3 e^{2i(e+fx)} \left( 4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2 \right)^{3/2} \\
 & \text{Log} \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a \left( 1 + e^{2i(e+fx)} \right)^2} \right) \right]
 \end{aligned}$$



$$\begin{aligned}
 & \frac{\operatorname{Sec}[e + f x]^5}{\left(1536 \sqrt{2} a^{7/2} (a + b)^2 \left(4 b e^{2 i (e + f x)} + a \left(1 + e^{2 i (e + f x)}\right)^2\right)^2\right)^{1/2}} \\
 & f \\
 & \left(a + b \operatorname{Sec}[e + f x]^2\right)^{5/2} + \\
 & \left(\left(2 a + 3 b + a \operatorname{Cos}[2 (e + f x)]\right) \left(a + 2 b + a \operatorname{Cos}[2 e + 2 f x]\right)\right)^{5/2} \\
 & \operatorname{Sec}[e + f x]^4 \\
 & \operatorname{Tan}[e + f x] \left/ \left(256\right.\right. \\
 & \left.\left.(a + b)^2\right.\right. \\
 & f \\
 & \left.\left.\left(a + 2 b + a \operatorname{Cos}[2 (e + f x)]\right)\right)^{3/2}\right. \\
 & \left.\left.\left(a + b \operatorname{Sec}[e + f x]^2\right)^{5/2}\right) - \right. \\
 & \left.\left(\left(b + (3 a + 2 b) \operatorname{Cos}[2 (e + f x)]\right) \left(a + 2 b + a \operatorname{Cos}[2 e + 2 f x]\right)\right)^{5/2}\right. \\
 & \operatorname{Sec}[e + f x]^4 \\
 & \operatorname{Tan}[e + f x] \left/ \left(384\right.\right. \\
 & \left.\left.(a + b)^2\right.\right. \\
 & f \\
 & \left.\left.\left(a + 2 b + a \operatorname{Cos}[2 (e + f x)]\right)\right)^{3/2}\right. \\
 & \left.\left.\left(a + b \operatorname{Sec}[e + f x]^2\right)^{5/2}\right) \right.
 \end{aligned}$$

**Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^2}{\left(a + b \operatorname{Sec}[e + f x]^2\right)^{5/2}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\begin{aligned}
 & \frac{(a + 5 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}\right]}{2 a^{7/2} f} - \frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{2 a f \left(a + b + b \operatorname{Tan}[e + f x]^2\right)^{3/2}} \\
 & \frac{5 b \operatorname{Tan}[e + f x]}{6 a^2 f \left(a + b + b \operatorname{Tan}[e + f x]^2\right)^{3/2}} - \frac{b \left(13 a + 15 b\right) \operatorname{Tan}[e + f x]}{6 a^3 \left(a + b\right) f \sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}
 \end{aligned}$$

Result (type 3, 3247 leaves):

$$\begin{aligned}
 & - \left( \left( i e^{i (e + f x)} \sqrt{4 b + a e^{-2 i (e + f x)} \left(1 + e^{2 i (e + f x)}\right)^2} \left(a + 2 b + a \operatorname{Cos}[2 e + 2 f x]\right)^{5/2} \right. \right. \\
 & \left. \left( -25 a^{7/2} - 58 a^{5/2} b - 32 a^{3/2} b^2 - 15 a^{7/2} e^{2 i (e + f x)} - 108 a^{5/2} b e^{2 i (e + f x)} - \right. \right. \\
 & \left. \left. 192 a^{3/2} b^2 e^{2 i (e + f x)} - 96 \sqrt{a} b^3 e^{2 i (e + f x)} + 15 a^{7/2} e^{4 i (e + f x)} + 108 a^{5/2} b e^{4 i (e + f x)} + \right. \right. \\
 & \left. \left. 192 a^{3/2} b^2 e^{4 i (e + f x)} + 96 \sqrt{a} b^3 e^{4 i (e + f x)} + 25 a^{7/2} e^{6 i (e + f x)} + 58 a^{5/2} b e^{6 i (e + f x)} + \right. \right. \\
 & \left. \left. 32 a^{3/2} b^2 e^{6 i (e + f x)} - 24 i a^2 \left(4 b e^{2 i (e + f x)} + a \left(1 + e^{2 i (e + f x)}\right)^2\right)^{3/2} f x - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 48 \, i \, a \, b \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 24 \, i \, b^2 \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} f x - 12 \, a^2 \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 \, i \, e} \left( a + 2 \, b + a \, e^{2 \, i \, (e+f x)} + \sqrt{a} \sqrt{4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right) \right] - \\
 & 24 \, a \, b \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 \, i \, e} \left( a + 2 \, b + a \, e^{2 \, i \, (e+f x)} + \sqrt{a} \sqrt{4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right) \right] - \\
 & 12 \, b^2 \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 \, i \, e} \left( a + 2 \, b + a \, e^{2 \, i \, (e+f x)} + \sqrt{a} \sqrt{4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right) \right] + \\
 & 12 \, a^2 \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 \, i \, e} \left( a + a \, e^{2 \, i \, (e+f x)} + 2 \, b \, e^{2 \, i \, (e+f x)} + \sqrt{a} \sqrt{4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right) \right] + \\
 & 24 \, a \, b \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 \, i \, e} \left( a + a \, e^{2 \, i \, (e+f x)} + 2 \, b \, e^{2 \, i \, (e+f x)} + \sqrt{a} \sqrt{4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right) \right] + \\
 & 12 \, b^2 \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \text{Log} \left[ e^{-2 \, i \, e} \left( a + a \, e^{2 \, i \, (e+f x)} + 2 \, b \, e^{2 \, i \, (e+f x)} + \sqrt{a} \sqrt{4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right) \right] \Bigg) \\
 & \text{Sec} [e + f x]^5 \Bigg) / \left( 128 \sqrt{2} \, a^{5/2} (a + b)^2 \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^2 \right. \\
 & \left. f (a + b \text{Sec} [e + f x]^2)^{5/2} \right) \Bigg) + \\
 & \left( i \, e^{-i \, (e+f x)} \sqrt{4 \, b + a \, e^{-2 \, i \, (e+f x)} \left( 1 + e^{2 \, i \, (e+f x)} \right)^2} \right. \\
 & \quad (a + 2 \, b + a \, \text{Cos} [2 \, e + 2 \, f x])^{5/2} \\
 & \quad \left( -12 \, a^{9/2} - 24 \, a^{7/2} \, b - 12 \, a^{5/2} \, b^2 - 113 \, a^{9/2} \, e^{2 \, i \, (e+f x)} - 532 \, a^{7/2} \, b \, e^{2 \, i \, (e+f x)} - \right. \\
 & \quad 740 \, a^{5/2} \, b^2 \, e^{2 \, i \, (e+f x)} - 320 \, a^{3/2} \, b^3 \, e^{2 \, i \, (e+f x)} - 87 \, a^{9/2} \, e^{4 \, i \, (e+f x)} - \\
 & \quad 690 \, a^{7/2} \, b \, e^{4 \, i \, (e+f x)} - 2040 \, a^{5/2} \, b^2 \, e^{4 \, i \, (e+f x)} - 2400 \, a^{3/2} \, b^3 \, e^{4 \, i \, (e+f x)} - \\
 & \quad 960 \sqrt{a} \, b^4 \, e^{4 \, i \, (e+f x)} + 87 \, a^{9/2} \, e^{6 \, i \, (e+f x)} + 690 \, a^{7/2} \, b \, e^{6 \, i \, (e+f x)} + \\
 & \quad 2040 \, a^{5/2} \, b^2 \, e^{6 \, i \, (e+f x)} + 2400 \, a^{3/2} \, b^3 \, e^{6 \, i \, (e+f x)} + 960 \sqrt{a} \, b^4 \, e^{6 \, i \, (e+f x)} + \\
 & \quad 113 \, a^{9/2} \, e^{8 \, i \, (e+f x)} + 532 \, a^{7/2} \, b \, e^{8 \, i \, (e+f x)} + 740 \, a^{5/2} \, b^2 \, e^{8 \, i \, (e+f x)} + \\
 & \quad 320 \, a^{3/2} \, b^3 \, e^{8 \, i \, (e+f x)} + 12 \, a^{9/2} \, e^{10 \, i \, (e+f x)} + 24 \, a^{7/2} \, b \, e^{10 \, i \, (e+f x)} + \\
 & \quad \left. 12 \, a^{5/2} \, b^2 \, e^{10 \, i \, (e+f x)} - 120 \, i \, a^3 \, e^{2 \, i \, (e+f x)} \left( 4 \, b \, e^{2 \, i \, (e+f x)} + a \left( 1 + e^{2 \, i \, (e+f x)} \right)^2 \right)^{3/2} f x - \right.
 \end{aligned}$$

$$\begin{aligned}
 & 480 i a^2 b e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 600 i a b^2 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 240 i b^3 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} f x - \\
 & 60 a^3 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] - \\
 & 240 a^2 b e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] - \\
 & 300 a b^2 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] - \\
 & 120 b^3 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 60 a^3 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 240 a^2 b e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 300 a b^2 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] + \\
 & 120 b^3 e^{2 i (e+f x)} \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^{3/2} \\
 & \quad \operatorname{Log} \left[ e^{-2 i e} \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2} \right) \right] \Bigg) \\
 & \operatorname{Sec}[e+f x]^5 \Bigg/ \left( 384 \sqrt{2} a^{7/2} (a+b)^2 \left( 4 b e^{2 i (e+f x)} + a \left( 1 + e^{2 i (e+f x)} \right)^2 \right)^2 \right. \\
 & \quad \left. f (a+b \operatorname{Sec}[e+f x]^2)^{5/2} \right) + \\
 & \left( 5 (2 a+3 b+a \operatorname{Cos}[2(e+f x)]) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^{5/2} \right. \\
 & \quad \left. \operatorname{Sec}[e+f x]^4 \operatorname{Tan}[e+f x] \right) \Bigg/ \\
 & \left( 384 (a+b)^2 f (a+2 b+a \operatorname{Cos}[2(e+f x)])^{3/2} \right)
 \end{aligned}$$

$$\frac{(a + b \operatorname{Sec}[e + f x]^2)^{5/2} - \left( (b + (3a + 2b) \operatorname{Cos}[2(e + f x)]) (a + 2b + a \operatorname{Cos}[2e + 2fx])^{5/2} \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x] \right)}{(384 (a + b)^2 f (a + 2b + a \operatorname{Cos}[2(e + f x)])^{3/2} (a + b \operatorname{Sec}[e + f x]^2)^{5/2}}$$

**Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{a^{5/2} f} - \frac{b \operatorname{Tan}[e + f x]}{3 a (a + b) f (a + b + b \operatorname{Tan}[e + f x]^2)^{3/2}} - \frac{b (5 a + 3 b) \operatorname{Tan}[e + f x]}{3 a^2 (a + b)^2 f \sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 6, 1927 leaves):

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^4 \operatorname{Sin}[e + f x] \right) / \left( 4 \sqrt{2} f (a + b \operatorname{Sec}[e + f x]^2)^{5/2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right. \\ \left. \left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) \right. \\ \left. \left( 15 a (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^5 \operatorname{Sin}[e + f x]^2 \right) / \left( 4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{7/2} \right. \\ \left. \left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right. \right.$$

$$\begin{aligned}
 & \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) + \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^5 \right) / \\
 & \left( 4 \sqrt{2} (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] + \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \quad \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^3 \right. \\
 & \quad \left. \sin[e+fx]^2 \right) / \left( \sqrt{2} (a+b - a \sin[e+fx]^2)^{5/2} \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) + \\
 & \left( 3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left( \frac{1}{3 (a+b)} 5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
 & \left( 4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] + \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \quad \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \right)
 \end{aligned}$$

$$\begin{aligned} & \left( \sin[e+fx] \left( 2f \left( 5a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right. \\ & \quad \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right. \\ & \quad \left. \cos[e+fx] \sin[e+fx] + 3(a+b) \left( \frac{1}{3(a+b)} 5af \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \right. \right. \right. \\ & \quad \quad \left. \left. \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right. \\ & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) + \\ & \left( \sin[e+fx]^2 \left( 5a \left( \frac{1}{5(a+b)} 21af \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[e+fx]^2, \right. \right. \right. \right. \\ & \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \right. \right. \\ & \quad \left. \left. \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) - 4(a+b) \right. \\ & \quad \left. \left( \frac{1}{a+b} 3af \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \right) \right) \\ & \left( \sin[e+fx] - \left( 6(a+b)^3 f \cot[e+fx] \csc[e+fx]^4 \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2 \right) \right. \\ & \quad \left( \frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right] \sin[e+fx]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} + \frac{a^2 \sin[e+fx]^4}{3(a+b)^2 \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2} + \right. \\ & \quad \left. \left. \frac{a \sin[e+fx]^2}{(a+b) \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)} \right) \right) \left/ \left( a^3 \left( 1 - \frac{a \sin[e+fx]^2}{a+b} \right)^{3/2} \right) \right) \right) \end{aligned}$$

$$\left( 4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \right. \\ \left. \left. \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right)^2 \right) \right)$$

### Problem 132: Result more than twice size of optimal antiderivative.

$$\int (a+b \operatorname{Sec}[e+fx]^2)^p (d \sin[e+fx])^m dx$$

Optimal (type 6, 123 leaves, ? steps):

$$\frac{1}{f(1+m)} \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{1}{2}+p, -p, \frac{3+m}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] (\cos[e+fx]^2)^{\frac{1}{2}+p} \\ (a+b \operatorname{Sec}[e+fx]^2)^p (d \sin[e+fx])^m \left( \frac{a+b - a \sin[e+fx]^2}{a+b} \right)^{-p} \tan[e+fx]$$

Result (type 6, 3356 leaves):

$$\left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\ \left. \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p \right. \\ \left. (a+b \operatorname{Sec}[e+fx]^2)^p \sin[e+fx] (d \sin[e+fx])^m \left( \frac{\tan[e+fx]}{\sqrt{\operatorname{Sec}[e+fx]^2}} \right)^m \right) / \\ \left( f(1+m) \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\ \left. \left( -2bp \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) (2+m) \right. \right. \\ \left. \left. \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \\ \left( \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \\ \left. \left. (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-1+p} \left( \frac{\tan[e+fx]}{\sqrt{\operatorname{Sec}[e+fx]^2}} \right)^m \right) / \left( (1+m) \right. \right)$$

$$\begin{aligned}
 & \left( (a+b) (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \left( -2 b p \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. (a+b) (2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, \right. \right. \\
 & \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Big) - \\
 & \left( (a+b) (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx]^2 \left( \frac{\tan[e+fx]}{\sqrt{\sec[e+fx]^2}} \right)^m \Big) / \left( (1+m) \right. \\
 & \left( (a+b) (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \left( -2 b p \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. (a+b) (2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, \right. \right. \\
 & \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Big) \Big) + \\
 & \left( 2 (a+b) (3+m) p \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx]^2 \left( \frac{\tan[e+fx]}{\sqrt{\sec[e+fx]^2}} \right)^m \Big) / \left( (1+m) \right. \\
 & \left( (a+b) (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \left( -2 b p \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. (a+b) (2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \\
 & \tan[e+fx]^2 \Big) - \left( 2 a (a+b) (3+m) p \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, \right. \right. \\
 & \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \cos[e+fx] (a+2b+a \cos[2(e+fx)])^{-1+p} \right. \\
 & \left. (\sec[e+fx]^2)^p \sin[e+fx] \sin[2(e+fx)] \left( \frac{\tan[e+fx]}{\sqrt{\sec[e+fx]^2}} \right)^m \right) / \left( (1+m) \right.
 \end{aligned}$$



$$\begin{aligned}
 & \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] - \right. \\
 & \left( -2 b p \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \\
 & \left. \operatorname{Tan}[e+fx]^2 \right) + \left( (a+b) (3+m) \operatorname{Cos}[e+fx] (a+2b+a \operatorname{Cos}[2(e+fx)])^p \right. \\
 & \left. (\operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[e+fx] \left( \frac{\operatorname{Tan}[e+fx]}{\sqrt{\operatorname{Sec}[e+fx]^2}} \right)^m \left( \frac{1}{(a+b) (3+m)} 2 b (1+m) p \right. \right. \\
 & \left. \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, \frac{2+m}{2}, 1-p, 1 + \frac{3+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right. \\
 & \left. \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{1}{3+m} (1+m) (2+m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, 1 + \frac{2+m}{2}, -p, \right. \right. \\
 & \left. \left. 1 + \frac{3+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \left. \right) / \left( (1+m) \right. \\
 & \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] - \right. \\
 & \left( -2 b p \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, \right. \right. \\
 & \left. \left. -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \operatorname{Tan}[e+fx]^2 \left. \right) + \\
 & \left( (a+b) m (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right. \\
 & \left. \operatorname{Cos}[e+fx] (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[e+fx] \right. \\
 & \left. \left( \frac{\operatorname{Tan}[e+fx]}{\sqrt{\operatorname{Sec}[e+fx]^2}} \right)^{-1+m} \left( \sqrt{\operatorname{Sec}[e+fx]^2} - \frac{\operatorname{Tan}[e+fx]^2}{\sqrt{\operatorname{Sec}[e+fx]^2}} \right) \right) \left. \right) / \left( (1+m) \right. \\
 & \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] - \right. \\
 & \left( -2 b p \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, \right. \right. \\
 & \left. \left. -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \operatorname{Tan}[e+fx]^2 \left. \right) \left. \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx] \left( \frac{\tan[e+fx]}{\sqrt{\sec[e+fx]^2}} \right)^m \\
 & \left. \left( -2 \left( -2bp \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \right. \\
 & \quad \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \right. \\
 & \quad \left. \sec[e+fx]^2 \tan[e+fx] + (a+b) (3+m) \left( \left( 2b(1+m)p \operatorname{AppellF1} \left[ 1+\frac{1+m}{2}, \frac{2+m}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 1-p, 1+\frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \\
 & \quad \left( (a+b) (3+m) - \frac{1}{3+m} (1+m) (2+m) \operatorname{AppellF1} \left[ 1+\frac{1+m}{2}, 1+\frac{2+m}{2}, -p, \right. \right. \\
 & \quad \left. \left. 1+\frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) - \\
 & \quad \tan[e+fx]^2 \left( -2bp \left( - \left( \left( 2b(3+m)(1-p) \operatorname{AppellF1} \left[ 1+\frac{3+m}{2}, \frac{2+m}{2}, 2-p, \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 1+\frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \\
 & \quad \left( (a+b) (5+m) \right) - \frac{1}{5+m} (2+m) (3+m) \operatorname{AppellF1} \left[ 1+\frac{3+m}{2}, 1+\frac{2+m}{2}, \right. \\
 & \quad \left. 1-p, 1+\frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) + \\
 & \quad (a+b) (2+m) \left( \left( 2b(3+m)p \operatorname{AppellF1} \left[ 1+\frac{3+m}{2}, \frac{4+m}{2}, 1-p, 1+\frac{5+m}{2}, \right. \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) / \\
 & \quad \left( (a+b) (5+m) - \frac{1}{5+m} (3+m) (4+m) \operatorname{AppellF1} \left[ 1+\frac{3+m}{2}, 1+\frac{4+m}{2}, -p, \right. \right. \\
 & \quad \left. \left. 1+\frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) \right) / \\
 & \left( (1+m) \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] - \left( -2bp \operatorname{AppellF1} \left[ \frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, \right. \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, \right. \right. \right.
 \end{aligned}$$

$$\left. \left. \left. \left. \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \tan[e+fx]^2 \right)^2 \right)^2 \right)^2 \right)$$

### Problem 136: Result more than twice size of optimal antiderivative.

$$\int \csc[e+fx] (a+b \sec[e+fx]^2)^p dx$$

Optimal (type 6, 77 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \sec[e+fx]^2, -\frac{b \sec[e+fx]^2}{a}\right] \sec[e+fx] (a+b \sec[e+fx]^2)^p \left(1 + \frac{b \sec[e+fx]^2}{a}\right)^{-p}$$

Result (type 6, 4417 leaves):

$$\begin{aligned} & \left( 2^p \sec[e+fx] (a+b \sec[e+fx]^2)^p \tan[e+fx] (1+\tan[e+fx]^2)^{-\frac{1}{2}+p} \left( \frac{a+b+b \tan[e+fx]^2}{1+\tan[e+fx]^2} \right)^p \right. \\ & \left( - \left( \left( 2(a+b) \text{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right. \right. \\ & \quad \left( 4(a+b) \text{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\ & \quad \left. \left( 2bp \text{AppellF1}\left[2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\ & \quad \left. \left. (a+b) \text{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \right) + \\ & \left( b(-1+2p) \text{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \right. \\ & \quad \left. (1+\tan[e+fx]^2) \right) / \left( (1+2p) \right. \\ & \quad \left( -2(a+b)p \text{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] - \right. \\ & \quad b \text{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + \\ & \quad \left. b(-1+2p) \text{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, \right. \right. \\ & \quad \left. \left. -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \tan[e+fx]^2 \right) \right) \right) / \\ & \left( f \left( 2^{1+p} \left( -\frac{1}{2} + p \right) \sec[e+fx]^2 \tan[e+fx]^3 (1+\tan[e+fx]^2)^{-\frac{3}{2}+p} \left( \frac{a+b+b \tan[e+fx]^2}{1+\tan[e+fx]^2} \right)^p \right. \right. \\ & \quad \left. \left( - \left( \left( 2(a+b) \text{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \left( 4 (a+b) \operatorname{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( 2 b p \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
 & \left( b (-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \right. \\
 & \quad \left. (1+\tan[e+fx]^2) \right) / \left( (1+2p) \left( -2(a+b) p \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}- \right. \right. \right. \\
 & \quad \left. \left. p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] - b \operatorname{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}- \right. \right. \\
 & \quad \left. \left. p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + b (-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}- \right. \right. \\
 & \quad \left. \left. p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \tan[e+fx]^2 \right) \right) + \\
 & 2^{1+p} \operatorname{Sec}[e+fx]^2 \tan[e+fx] (1+\tan[e+fx]^2)^{-\frac{1}{2}+p} \left( \frac{a+b+b \tan[e+fx]^2}{1+\tan[e+fx]^2} \right)^p \\
 & \left( - \left( \left( 2(a+b) \operatorname{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right. \right. \\
 & \quad \left( 4(a+b) \operatorname{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \right) + \\
 & \left( b (-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \right. \\
 & \quad \left. (1+\tan[e+fx]^2) \right) / \left( (1+2p) \left( -2(a+b) p \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}- \right. \right. \right. \\
 & \quad \left. \left. p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] - b \operatorname{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}- \right. \right. \\
 & \quad \left. \left. p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + b (-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}- \right. \right. \\
 & \quad \left. \left. p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \tan[e+fx]^2 \right) \right) + \\
 & 2^p p \tan[e+fx]^2 (1+\tan[e+fx]^2)^{-\frac{1}{2}+p} \left( \frac{a+b+b \tan[e+fx]^2}{1+\tan[e+fx]^2} \right)^{-1+p} \\
 & \left( \frac{2 b \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{1+\tan[e+fx]^2} - \frac{2 \operatorname{Sec}[e+fx]^2 \tan[e+fx] (a+b+b \tan[e+fx]^2)}{(1+\tan[e+fx]^2)^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( - \left( \left( 2 (a+b) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) / \right. \right. \\
 & \quad \left( 4 (a+b) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. \left( 2 b p \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ 2, \frac{3}{2}, -p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \right) + \\
 & \left( b (-1+2p) \operatorname{AppellF1} \left[ -\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b} \right] \right. \\
 & \quad \left. (1+\tan[e+fx]^2) \right) / \left( (1+2p) \left( -2 (a+b) p \operatorname{AppellF1} \left[ \frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}- \right. \right. \right. \\
 & \quad \left. \left. p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b} \right] - b \operatorname{AppellF1} \left[ \frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}- \right. \right. \\
 & \quad \left. \left. p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b} \right] + b (-1+2p) \operatorname{AppellF1} \left[ -\frac{1}{2}- \right. \right. \\
 & \quad \left. \left. p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b} \right] \tan[e+fx]^2 \right) \right) \right) + \\
 & 2^p \tan[e+fx]^2 (1+\tan[e+fx]^2)^{-\frac{1}{2}+p} \left( \frac{a+b+b \tan[e+fx]^2}{1+\tan[e+fx]^2} \right)^p \\
 & \left( - \left( \left( 2 (a+b) \left( \frac{1}{a+b} b p \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{2} \operatorname{AppellF1} \left[ 2, \frac{3}{2}, -p, 3, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \right) / \\
 & \quad \left( 4 (a+b) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ 2, \frac{3}{2}, -p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \right) + \\
 & \left( 2 b (-1+2p) \operatorname{AppellF1} \left[ -\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\cot[e+fx]^2, \right. \right. \\
 & \quad \left. \left. -\frac{(a+b) \cot[e+fx]^2}{b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) / \\
 & \quad \left( (1+2p) \left( -2 (a+b) p \operatorname{AppellF1} \left[ \frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\cot[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{(a+b) \cot[e+fx]^2}{b} \right] - b \operatorname{AppellF1} \left[ \frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\cot[e+fx]^2, \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \Big] + b(-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-\right. \\
 & \left. p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Tan}[e+fx]^2 \Big) + \\
 & \left( b(-1+2p) \left( -\frac{1}{b\left(\frac{1}{2}-p\right)} 2(a+b) \left(-\frac{1}{2}-p\right) p \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, \right. \right. \right. \\
 & \left. \left. -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 - \frac{1}{\frac{1}{2}-p} \right. \right. \\
 & \left. \left. \left(-\frac{1}{2}-p\right) \operatorname{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \right. \right. \\
 & \left. \left. \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 \right) (1+\operatorname{Tan}[e+fx]^2) \right) \Big/ \\
 & \left( (1+2p) \left( -2(a+b) p \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] - b \operatorname{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, \right. \right. \\
 & \left. \left. -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] + b(-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-\right. \right. \\
 & \left. \left. p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Tan}[e+fx]^2 \right) \Big) - \\
 & \left( b(-1+2p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \right. \\
 & (1+\operatorname{Tan}[e+fx]^2) \left( -2(a+b) p \left( \frac{1}{b\left(\frac{3}{2}-p\right)} 2(a+b) \left(\frac{1}{2}-p\right) (1-p) \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-p, -\frac{1}{2}, 2-p, \frac{5}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \right. \right. \\
 & \left. \left. \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 - \frac{1}{\frac{3}{2}-p} \left(\frac{1}{2}-p\right) \operatorname{AppellF1}\left[\frac{3}{2}-p, \frac{1}{2}, 1-p, \frac{5}{2}-p, \right. \right. \right. \\
 & \left. \left. -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 \right) \Big) - \\
 & b \left( -\frac{1}{b\left(\frac{3}{2}-p\right)} 2(a+b) \left(\frac{1}{2}-p\right) p \operatorname{AppellF1}\left[\frac{3}{2}-p, \frac{1}{2}, 1-p, \frac{5}{2}-p, \right. \right. \\
 & \left. \left. -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 + \frac{1}{\frac{3}{2}-p} \right. \\
 & \left. \left. \left(\frac{1}{2}-p\right) \operatorname{AppellF1}\left[\frac{3}{2}-p, \frac{3}{2}, -p, \frac{5}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b) \operatorname{Cot}[e+fx]^2}{b} \right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned}
 & \cot [e+f x] \operatorname{Csc}[e+f x]^2 \right) + 2 b(-1+2 p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-\right. \\
 & \left. p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] + \\
 & b(-1+2 p)\left(-\frac{1}{b\left(\frac{1}{2}-p\right)} 2(a+b)\left(-\frac{1}{2}-p\right) p \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \right. \right. \\
 & \left. \left. \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b}\right] \cot [e+f x] \operatorname{Csc}[e+f x]^2 - \right. \\
 & \left. \frac{1}{\frac{1}{2}-p}\left(-\frac{1}{2}-p\right) \operatorname{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, \right. \right. \\
 & \left. \left. -\frac{(a+b) \cot [e+f x]^2}{b}\right] \cot [e+f x] \operatorname{Csc}[e+f x]^2\right) \tan [e+f x]^2 \left. \right) / \\
 & \left( (1+2 p)\left(-2(a+b) p \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\cot [e+f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a+b) \cot [e+f x]^2}{b}\right] -b \operatorname{AppellF1}\left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, \right. \right. \\
 & \left. \left. -\frac{(a+b) \cot [e+f x]^2}{b}\right] +b(-1+2 p) \operatorname{AppellF1}\left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \right. \right. \\
 & \left. \left. \frac{1}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b}\right] \tan [e+f x]^2\right)^2 \left. \right) + \\
 & \left( 2(a+b) \operatorname{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b}\right] \right. \\
 & \left( 2\left( 2 b p \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-p, 3, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. (a+b) \operatorname{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b}\right] \right) \right) \\
 & \operatorname{Sec}[e+f x]^2 \tan [e+f x] + 4(a+b)\left(\frac{1}{a+b} b p \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-p, 3, \right. \right. \\
 & \left. \left. -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] -\frac{1}{2} \operatorname{AppellF1}\left[2, \right. \right. \\
 & \left. \left. \frac{3}{2}, -p, 3, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\right) + \\
 & \tan [e+f x]^2\left( 2 b p\left(-\frac{1}{3(a+b)} 4 b(1-p) \operatorname{AppellF1}\left[3, \frac{1}{2}, 2-p, 4, -\tan [e+f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] -\frac{2}{3} \operatorname{AppellF1}\left[3, \frac{3}{2}, 1-p, \right. \right. \\
 & \left. \left. 4, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\right) -
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 & (a+b) \left( \frac{1}{3(a+b)} 4 b p \operatorname{AppellF1} \left[ 3, \frac{3}{2}, 1-p, 4, -\tan [e+f x]^2, \right. \right. \\
 & \quad \left. \left. -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] - 2 \operatorname{AppellF1} \left[ 3, \frac{5}{2}, -p, \right. \right. \\
 & \quad \left. \left. 4, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
 & \left( 4(a+b) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, -p, 2, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 1-p, 3, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] - (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ 2, \frac{3}{2}, -p, 3, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \tan [e+f x]^2 \right)^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
 \end{aligned}$$

**Problem 137: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+f x]^3 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 81 leaves, 3 steps):

$$\frac{1}{3 f} \operatorname{AppellF1} \left[ \frac{3}{2}, 2, -p, \frac{5}{2}, \operatorname{Sec}[e+f x]^2, -\frac{b \operatorname{Sec}[e+f x]^2}{a} \right] \operatorname{Sec}[e+f x]^3 (a+b \operatorname{Sec}[e+f x]^2)^p \left( 1 + \frac{b \operatorname{Sec}[e+f x]^2}{a} \right)^{-p}$$

Result (type 6, 2081 leaves):

$$\begin{aligned}
 & - \left( \left( b(-3+2 p) \operatorname{AppellF1} \left[ \frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \right. \right. \\
 & \quad \left. \left. (a+2 b+a \cos [2(e+f x)])^p \operatorname{Csc}[e+f x]^3 \right. \right. \\
 & \quad \left. \left. (\operatorname{Sec}[e+f x]^2)^{\frac{1}{2}+p} (a+b \operatorname{Sec}[e+f x]^2)^p \right) \Bigg) / \left( f(-1+2 p) \right. \\
 & \quad \left( 2(a+b) p \operatorname{AppellF1} \left[ \frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] + \right. \\
 & \quad b \left( \operatorname{AppellF1} \left[ \frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] + \right. \\
 & \quad \left. (3-2 p) \operatorname{AppellF1} \left[ \frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, \right. \right. \\
 & \quad \left. \left. -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \tan [e+f x]^2 \right) \Bigg) \Bigg) \\
 & \left( - \left( \left( b(-3+2 p) (a+2 b+a \cos [2(e+f x)])^p \left( -\frac{1}{b \left( \frac{3}{2}-p \right)} 2(a+b) \left( \frac{1}{2}-p \right) p \operatorname{AppellF1} \left[ \right. \right. \right. \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & \left. \left( \frac{3}{2} - p, -\frac{1}{2}, 1 - p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right) \text{Cot}[e + f x] \right. \\
 & \left. \text{Csc}[e + f x]^2 - \frac{1}{\frac{3}{2} - p} \left( \frac{1}{2} - p \right) \text{AppellF1} \left[ \frac{3}{2} - p, \frac{1}{2}, -p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] \text{Cot}[e + f x] \text{Csc}[e + f x]^2 \right) \left( \text{Sec}[e + f x]^2 \right)^{\frac{1}{2} + p} \Bigg/ \\
 & \left( (-1 + 2p) \left( 2(a + b)p \text{AppellF1} \left[ \frac{3}{2} - p, -\frac{1}{2}, 1 - p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] + b \left( \text{AppellF1} \left[ \frac{3}{2} - p, \frac{1}{2}, -p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] + (3 - 2p) \text{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \frac{3}{2} - p, \right. \right. \\
 & \left. \left. -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] \text{Tan}[e + f x]^2 \right) \right) \Bigg) + \\
 & \left( 2abp(-3 + 2p) \text{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \frac{3}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] \right. \\
 & \left. (a + 2b + a \text{Cos}[2(e + f x)])^{-1 + p} (\text{Sec}[e + f x]^2)^{\frac{1}{2} + p} \text{Sin}[2(e + f x)] \right) \Bigg/ \\
 & \left( (-1 + 2p) \left( 2(a + b)p \text{AppellF1} \left[ \frac{3}{2} - p, -\frac{1}{2}, 1 - p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] + b \left( \text{AppellF1} \left[ \frac{3}{2} - p, \frac{1}{2}, -p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] + (3 - 2p) \text{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \right. \right. \\
 & \left. \left. \frac{3}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] \text{Tan}[e + f x]^2 \right) \right) \Bigg) - \\
 & \left( 2b \left( \frac{1}{2} + p \right) (-3 + 2p) \text{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \frac{3}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] (a + 2b + a \text{Cos}[2(e + f x)])^p (\text{Sec}[e + f x]^2)^{\frac{1}{2} + p} \text{Tan}[e + f x] \right) \Bigg/ \\
 & \left( (-1 + 2p) \left( 2(a + b)p \text{AppellF1} \left[ \frac{3}{2} - p, -\frac{1}{2}, 1 - p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] + b \left( \text{AppellF1} \left[ \frac{3}{2} - p, \frac{1}{2}, -p, \frac{5}{2} - p, -\text{Cot}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] + (3 - 2p) \text{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \right. \right. \\
 & \left. \left. \frac{3}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b} \right] \text{Tan}[e + f x]^2 \right) \right) \Bigg) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( b (-3 + 2 p) \operatorname{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \frac{3}{2} - p, -\operatorname{Cot}[e + f x]^2, -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \right. \\
 & \quad (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^p (\operatorname{Sec}[e + f x]^2)^{\frac{1}{2} + p} \\
 & \quad \left( 2 (a + b) p \left( \frac{1}{b \left( \frac{5}{2} - p \right)} 2 (a + b) (1 - p) \left( \frac{3}{2} - p \right) \operatorname{AppellF1} \left[ \frac{5}{2} - p, -\frac{1}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. 2 - p, \frac{7}{2} - p, -\operatorname{Cot}[e + f x]^2, -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \operatorname{Cot}[e + f x] \right. \right. \\
 & \quad \left. \left. \operatorname{Csc}[e + f x]^2 - \frac{1}{\frac{5}{2} - p} \left( \frac{3}{2} - p \right) \operatorname{AppellF1} \left[ \frac{5}{2} - p, \frac{1}{2}, 1 - p, \frac{7}{2} - p, \right. \right. \right. \\
 & \quad \left. \left. \left. -\operatorname{Cot}[e + f x]^2, -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]^2 \right) \right) + \\
 & \quad b \left( -\frac{1}{b \left( \frac{5}{2} - p \right)} 2 (a + b) \left( \frac{3}{2} - p \right) p \operatorname{AppellF1} \left[ \frac{5}{2} - p, \frac{1}{2}, 1 - p, \frac{7}{2} - p, -\operatorname{Cot}[e + f x]^2, \right. \right. \\
 & \quad \left. \left. -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]^2 + \frac{1}{\frac{5}{2} - p} \left( \frac{3}{2} - p \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[ \frac{5}{2} - p, \frac{3}{2}, -p, \frac{7}{2} - p, -\operatorname{Cot}[e + f x]^2, -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \right. \\
 & \quad \left. \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]^2 + 2 (3 - 2 p) \operatorname{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \frac{3}{2} - p, \right. \right. \\
 & \quad \left. \left. -\operatorname{Cot}[e + f x]^2, -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + (3 - 2 p) \right. \\
 & \quad \left( -\frac{1}{b \left( \frac{3}{2} - p \right)} 2 (a + b) \left( \frac{1}{2} - p \right) p \operatorname{AppellF1} \left[ \frac{3}{2} - p, -\frac{1}{2}, 1 - p, \frac{5}{2} - p, \right. \right. \\
 & \quad \left. \left. -\operatorname{Cot}[e + f x]^2, -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]^2 - \right. \\
 & \quad \left. \frac{1}{\frac{3}{2} - p} \left( \frac{1}{2} - p \right) \operatorname{AppellF1} \left[ \frac{3}{2} - p, \frac{1}{2}, -p, \frac{5}{2} - p, -\operatorname{Cot}[e + f x]^2, \right. \right. \\
 & \quad \left. \left. -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]^2 \right) \operatorname{Tan}[e + f x]^2 \left. \right) \left. \right) \left. \right) / \\
 & \quad \left( (-1 + 2 p) \left( 2 (a + b) p \operatorname{AppellF1} \left[ \frac{3}{2} - p, -\frac{1}{2}, 1 - p, \frac{5}{2} - p, -\operatorname{Cot}[e + f x]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] + b \left( \operatorname{AppellF1} \left[ \frac{3}{2} - p, \frac{1}{2}, -p, \frac{5}{2} - p, -\operatorname{Cot}[e + f x]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{(a + b) \operatorname{Cot}[e + f x]^2}{b} \right] + (3 - 2 p) \operatorname{AppellF1} \left[ \frac{1}{2} - p, -\frac{1}{2}, -p, \frac{3}{2} - p, \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & \left( f \left( 3 \times 2^{1+p} (a+b) (-3+p) \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]^2 (1+\operatorname{Tan}[e+fx]^2)^{-4+p} \right. \right. \\
 & \quad \left. \left. \left( \frac{a+b+b \operatorname{Tan}[e+fx]^2}{1+\operatorname{Tan}[e+fx]^2} \right)^p \right. \right. \\
 & \quad \left. \left( \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] (1+\operatorname{Tan}[e+fx]^2) \right) \right) / \right. \\
 & \quad \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + 2 (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \operatorname{Tan}[e+fx]^2 \right) + \\
 & \quad \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] / \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] - 3 (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \operatorname{Tan}[e+fx]^2 \right) - \\
 & \quad \left( \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] (1+\operatorname{Tan}[e+fx]^2)^2 \right) / \\
 & \quad \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \\
 & \quad \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + (a+b) \operatorname{AppellF1} \left[ \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Tan}[e+fx]^2 \right) \right) \right) + \\
 & \quad 3 \times 2^p (a+b) \operatorname{Sec}[e+fx]^2 (1+\operatorname{Tan}[e+fx]^2)^{-3+p} \left( \frac{a+b+b \operatorname{Tan}[e+fx]^2}{1+\operatorname{Tan}[e+fx]^2} \right)^p \\
 & \quad \left( \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] (1+\operatorname{Tan}[e+fx]^2) \right) \right) / \\
 & \quad \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + 2 (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \operatorname{Tan}[e+fx]^2 \right) + \\
 & \quad \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] / \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - 3(a+b) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \tan[e+fx]^2 \right) - \\
 & \left( \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] (1 + \tan[e+fx]^2)^2 \right) / \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \quad 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \Bigg) + \\
 & 3 \times 2^p (a+b) p \tan[e+fx] (1 + \tan[e+fx]^2)^{-3+p} \left( \frac{a+b+b \tan[e+fx]^2}{1 + \tan[e+fx]^2} \right)^{-1+p} \\
 & \left( \frac{2 b \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{1 + \tan[e+fx]^2} - \frac{2 \operatorname{Sec}[e+fx]^2 \tan[e+fx] (a+b+b \tan[e+fx]^2)}{(1 + \tan[e+fx]^2)^2} \right) \\
 & \left( \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (1 + \tan[e+fx]^2) \right) / \right. \\
 & \quad \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2(a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
 & \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] / \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - 3(a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
 & \left( \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] (1 + \tan[e+fx]^2)^2 \right) / \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \quad 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \Bigg) + \\
 & 3 \times 2^p (a+b) \tan[e+fx] (1 + \tan[e+fx]^2)^{-3+p} \left( \frac{a+b+b \tan[e+fx]^2}{1 + \tan[e+fx]^2} \right)^p
 \end{aligned}$$

$$\begin{aligned}
 & \left( \left( 4 \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) / \right. \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2(a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 2 \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \\
 & \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{4}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
 & \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) (1 + \tan[e+fx]^2) \right) / \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2(a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
 & \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - 2 \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, \right. \right. \\
 & \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) / \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - 3(a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 4 \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \right. \\
 & \left. \tan[e+fx] (1 + \tan[e+fx]^2) \right) / \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \right. \\
 & \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \right. \right. \\
 & \quad \left. \left. \operatorname{Sec}[e+f x]^2 \tan [e+f x] - \frac{2}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \right. \right. \\
 & \quad \left. \left. - \tan [e+f x]^2 \right) \operatorname{Sec}[e+f x]^2 \tan [e+f x] \right) (1 + \tan [e+f x]^2)^2 \Big/ \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] + \right. \\
 & \quad 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \right) \tan [e+f x]^2 \Big) - \\
 & \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] (1 + \tan [e+f x]^2) \right. \\
 & \quad \left( 4 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \right. \\
 & \quad \left. \left. 2(a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \right. \\
 & \quad \left. \operatorname{Sec}[e+f x]^2 \tan [e+f x] - 3(a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] - \frac{4}{3} \operatorname{AppellF1} \left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] \right) + \\
 & \quad 2 \tan [e+f x]^2 \left( -b p \left( -\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 2, 2-p, \frac{7}{2}, -\tan [e+f x]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] - \frac{12}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 3, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] \right) + \\
 & \quad 2(a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 3, 1-p, \frac{7}{2}, -\tan [e+f x]^2, \right. \right. \\
 & \quad \left. \left. -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] - \frac{18}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 4, -p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \tan [e+f x] \right) \Big) \Big) \Big) \Big) \Big/ \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + 2(a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \tan [e+f x]^2 \right) \Big) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( \text{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right. \\
 & \left( 4 \left( b p \text{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \quad \left. \left. 3 (a+b) \text{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \right. \\
 & \quad \left. \text{Sec}[e+fx]^2 \text{Tan}[e+fx] + 3 (a+b) \left( \frac{1}{3 (a+b)} 2 b p \text{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - 2 \text{AppellF1} \left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, 4, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right) + \\
 & \quad 2 \text{Tan}[e+fx]^2 \left( b p \left( -\frac{1}{5 (a+b)} 6 b (1-p) \text{AppellF1} \left[ \frac{5}{2}, 3, 2-p, \frac{7}{2}, -\text{Tan}[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - \frac{18}{5} \text{AppellF1} \left[ \frac{5}{2}, 4, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right) - \\
 & \quad 3 (a+b) \left( \frac{1}{5 (a+b)} 6 b p \text{AppellF1} \left[ \frac{5}{2}, 4, 1-p, \frac{7}{2}, -\text{Tan}[e+fx]^2, \right. \right. \\
 & \quad \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - \frac{24}{5} \text{AppellF1} \left[ \frac{5}{2}, 5, -p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right) \right) \Big/ \\
 & \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( b p \text{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] - 3 (a+b) \right. \\
 & \quad \left. \left. \text{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Tan}[e+fx]^2 \right)^2 + \\
 & \left( \text{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] (1 + \text{Tan}[e+fx]^2)^2 \right. \\
 & \quad \left( 4 \left( -b p \text{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] + \right. \right. \\
 & \quad \left. \left. (a+b) \text{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \right) \text{Sec}[e+fx]^2 \right. \\
 & \quad \left. \text{Tan}[e+fx] - 3 (a+b) \left( \frac{1}{3 (a+b)} 2 b p \text{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, \right. \right. \right. \\
 & \quad \left. \left. -\text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - \frac{2}{3} \text{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & \left. \left. \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + 2 \operatorname{Tan}[e+f x]^2 \right. \\
 & \left. \left( -b p \left( -\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right. \right. \right. \\
 & \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5(a+b)} 6 b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2-p, 1, \right. \right. \\
 & \left. \left. \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \right. \\
 & \left. (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
 & \left. \left. -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, -p, 3, \right. \right. \\
 & \left. \left. \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) \right) / \\
 & \left( -3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \right. \\
 & \left. 2 \left( -b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + (a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right)^2 \right) \right)
 \end{aligned}$$

**Problem 139: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x]^2 dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$\frac{1}{3 f} \operatorname{AppellF1}\left[\frac{3}{2}, 2, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Tan}[e+f x]^3 (a+b+b \operatorname{Tan}[e+f x]^2)^p \left(1+\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p}$$

Result (type 6, 3781 leaves):

$$\begin{aligned}
 & \left( 3(a+b) (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-2+p} (a+b \operatorname{Sec}[e+f x]^2)^p \right. \\
 & \left. \operatorname{Sin}[e+f x]^2 \operatorname{Tan}[e+f x] \left( \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \right) / \\
 & \left( -3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
 & \left. 2 \left( -b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \right. \\
 & \left. \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2 \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \text{Sec}[e+fx]^2 \right) / \\
 & \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \\
 & \quad \quad \left. (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \left. \right) \Bigg) / \\
 & \left( f \left( 3(a+b)(a+2b+a \cos[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-1+p} \right. \right. \\
 & \quad \left( \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] / \right. \\
 & \quad \left( -3(a+b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \quad 2 \left( -b p \text{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 2(a+b) \right. \\
 & \quad \quad \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \right. \\
 & \quad \left. \left( \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \text{Sec}[e+fx]^2 \right) / \right. \\
 & \quad \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \quad 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \text{AppellF1}\left[ \right. \right. \\
 & \quad \quad \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \left. \right) \Bigg) - \\
 & 6 a (a+b) p (a+2 b+a \cos [2(e+f x)])^{-1+p} (\text{Sec}[e+f x]^2)^{-2+p} \sin [2(e+f x)] \\
 & \tan [e+f x] \left( \text{AppellF1}\left[\frac{1}{2}, 2,-p, \frac{3}{2}, -\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] / \right. \\
 & \quad \left( -3(a+b) \text{AppellF1}\left[\frac{1}{2}, 2,-p, \frac{3}{2}, -\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] + \right. \\
 & \quad \quad 2 \left( -b p \text{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] + 2(a+b) \right. \\
 & \quad \quad \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}, 3,-p, \frac{5}{2}, -\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \right) \tan [e+f x]^2 \right) + \right. \\
 & \quad \left( \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2\right] \text{Sec}[e+f x]^2 \right) / \\
 & \quad \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2\right] + \right. \\
 & \quad \quad 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2\right] - (a+b) \text{AppellF1}\left[ \right. \right. \\
 & \quad \quad \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2\right] \right) \tan [e+f x]^2 \left. \right) \Bigg)
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right) \tan[e+fx]^2 \right) \right) + \\
 & 6 (a+b) (-2+p) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-2+p} \tan[e+fx]^2 \\
 & \left( \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] / \right. \\
 & \quad \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2 (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
 & \left. \left( \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \right) / \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - (a+b) \operatorname{AppellF1} \left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \right) + \\
 & 3 (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-2+p} \tan[e+fx] \\
 & \left( \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \\
 & \quad \left. \left. \sec[e+fx]^2 \tan[e+fx] - \frac{4}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) / \right. \\
 & \quad \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( -b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2 (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) / \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) + \\
 & \left( \sec[e+fx]^2 \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -\tan [e+f x]^2] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2},-p, 2,\right. \\
 & \left.\frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right) \Bigg) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right. \\
 & 2\left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]- \right. \\
 & \left.(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]\right) \tan [e+f x]^2\left.\right)- \\
 & \left(\operatorname{AppellF1}\left[\frac{1}{2}, 2,-p, \frac{3}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \right. \\
 & \left(4\left(-b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right]+ \right. \right. \\
 & \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 3,-p, \frac{5}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right]\right) \\
 & \operatorname{Sec}[e+f x]^2 \tan [e+f x]-3(a+b)\left(\frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \right. \right. \\
 & \left.\frac{5}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{4}{3} \operatorname{AppellF1}\left[ \right. \\
 & \left.\frac{3}{2}, 3,-p, \frac{5}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right)+ \\
 & 2 \tan [e+f x]^2\left(-b p\left(-\frac{1}{5(a+b)} 6 b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2, 2-p, \frac{7}{2},-\tan [e+f x]^2, \right. \right. \right. \\
 & \left.\left.-\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 3, 1-p, \right. \right. \\
 & \left.\frac{7}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right)+ \\
 & 2(a+b)\left(\frac{1}{5(a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 3, 1-p, \frac{7}{2},-\tan [e+f x]^2, \right. \right. \\
 & \left.\left.-\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{18}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 4,-p, \right. \right. \\
 & \left.\frac{7}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right)\left.\right)\left.\right) \Bigg) / \\
 & \left(-3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2,-p, \frac{3}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right]+ \right. \\
 & 2\left(-b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right]+2(a+b) \right. \\
 & \left.\operatorname{AppellF1}\left[\frac{3}{2}, 3,-p, \frac{5}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right]\right) \tan [e+f x]^2\left.\right)^2- \\
 & \left(\operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( 4 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \right. \\
 & \quad \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{2}{3} \operatorname{AppellF1} \left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \\
 & \quad 2 \tan[e+fx]^2 \left( b p \left( -\frac{6}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right. \right. \\
 & \quad \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 2-p, 1, \right. \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) - \\
 & \quad \left. (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{12}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, -p, 3, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \Big/ \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \Big) \Big)
 \end{aligned}$$

**Problem 140: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1} \left[ \frac{1}{2}, 1, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \tan[e+fx] (a+b + b \tan[e+fx]^2)^p \left( 1 + \frac{b \tan[e+fx]^2}{a+b} \right)^{-p}$$

Result (type 6, 2137 leaves):

$$\left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \cos[e+fx] \right.$$

$$\begin{aligned}
 & \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p (a + b \sec[e + fx]^2)^p \sin[e + fx] \right) / \\
 & \left( f \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \right. \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) \\
 & \left( \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \right. \\
 & \quad \left. \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{-1+p} \right) / \right. \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) - \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \\
 & \quad \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p \sin[e + fx]^2 \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) + \\
 & \left( 6(a + b) p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \\
 & \quad \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p \sin[e + fx]^2 \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) - \\
 & \left( 6a(a + b) p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \cos[e + fx] \right. \\
 & \quad \left. (a + 2b + a \cos[2(e + fx)])^{-1+p} (\sec[e + fx]^2)^p \sin[e + fx] \sin[2(e + fx)] \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \Big) + \\
 & \left( 3 (a+b) \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx] \right. \\
 & \left. \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right. \right. \\
 & \quad \left. \left. \sec[e+fx]^2 \tan[e+fx] - \frac{2}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) \Big) / \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \Big) - \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \cos[e+fx] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx] \right. \\
 & \left. \left( 4 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - \right. \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \right. \\
 & \quad \left. \sec[e+fx]^2 \tan[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] - \frac{2}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) + \\
 & 2 \tan[e+fx]^2 \left( b p \left( -\frac{6}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right. \right. \\
 & \quad \left. \left. \sec[e+fx]^2 \tan[e+fx] - \frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 2-p, 1, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) - \\
 & (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2 \right] \right)
 \end{aligned}$$

$$\begin{aligned} & -\tan [e+f x]^2] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2},-p, 3,\right. \\ & \left.\frac{7}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\right)\right)\right)\right) / \\ & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right. \\ & \left.2\left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]- \right. \right. \\ & \left. \left.(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]\right) \tan [e+f x]^2\right)^2\right)\right) \end{aligned}$$

**Problem 152: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+f x]^5(a+b \operatorname{Sec}[e+f x]^2) d x$$

Optimal (type 3, 98 leaves, 4 steps):

$$\begin{aligned} & \frac{(6 a+5 b) \operatorname{ArcTanh}[\operatorname{Sin}[e+f x]]}{16 f}+\frac{(6 a+5 b) \operatorname{Sec}[e+f x] \tan [e+f x]}{16 f}+ \\ & \frac{(6 a+5 b) \operatorname{Sec}[e+f x]^3 \tan [e+f x]}{24 f}+\frac{b \operatorname{Sec}[e+f x]^5 \tan [e+f x]}{6 f} \end{aligned}$$

Result (type 3, 445 leaves):

$$\begin{aligned} & -\frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f}-\frac{5 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{16 f}+ \\ & \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f}+\frac{5 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{16 f}+ \\ & \frac{48 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6}{b}+\frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{a}+ \\ & \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{5 b}+\frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{3 a}+ \\ & \frac{32 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{a}-\frac{48 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6}{b}- \\ & \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{3 a}-\frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{5 b}- \\ & \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{32 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2} \end{aligned}$$



### Problem 155: Result more than twice size of optimal antiderivative.

$$\int \cos[e + f x] (a + b \sec[e + f x]^2) dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{b \operatorname{ArcTanh}[\sin[e + f x]]}{f} + \frac{a \sin[e + f x]}{f}$$

Result (type 3, 92 leaves):

$$-\frac{b \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{a \cos[fx] \sin[e]}{f} + \frac{a \cos[e] \sin[fx]}{f}$$

### Problem 165: Result more than twice size of optimal antiderivative.

$$\int \sec[e + f x]^5 (a + b \sec[e + f x]^2)^2 dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\frac{(48 a^2 + 80 a b + 35 b^2) \operatorname{ArcTanh}[\sin[e + f x]]}{128 f} + \frac{(48 a^2 + 80 a b + 35 b^2) \sec[e + f x] \tan[e + f x]}{128 f} + \frac{(48 a^2 + 80 a b + 35 b^2) \sec[e + f x]^3 \tan[e + f x]}{192 f} + \frac{b (10 a + 7 b) \sec[e + f x]^5 \tan[e + f x]}{48 f} + \frac{b \sec[e + f x]^7 (a + b - a \sin[e + f x]^2) \tan[e + f x]}{8 f}$$

Result (type 3, 803 leaves):

$$\begin{aligned}
 & - \frac{3 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f} - \frac{5 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f} \\
 & - \frac{35 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{128 f} + \frac{3 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f} + \\
 & + \frac{5 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f} + \frac{35 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{128 f} + \\
 & + \frac{128 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^8}{5 b^2} + \frac{24 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6}{a^2} + \\
 & + \frac{192 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6}{a b} + \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{15 b^2} + \\
 & + \frac{8 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{3 a^2} + \frac{256 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{5 a b} + \\
 & + \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{35 b^2} + \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{b^2} + \\
 & - \frac{256 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{a b} - \frac{128 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^8}{5 b^2} - \\
 & - \frac{24 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6}{a^2} - \frac{192 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6}{a b} - \\
 & - \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{15 b^2} - \frac{8 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{3 a^2} - \\
 & - \frac{256 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4}{5 a b} - \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{35 b^2} - \\
 & - \frac{16 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{256 f} - \frac{256 f\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2}{256 f}
 \end{aligned}$$

**Problem 166: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+f x]^3 (a+b \operatorname{Sec}[e+f x]^2)^2 dx$$

Optimal (type 3, 129 leaves, 5 steps):

$$\frac{(8 a^2+12 a b+5 b^2) \operatorname{ArcTanh}[\operatorname{Sin}[e+f x]]}{16 f} + \frac{(8 a^2+12 a b+5 b^2) \operatorname{Sec}[e+f x] \operatorname{Tan}[e+f x]}{16 f} + \\
 \frac{b(8 a+5 b) \operatorname{Sec}[e+f x]^3 \operatorname{Tan}[e+f x]}{24 f} + \frac{b \operatorname{Sec}[e+f x]^5 (a+b-a \operatorname{Sin}[e+f x]^2) \operatorname{Tan}[e+f x]}{6 f}$$

Result (type 3, 601 leaves):

$$\begin{aligned}
 & - \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right]}{2f} - \frac{3ab \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right]}{4f} - \\
 & \frac{5b^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right]}{16f} + \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right]}{2f} + \\
 & \frac{3ab \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right]}{4f} + \frac{5b^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right]}{16f} + \\
 & \frac{48f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^6}{b^2} + \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{a^2} + \\
 & \frac{16f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{3ab} + \frac{4f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{5b^2} + \\
 & \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{b^2} - \frac{32f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{ab} - \\
 & \frac{48f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^6}{b^2} - \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{a^2} - \\
 & \frac{16f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{3ab} - \frac{4f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{5b^2} - \\
 & \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{b^2} - \frac{32f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{ab}
 \end{aligned}$$

**Problem 169: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^2 dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{b^2 \operatorname{ArcTanh}\left[\sin[e+fx]\right]}{f} + \frac{a(a+2b) \sin[e+fx]}{f} - \frac{a^2 \sin[e+fx]^3}{3f}$$

Result (type 3, 134 leaves):

$$\begin{aligned}
 & - \frac{b^2 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b^2 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \\
 & \frac{2ab \cos[fx] \sin[e]}{f} + \frac{2ab \cos[e] \sin[fx]}{f} + \frac{3a^2 \sin[e+fx]}{4f} + \frac{a^2 \sin[3(e+fx)]}{12f}
 \end{aligned}$$

**Problem 170: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^2 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{(a+b)^2 \sin[e+fx]}{f} - \frac{2a(a+b) \sin[e+fx]^3}{3f} + \frac{a^2 \sin[e+fx]^5}{5f}$$

Result (type 3, 111 leaves):

$$\frac{b^2 \cos[fx] \sin[e]}{f} + \frac{b^2 \cos[e] \sin[fx]}{f} + \frac{5a^2 \sin[e+fx]}{8f} + \frac{3ab \sin[e+fx]}{2f} + \frac{5a^2 \sin[3(e+fx)]}{48f} + \frac{ab \sin[3(e+fx)]}{6f} + \frac{a^2 \sin[5(e+fx)]}{80f}$$

**Problem 171: Result more than twice size of optimal antiderivative.**

$$\int \sec[e+fx]^6 (a+b \sec[e+fx]^2)^2 dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$\frac{(a+b)^2 \tan[e+fx]}{f} + \frac{2(a+b)(a+2b) \tan[e+fx]^3}{3f} + \frac{(a^2+6ab+6b^2) \tan[e+fx]^5}{5f} + \frac{2b(a+2b) \tan[e+fx]^7}{7f} + \frac{b^2 \tan[e+fx]^9}{9f}$$

Result (type 3, 261 leaves):

$$\frac{8a^2 \tan[e+fx]}{15f} + \frac{32ab \tan[e+fx]}{35f} + \frac{128b^2 \tan[e+fx]}{315f} + \frac{4a^2 \sec[e+fx]^2 \tan[e+fx]}{15f} + \frac{16ab \sec[e+fx]^2 \tan[e+fx]}{35f} + \frac{64b^2 \sec[e+fx]^2 \tan[e+fx]}{315f} + \frac{a^2 \sec[e+fx]^4 \tan[e+fx]}{5f} + \frac{12ab \sec[e+fx]^4 \tan[e+fx]}{35f} + \frac{16b^2 \sec[e+fx]^4 \tan[e+fx]}{105f} + \frac{2ab \sec[e+fx]^6 \tan[e+fx]}{7f} + \frac{8b^2 \sec[e+fx]^6 \tan[e+fx]}{63f} + \frac{b^2 \sec[e+fx]^8 \tan[e+fx]}{9f}$$

**Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \sec[e+fx]^4 (a+b \sec[e+fx]^2)^2 dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$\frac{(a+b)^2 \tan[e+fx]}{f} + \frac{(a+b)(a+3b) \tan[e+fx]^3}{3f} + \frac{b(2a+3b) \tan[e+fx]^5}{5f} + \frac{b^2 \tan[e+fx]^7}{7f}$$

Result (type 3, 190 leaves):

$$\frac{2a^2 \tan[e+fx]}{3f} + \frac{16ab \tan[e+fx]}{15f} + \frac{16b^2 \tan[e+fx]}{35f} + \frac{a^2 \sec[e+fx]^2 \tan[e+fx]}{3f} + \frac{8ab \sec[e+fx]^2 \tan[e+fx]}{15f} + \frac{8b^2 \sec[e+fx]^2 \tan[e+fx]}{35f} + \frac{2ab \sec[e+fx]^4 \tan[e+fx]}{5f} + \frac{6b^2 \sec[e+fx]^4 \tan[e+fx]}{35f} + \frac{b^2 \sec[e+fx]^6 \tan[e+fx]}{7f}$$

**Problem 173: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^2 (a + b \text{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{(a + b)^2 \text{Tan}[e + f x]}{f} + \frac{2 b (a + b) \text{Tan}[e + f x]^3}{3 f} + \frac{b^2 \text{Tan}[e + f x]^5}{5 f}$$

Result (type 3, 116 leaves):

$$\frac{a^2 \text{Tan}[e + f x]}{f} + \frac{4 a b \text{Tan}[e + f x]}{3 f} + \frac{8 b^2 \text{Tan}[e + f x]}{15 f} + \frac{2 a b \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{3 f} + \frac{4 b^2 \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{15 f} + \frac{b^2 \text{Sec}[e + f x]^4 \text{Tan}[e + f x]}{5 f}$$

**Problem 174: Result more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b (2 a + b) \text{Tan}[e + f x]}{f} + \frac{b^2 \text{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 106 leaves):

$$\left( 4 (b + a \text{Cos}[e + f x]^2)^2 \text{Sec}[e + f x]^3 \right. \\ \left. (3 a^2 f x \text{Cos}[e + f x]^3 + b^2 \text{Sec}[e] \text{Sin}[f x] + 2 b (3 a + b) \text{Cos}[e + f x]^2 \text{Sec}[e] \text{Sin}[f x] + \right. \\ \left. b^2 \text{Cos}[e + f x] \text{Tan}[e]) \right) / \left( 3 f (a + 2 b + a \text{Cos}[2 (e + f x)])^2 \right)$$

**Problem 178: Result more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[c + d x]^2)^3 dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a^3 x + \frac{b (3 a^2 + 3 a b + b^2) \text{Tan}[c + d x]}{d} + \frac{b^2 (3 a + 2 b) \text{Tan}[c + d x]^3}{3 d} + \frac{b^3 \text{Tan}[c + d x]^5}{5 d}$$

Result (type 3, 268 leaves):

$$\frac{1}{480 d} \text{Sec}[c] \text{Sec}[c + d x]^5 \\ (150 a^3 d x \text{Cos}[d x] + 150 a^3 d x \text{Cos}[2 c + d x] + 75 a^3 d x \text{Cos}[2 c + 3 d x] + 75 a^3 d x \text{Cos}[4 c + 3 d x] + \\ 15 a^3 d x \text{Cos}[4 c + 5 d x] + 15 a^3 d x \text{Cos}[6 c + 5 d x] + 540 a^2 b \text{Sin}[d x] + 420 a b^2 \text{Sin}[d x] + \\ 160 b^3 \text{Sin}[d x] - 360 a^2 b \text{Sin}[2 c + d x] - 180 a b^2 \text{Sin}[2 c + d x] + 360 a^2 b \text{Sin}[2 c + 3 d x] + \\ 300 a b^2 \text{Sin}[2 c + 3 d x] + 80 b^3 \text{Sin}[2 c + 3 d x] - 90 a^2 b \text{Sin}[4 c + 3 d x] + \\ 90 a^2 b \text{Sin}[4 c + 5 d x] + 60 a b^2 \text{Sin}[4 c + 5 d x] + 16 b^3 \text{Sin}[4 c + 5 d x])$$

**Problem 179: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x]^2)^4 dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$a^4 x + \frac{b (2 a + b) (2 a^2 + 2 a b + b^2) \operatorname{Tan}[c + d x]}{d} + \frac{b^2 (6 a^2 + 8 a b + 3 b^2) \operatorname{Tan}[c + d x]^3}{3 d} + \frac{b^3 (4 a + 3 b) \operatorname{Tan}[c + d x]^5}{5 d} + \frac{b^4 \operatorname{Tan}[c + d x]^7}{7 d}$$

Result (type 3, 455 leaves):

$$\frac{1}{13440 d} \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^7 (3675 a^4 d x \operatorname{Cos}[d x] + 3675 a^4 d x \operatorname{Cos}[2 c + d x] + 2205 a^4 d x \operatorname{Cos}[2 c + 3 d x] + 2205 a^4 d x \operatorname{Cos}[4 c + 3 d x] + 735 a^4 d x \operatorname{Cos}[4 c + 5 d x] + 735 a^4 d x \operatorname{Cos}[6 c + 5 d x] + 105 a^4 d x \operatorname{Cos}[6 c + 7 d x] + 105 a^4 d x \operatorname{Cos}[8 c + 7 d x] + 16800 a^3 b \operatorname{Sin}[d x] + 18480 a^2 b^2 \operatorname{Sin}[d x] + 11200 a b^3 \operatorname{Sin}[d x] + 3360 b^4 \operatorname{Sin}[d x] - 12600 a^3 b \operatorname{Sin}[2 c + d x] - 10920 a^2 b^2 \operatorname{Sin}[2 c + d x] - 4480 a b^3 \operatorname{Sin}[2 c + d x] + 12600 a^3 b \operatorname{Sin}[2 c + 3 d x] + 15120 a^2 b^2 \operatorname{Sin}[2 c + 3 d x] + 9408 a b^3 \operatorname{Sin}[2 c + 3 d x] + 2016 b^4 \operatorname{Sin}[2 c + 3 d x] - 5040 a^3 b \operatorname{Sin}[4 c + 3 d x] - 2520 a^2 b^2 \operatorname{Sin}[4 c + 3 d x] + 5040 a^3 b \operatorname{Sin}[4 c + 5 d x] + 5880 a^2 b^2 \operatorname{Sin}[4 c + 5 d x] + 3136 a b^3 \operatorname{Sin}[4 c + 5 d x] + 672 b^4 \operatorname{Sin}[4 c + 5 d x] - 840 a^3 b \operatorname{Sin}[6 c + 5 d x] + 840 a^3 b \operatorname{Sin}[6 c + 7 d x] + 840 a^2 b^2 \operatorname{Sin}[6 c + 7 d x] + 448 a b^3 \operatorname{Sin}[6 c + 7 d x] + 96 b^4 \operatorname{Sin}[6 c + 7 d x])$$

**Problem 180: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^5}{a + b \operatorname{Sec}[e + f x]^2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$-\frac{(2 a - b) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 b^2 f} + \frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e + f x]}{\sqrt{a + b}}\right]}{b^2 \sqrt{a + b} f} + \frac{\operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 b f}$$

Result (type 3, 2519 leaves):

$$\left( (2 a - b) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] \operatorname{Sec}[e + f x]^2 \right) / (4 b^2 f (a + b \operatorname{Sec}[e + f x]^2)) + \left( (-2 a + b) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] \operatorname{Sec}[e + f x]^2 \right) / (4 b^2 f (a + b \operatorname{Sec}[e + f x]^2)) + \frac{1}{4 b^2 \sqrt{a + b} f (a + b \operatorname{Sec}[e + f x]^2) \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]}} i a^{3/2} \operatorname{ArcTan}\left[ \left( -i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3 e] + i b \operatorname{Cos}[3 e] + a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a + b} \operatorname{Cos}[e - f x] \right) \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right] +$$

$$\begin{aligned}
 & \frac{\sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} + a \sin[3e] + b \sin[3e] -}{i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]} \\
 & \left( (a \cos[e] + 3b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3ia \sin[e] - ib \sin[e] - ia \sin[3e] - ib \sin[3e] - ia \sin[e+2fx] + ia \sin[3e+2fx]) \right) \\
 & \cos[e] (a+2b+a \cos[2e+2fx]) \sec[e+fx]^2 - \left( a^{3/2} \operatorname{ArcTanh} \left[ \frac{2(a+b) \sin[e]}{\sqrt{-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}} \right] \right) \\
 & \cos[e] (a+2b+a \cos[2e+2fx]) \sec[e+fx]^2 \Big/ \\
 & \left( 4b^2 \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{\cos[2e]-i \sin[2e]} \right) + \\
 & \left( a^{3/2} \cos[e] (a+2b+a \cos[2e+2fx]) \log \left[ \frac{a+2a \cos[2e]+2b \cos[2e]-a \cos[2e+2fx]-2ia \sin[2e]-2ib \sin[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[fx]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[2e+fx]}{\sqrt{-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}} \right] \right) \\
 & \left( 8b^2 \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{\cos[2e]-i \sin[2e]} \right) - \\
 & \left( a^{3/2} \cos[e] (a+2b+a \cos[2e+2fx]) \log \left[ \frac{-a-2a \cos[2e]-2b \cos[2e]+a \cos[2e+2fx]+2ia \sin[2e]+2ib \sin[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[fx]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[2e+fx]}{\sqrt{-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}} \right] \right) \\
 & \left( 8b^2 \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{\cos[2e]-i \sin[2e]} \right) + \\
 & \frac{1}{4b^2 \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{\cos[2e]-i \sin[2e]}} \\
 & a^{3/2} \operatorname{ArcTan} \left[ \frac{-ia \cos[e] - ib \cos[e] + ia \cos[3e] + ib \cos[3e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}{(a \cos[e] + 3b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3ia \sin[e] - ib \sin[e] - ia \sin[3e] - ib \sin[3e] - ia \sin[e+2fx] + ia \sin[3e+2fx])}{(a+2b+a \cos[2e+2fx]) \sec[e+fx]^2 \sin[e] + \left( ia^{3/2} \operatorname{ArcTanh} \left[ \frac{2(a+b) \sin[e]}{\sqrt{-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}} \right] \right)} \right] \\
 & \left( 4b^2 \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{\cos[2e]-i \sin[2e]} \right) - \\
 & \left( ia^{3/2} (a+2b+a \cos[2e+2fx]) \log \left[ \frac{a+2a \cos[2e]+2b \cos[2e]-a \cos[2e+2fx]-2ia \sin[2e]-2ib \sin[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[fx]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[2e+fx]}{\sqrt{-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}} \right] \right) \\
 & \left( 8b^2 \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{\cos[2e]-i \sin[2e]} \right) + \\
 & \left( ia^{3/2} (a+2b+a \cos[2e+2fx]) \log \left[ \frac{-a-2a \cos[2e]-2b \cos[2e]+a \cos[2e+2fx]+2ia \sin[2e]+2ib \sin[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[fx]+2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[2e+fx]}{\sqrt{-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e]-i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e]-i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e]-i \sin[2e]} \sin[3e+fx]}} \right] \right)
 \end{aligned}$$

$$\frac{2 i a \sin [2 e]+2 i b \sin [2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{\cos [2 e]-i \sin [2 e]} \sin [f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{\cos [2 e]-i \sin [2 e]} \sin [2 e+f x]}{\left(8 b^2 \sqrt{a+b} f\left(a+b \sec [e+f x]^2\right) \sqrt{\cos [2 e]-i \sin [2 e]}\right)+\left(a+2 b+a \cos [2 e+2 f x]\right) \sec [e+f x]^2} + \frac{8 b f\left(a+b \sec [e+f x]^2\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2}{\left(a+2 b+a \cos [2 e+2 f x]\right) \sec [e+f x]^2} - \frac{8 b f\left(a+b \sec [e+f x]^2\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2}{\left(a+2 b+a \cos [2 e+2 f x]\right) \sec [e+f x]^2}$$

**Problem 181: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [e+f x]^3}{a+b \sec [e+f x]^2} d x$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin [e+f x]]}{b f} - \frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [e+f x]}{\sqrt{a+b}}\right]}{b \sqrt{a+b} f}$$

Result (type 3, 1022 leaves):



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$$\begin{aligned}
 & 8 b \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2 \left(-\sqrt{a} \operatorname{Cos}[e] \right. \\
 & \quad \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \right. \\
 & \quad \left. \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right]+ \\
 & \quad \sqrt{a} \operatorname{Cos}[e] \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+ \right. \\
 & \quad \left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \\
 & \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right]- \\
 & \quad 2 i \sqrt{a} \operatorname{ArcTan}\left[\left(2 \operatorname{Sin}[e]\left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \right. \right. \right. \\
 & \quad \left. \left. \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \right. \right. \\
 & \quad \left. \left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]- \right. \right. \\
 & \quad \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right)\right] / \\
 & \quad \left(i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+ \right. \\
 & \quad \left. 3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x]\right) \\
 & \quad \left(\operatorname{Cos}[e]-i \operatorname{Sin}[e]\right)-4 \sqrt{a+b} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] \\
 & \quad \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \\
 & \quad 4 \sqrt{a+b} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \\
 & \quad i \sqrt{a} \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]- \right. \\
 & \quad \left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \\
 & \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right] \operatorname{Sin}[e]- \\
 & \quad i \sqrt{a} \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+ \right. \\
 & \quad \left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \\
 & \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right] \operatorname{Sin}[e]+ \\
 & \quad 2 \sqrt{a} \operatorname{ArcTan}\left[\left((a+b) \operatorname{Sin}[e]\right) / \left((a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \right. \right. \\
 & \quad \left. \left. (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]\right)\right] (i \operatorname{Cos}[e]+\operatorname{Sin}[e]) \Big)
 \end{aligned}$$

**Problem 182: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}}\right]}{\sqrt{a} \sqrt{a+b} f}$$

Result (type 3, 653 leaves):

$$\frac{1}{8 \sqrt{a} \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}}$$

$$\left( (a+2 b+a \operatorname{Cos}[2(e+f x)]) \left( -2 i \operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Sin}[e]}{(a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]}}\right] + \right. \right.$$

$$\left. 2 i \operatorname{ArcTan}\left[\frac{2 \operatorname{Sin}[e] \left( i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \right)}{\sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x]} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+} \right. \right.$$

$$\left. \left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]}\right] \right) /$$

$$\left( i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+ \right.$$

$$\left. 3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x] \right) +$$

$$\operatorname{Log}\left[ a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+ \right.$$

$$\left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right.$$

$$\left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] -$$

$$\operatorname{Log}\left[ -a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+ \right.$$

$$\left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right.$$

$$\left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sec}[e+f x]^2 (\operatorname{Cos}[e]-i \operatorname{Sin}[e])$$

**Problem 183: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+f x]}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}}\right]}{a^{3/2} \sqrt{a+b} f} + \frac{\operatorname{Sin}[e+f x]}{a f}$$

Result (type 3, 941 leaves):

1

$$\begin{aligned}
 & 8 a^{3/2} \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2 \\
 & \left( -b \operatorname{Cos}[e] \operatorname{Log}[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+ \right. \\
 & \quad 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \\
 & \quad 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]]+b \operatorname{Cos}[e] \\
 & \operatorname{Log}[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \\
 & \quad \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]]+ \\
 & i b \operatorname{Log}[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \\
 & \quad \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]] \\
 & \operatorname{Sin}[e]-i b \operatorname{Log}[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]]+2 i a \operatorname{Sin}[2 e]+ \\
 & \quad 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \\
 & \quad 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]] \operatorname{Sin}[e]+ \\
 & 2 b \operatorname{ArcTan}\left[\frac{((a+b) \operatorname{Sin}[e])}{((a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} \right. \\
 & \quad \left. (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x])\right](i \operatorname{Cos}[e]+\operatorname{Sin}[e])+ \\
 & \operatorname{ArcTan}\left[\frac{2 \operatorname{Sin}[e]\left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x]\right) \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\right. \\
 & \quad \sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+a \operatorname{Sin}[2 e]+ \\
 & \quad b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]- \\
 & \quad \left. i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x])\right]}{ \\
 & \quad (i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+ \\
 & \quad 3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x])} \\
 & \quad \left. (-2 i b \operatorname{Cos}[e]-2 b \operatorname{Sin}[e])+4 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[e+f x]\right)
 \end{aligned}$$

**Problem 186: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]^6}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{b^{5/2} \sqrt{a+b} f} - \frac{(a-b) \operatorname{Tan}[e+f x]}{b^2 f} + \frac{\operatorname{Tan}[e+f x]^3}{3 b f}$$

Result (type 3, 224 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^2 \right. \\ \left. \left( -3a^2 \operatorname{ArcTan}\left[ \frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] \right) \right. \\ \left. \left( \cos[2e] - i \sin[2e] + \sqrt{a+b} \sec[e + fx] \sqrt{b (i \cos[e] + \sin[e])^4} \right) \right. \\ \left. \left( \sec[e] (-3a + 2b + b \sec[e + fx]^2) \sin[fx] + b \sec[e + fx] \tan[e] \right) \right) \Big/ \\ \left( 6b^2 \sqrt{a+b} f (a + b \sec[e + fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right)$$

**Problem 187: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^4}{a + b \sec[e + fx]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{a \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{b^{3/2} \sqrt{a+b} f} + \frac{\tan[e + fx]}{bf}$$

Result (type 3, 192 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^2 \right. \\ \left( a \operatorname{ArcTan}\left[ \frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] \right) \Big/ \\ \left( 2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \left( \cos[2e] - i \sin[2e] + \right. \\ \left. \sqrt{a+b} \sec[e] \sec[e + fx] \sqrt{b (i \cos[e] + \sin[e])^4} \sin[fx] \right) \Big/ \\ \left( 2b \sqrt{a+b} f (a + b \sec[e + fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right)$$

**Problem 189: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + b \sec[e + fx]^2} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{x}{a} + \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \cot[e + fx]}{\sqrt{b}}\right]}{a \sqrt{a+b} f}$$

Result (type 3, 182 leaves):

$$\begin{aligned} & \left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^2 \left( \sqrt{a+b} f x \sqrt{b (\cos[e] - i \sin[e])^4} + \right. \right. \\ & \quad \left. \left. b \operatorname{ArcTan} \left[ \left( \sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx]) \right) \right] \right) \right) / \\ & \quad \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (\cos[2e] - i \sin[2e]) \Big) / \\ & \left( 2 a \sqrt{a+b} f (a + b \sec[e + fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right) \end{aligned}$$

**Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^5}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 102 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}[\sin[e + fx]]}{b^2 f} - \frac{\sqrt{a} (2a + 3b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[e + fx]}{\sqrt{a+b}}\right]}{2b^2 (a+b)^{3/2} f} - \frac{a \sin[e + fx]}{2b (a+b) f (a+b - a \sin[e + fx]^2)}$$

Result (type 3, 2333 leaves):

$$\begin{aligned} & - \left( \left( (a + 2b + a \cos[2e + 2fx])^2 \operatorname{Log} \left[ \cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right] \sec[e + fx]^4 \right) / \right. \\ & \quad \left. (4b^2 f (a + b \sec[e + fx]^2)^2) \right) + \\ & \left( (a + 2b + a \cos[2e + 2fx])^2 \operatorname{Log} \left[ \cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right] \sec[e + fx]^4 \right) / \\ & \quad (4b^2 f (a + b \sec[e + fx]^2)^2) + \\ & \frac{1}{(a + b) (a + b \sec[e + fx]^2)^2} (-2a^2 - 3ab) (a + 2b + a \cos[2e + 2fx])^2 \\ & \quad \sec[e + fx]^4 \left( i \operatorname{ArcTan} \left[ (-i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + \right. \right. \\ & \quad a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e - fx] \sqrt{\cos[2e] - i \sin[2e]} + \\ & \quad \sqrt{a} \sqrt{a+b} \cos[3e + fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \\ & \quad \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e - fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \\ & \quad \left. \left. \sin[e + fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + fx] \right) \right] / (a \cos[e] + 3b \\ & \quad \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e + 2fx] + a \cos[3e + 2fx] - 3i a \sin[e] - \\ & \quad i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e + 2fx] + i a \sin[3e + 2fx]) \Big] \\ & \quad \cos[e] \Big) / (16 \sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]}) + \\ & \left( \operatorname{ArcTan} \left[ (-i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + \right. \right. \\ & \quad b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e - fx] \sqrt{\cos[2e] - i \sin[2e]} + \\ & \quad \sqrt{a} \sqrt{a+b} \cos[3e + fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \\ & \quad \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e - fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \\ & \quad \left. \left. \sin[e + fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + fx] \right) \right] / (a \cos[e] + 3b \\ & \quad \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e + 2fx] + a \cos[3e + 2fx] - 3i a \sin[e] - \\ & \quad i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e + 2fx] + i a \sin[3e + 2fx]) \Big] \end{aligned}$$

$$\begin{aligned}
 & \frac{\sin[e]}{1} \Big/ \left( 16 \sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\
 & \frac{1}{(a+b) (a+b \sec[e+fx])^2} (2a+3b) (a+2b+a \cos[2e+2fx])^2 \\
 & \sec[e+fx]^4 \\
 & \left( \left( \sqrt{a} \operatorname{ArcTanh}[(2(a+b) \sin[e])] \Big/ \right. \right. \\
 & \quad \left( -2 i a \cos[e] - 2 i b \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \right. \\
 & \quad \left. \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \right. \\
 & \quad \left. \left. \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \cos[e] \right) \Big/ \\
 & \left( 16 b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \left( i \sqrt{a} \operatorname{ArcTanh}[(2(a+b) \sin[e])] \Big/ \right. \\
 & \quad \left( -2 i a \cos[e] - 2 i b \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \right. \\
 & \quad \left. \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \right. \\
 & \quad \left. \left. \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \sin[e] \right) \Big/ \\
 & \left( 16 b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \Big) + \frac{1}{(a+b) (a+b \sec[e+fx])^2} \\
 & (-2a^2 - 3ab) (a+2b+a \cos[2e+2fx])^2 \\
 & \sec[e+fx]^4 \\
 & \left( \left( \cos[e] \log[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - \right. \right. \\
 & \quad \left. \left. 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \right. \\
 & \quad \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right) \Big/ \\
 & \left( 32 \sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \left( i \log[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - \right. \\
 & \quad \left. 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \\
 & \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \sin[e] \Big/ \\
 & \left( 32 \sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \Big) + \\
 & \frac{1}{(a+b) (a+b \sec[e+fx])^2} (2a+3b) \\
 & (a+2b+a \cos[2e+2fx])^2 \\
 & \sec[e+fx]^4 \\
 & \left( \left( \sqrt{a} \cos[e] \log[-a-2a \cos[2e] - 2b \cos[2e] + a \cos[2e+2fx] + 2 i a \sin[2e] + \right. \right. \\
 & \quad \left. \left. 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2 \sqrt{a} \sqrt{a+b} \right. \right. \\
 & \quad \left. \left. \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right) \Big/ \left( 32 b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \left( i \sqrt{a} \log[-a-2a \cos[2e] - 2b \cos[2e] + a \cos[2e+2fx] + 2 i a \sin[2e] + \right. \\
 & \quad \left. 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \\
 & \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \\
 & \sin[e] \Big/ \left( 32 b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \Big) - \\
 & \frac{a (a+2b+a \cos[2e+2fx]) \sec[e+fx]^3 \tan[e+fx]}{4b (a+b) f (a+b \sec[e+fx])^2}
 \end{aligned}$$

Problem 194: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a} \text{Sin}[e + f x]}{\sqrt{a + b}}\right]}{2 \sqrt{a} (a + b)^{3/2} f} + \frac{\text{Sin}[e + f x]}{2 (a + b) f (a + b - a \text{Sin}[e + f x]^2)}$$

Result (type 3, 798 leaves):

$$\frac{1}{32 \sqrt{a} (a + b)^{3/2} f (a + b \text{Sec}[e + f x]^2)^2 \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \left( (a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x]^3 \left( -2i \text{ArcTan}\left[\frac{(a + b) \text{Sin}[e]}{(a + b) \text{Cos}[e] - \sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} (\text{Cos}[2e] + i \text{Sin}[2e]) \text{Sin}[e + f x]}\right]} \right) \right. \\ \left. \left( (a + b) \text{Cos}[e] - \sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} (\text{Cos}[2e] + i \text{Sin}[2e]) \text{Sin}[e + f x] \right) \right) + \\ (a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x] (\text{Cos}[e] - i \text{Sin}[e]) + \\ (a + 2b + a \text{Cos}[2(e + f x)]) \text{Log}[a + 2(a + b) \text{Cos}[2e] - a \text{Cos}[2(e + f x)]] - \\ 2i a \text{Sin}[2e] - 2i b \text{Sin}[2e] + 2\sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Sin}[f x] + \\ 2\sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Sin}[2e + f x] \text{Sec}[e + f x] (\text{Cos}[e] - i \text{Sin}[e]) - \\ (a + 2b + a \text{Cos}[2(e + f x)]) \text{Log}[-a - 2(a + b) \text{Cos}[2e] + a \text{Cos}[2(e + f x)]] + \\ 2i a \text{Sin}[2e] + 2i b \text{Sin}[2e] + 2\sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Sin}[f x] + \\ 2\sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Sin}[2e + f x] \text{Sec}[e + f x] (\text{Cos}[e] - i \text{Sin}[e]) + \\ 2 \text{ArcTan}\left[\frac{2 \text{Sin}[e] (i a + i b + i (a + b) \text{Cos}[2e] + \sqrt{a} \sqrt{a + b} \text{Cos}[f x]}{\sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} - \sqrt{a} \sqrt{a + b} \text{Cos}[2e + f x]} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} + a \text{Sin}[2e] + b \text{Sin}[2e] - i \sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Sin}[f x] - i \sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Sin}[2e + f x]}\right]} \right) \\ \left. \left( i (a + 3b) \text{Cos}[e] + i (a + b) \text{Cos}[3e] + i a \text{Cos}[e + 2f x] + i a \text{Cos}[3e + 2f x] + 3a \text{Sin}[e] + b \text{Sin}[e] + a \text{Sin}[3e] + b \text{Sin}[3e] + a \text{Sin}[e + 2f x] - a \text{Sin}[3e + 2f x] \right) \right) \\ (a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x] (i \text{Cos}[e] + \text{Sin}[e]) + \\ 8 \sqrt{a} \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan}[e + f x] \right)$$

Problem 195: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 83 leaves, 3 steps):

$$\frac{(2 a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e + f x]}{\sqrt{a+b}}\right]}{2 a^{3/2} (a+b)^{3/2} f} - \frac{b \operatorname{Sin}[e + f x]}{2 a (a+b) f (a+b - a \operatorname{Sin}[e + f x]^2)}$$

Result (type 3, 819 leaves):

$$\frac{1}{32 a^{3/2} (a+b)^{3/2} f (a+b \operatorname{Sec}[e + f x]^2)^2 \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \left( (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^3 \left( -2 i (2 a+b) \operatorname{ArcTan}\left[\left(\frac{(a+b) \operatorname{Sin}[e]}{\sqrt{(a+b) \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} (\operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]}}\right)}\right] \right. \right. \\ \left. \left. + (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) + (2 a+b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e] - a \operatorname{Cos}[2(e+f x)]\right] - 2 i a \operatorname{Sin}[2 e] - 2 i b \operatorname{Sin}[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x] + 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) - (2 a+b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e] + a \operatorname{Cos}[2(e+f x)]\right] + 2 i a \operatorname{Sin}[2 e] + 2 i b \operatorname{Sin}[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x] + 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) + 2(2 a+b) \operatorname{ArcTan}\left[\left(\frac{2 \operatorname{Sin}[e] (i a+i b+i(a+b) \operatorname{Cos}[2 e] + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] + \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] + \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} + a \operatorname{Sin}[2 e] + b \operatorname{Sin}[2 e] - i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x] - i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]}}\right)}\right] \right) \left. \right) \\ \left. + (i(a+3 b) \operatorname{Cos}[e] + i(a+b) \operatorname{Cos}[3 e] + i a \operatorname{Cos}[e+2 f x] + i a \operatorname{Cos}[3 e+2 f x] + 3 a \operatorname{Sin}[e] + b \operatorname{Sin}[e] + a \operatorname{Sin}[3 e] + b \operatorname{Sin}[3 e] + a \operatorname{Sin}[e+2 f x] - a \operatorname{Sin}[3 e+2 f x]) \right) \\ \left. (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (i \operatorname{Cos}[e] + \operatorname{Sin}[e]) - 8 \sqrt{a} b \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}[e+f x] \right)$$

**Problem 196: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e + f x]}{(a + b \operatorname{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$-\frac{b(4 a+3 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}}\right]}{2 a^{5/2} (a+b)^{3/2} f} + \frac{\operatorname{Sin}[e+f x]}{a^2 f} + \frac{b^2 \operatorname{Sin}[e+f x]}{2 a^2 (a+b) f (a+b - a \operatorname{Sin}[e+f x]^2)}$$

Result (type 3, 945 leaves):



$$\begin{aligned}
 & \frac{1}{32 a^{5/2} (a+b)^{3/2} f (a+b \operatorname{Sec}[e+f x])^2 \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^3 \\
 & \left( -2 i b (4 a+3 b) \operatorname{ArcTan}\left[2 \operatorname{Sin}[e] \left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x]\right.\right.\right. \\
 & \quad \left.\left.\left.\sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x]} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \right.\right.\right. \\
 & \quad \left.\left.\left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]- \right.\right.\right. \\
 & \quad \left.\left.\left. i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]\right)\right)\right] / \\
 & \left( i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+ \right. \\
 & \quad \left. 3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x]\right) ] \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) - \\
 & b(4 a+3 b)(a+2 b+a \operatorname{Cos}[2(e+f x)]) \\
 & \operatorname{Log}[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]]-2 i a \operatorname{Sin}[2 e]- \\
 & \quad 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \\
 & \quad 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) + \\
 & b(4 a+3 b)(a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log}[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]] + \\
 & \quad 2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \\
 & \quad 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x]] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) + \\
 & 8 \sqrt{a}(a+b)^{3/2} \operatorname{Cos}[f x](a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] \\
 & \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[e]+2 b(4 a+3 b) \operatorname{ArcTan}\left[\left((a+b) \operatorname{Sin}[e]\right)\right] / \\
 & \left( (a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]\right) ] \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (i \operatorname{Cos}[e]+\operatorname{Sin}[e]) + \\
 & 8 \sqrt{a}(a+b)^{3/2} \operatorname{Cos}[e](a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] \\
 & \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \\
 & 8 \sqrt{a} b^2 \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Tan}[e+f x] )
 \end{aligned}$$

**Problem 199: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{a(3 a+4 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{2 b^{5/2}(a+b)^{3/2} f} + \frac{\operatorname{Tan}[e+f x]}{b^2 f} + \frac{a^2 \operatorname{Tan}[e+f x]}{2 b^2(a+b) f(a+b+b \operatorname{Tan}[e+f x]^2)}$$

Result (type 3, 248 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx] \right)^4 \left( \left( a(3a + 4b) \operatorname{ArcTan}[(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx]))] \right) / \left( 2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (a + 2b + a \cos[2(e + fx)]) (\cos[2e] - i \sin[2e]) \right) / \left( (a+b)^{3/2} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + 2(a + 2b + a \cos[2(e + fx)]) \sec[e] \sec[e + fx] \sin[fx] + \frac{a(- (a + 2b) \sin[2e] + a \sin[2fx])}{(a+b) (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) / \left( 8b^2 f (a + b \sec[e + fx]^2)^2 \right)$$

**Problem 201: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^2}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{2\sqrt{b} (a+b)^{3/2} f} + \frac{\tan[e + fx]}{2(a+b) f (a+b + b \tan[e + fx]^2)}$$

Result (type 3, 211 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx] \right)^4 \left( - \left( \left( \operatorname{ArcTan}[(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx]))] \right) / \left( 2\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (a + 2b + a \cos[2(e + fx)]) (\cos[2e] - i \sin[2e]) \right) / \left( \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \frac{- (a + 2b) \sin[2e] + a \sin[2fx]}{a (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) / \left( 8(a+b) f (a + b \sec[e + fx]^2)^2 \right)$$

**Problem 202: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{3/2} f} - \frac{b \operatorname{Tan}[e+fx]}{2a (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 240 leaves):

$$\left( (a + 2b + a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^4 \right. \\ \left. \left( 2x (a + 2b + a \operatorname{Cos}[2(e+fx)]) + (b (3a + 2b) \operatorname{ArcTan}\left[ (\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right. \right. \right. \right. \\ \left. \left. \left. (- (a + 2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]) \right) \right] / \left( 2\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) \right) \right. \\ \left. (a + 2b + a \operatorname{Cos}[2(e+fx)]) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) / \\ \left( (a+b)^{3/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} + \right. \\ \left. \frac{b ((a+2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{(a+b) f (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} \right) \left. \right) / \left( 8a^2 (a+b \operatorname{Sec}[e+fx]^2)^2 \right)$$

**Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+fx]^6}{(a+b \operatorname{Sec}[e+fx]^2)^2} dx$$

Optimal (type 3, 278 leaves, 8 steps):

$$\frac{(5a^3 - 12a^2b + 24ab^2 - 64b^3)x}{16a^5} + \frac{b^{7/2} (9a + 8b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^5 (a+b)^{3/2} f} + \\ \frac{(15a^2 - 26ab + 48b^2) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48a^3 f (a+b+b \operatorname{Tan}[e+fx]^2)} + \frac{(5a - 8b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24a^2 f (a+b+b \operatorname{Tan}[e+fx]^2)} + \\ \frac{\operatorname{Cos}[e+fx]^5 \operatorname{Sin}[e+fx]}{6af (a+b+b \operatorname{Tan}[e+fx]^2)} + \frac{b (5a^3 - 7a^2b + 12ab^2 + 32b^3) \operatorname{Tan}[e+fx]}{16a^4 (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 921 leaves):

$$\begin{aligned}
 & \left( (5 a^3 - 12 a^2 b + 24 a b^2 - 64 b^3) x (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \right) / \\
 & \left( 64 a^5 (a + b \operatorname{Sec}[e + f x]^2)^2 \right) + \\
 & \left( (15 a^2 - 32 a b + 48 b^2) \operatorname{Cos}[2 f x] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}[2 e] \right) / \\
 & \left( 256 a^4 f (a + b \operatorname{Sec}[e + f x]^2)^2 \right) + \\
 & \left( (3 a - 4 b) \operatorname{Cos}[4 f x] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}[4 e] \right) / \\
 & \left( 256 a^3 f (a + b \operatorname{Sec}[e + f x]^2)^2 \right) + \\
 & \left( (9 a + 8 b) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \left( - \left( \left( b^4 \operatorname{ArcTan}[\operatorname{Sec}[f x] \right. \right. \right. \right. \\
 & \left. \left. \left. \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right. \right. \right. \right. \\
 & \left. \left. \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Cos}[2 e] \right) \right) \right) / \\
 & \left( 8 a^5 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) + \left( i b^4 \operatorname{ArcTan} \left[ \right. \right. \\
 & \operatorname{Sec}[f x] \left. \left. \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right. \right. \\
 & \left. \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Sin}[2 e] \right) \right) / \\
 & \left. \left. \left. \left. \left( 8 a^5 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) \right) \right) \right) / \left( (a+b) (a+b \operatorname{Sec}[e + f x]^2)^2 \right) + \\
 & \frac{\operatorname{Cos}[6 f x] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}[6 e]}{768 a^2 f (a + b \operatorname{Sec}[e + f x]^2)^2} + \\
 & \left( (15 a^2 - 32 a b + 48 b^2) \operatorname{Cos}[2 e] \right. \\
 & \left. (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}[2 f x] \right) / \\
 & \left( 256 a^4 f (a + b \operatorname{Sec}[e + f x]^2)^2 \right) + \\
 & \left( (a + 2 b + a \operatorname{Cos}[2 e + 2 f x]) \operatorname{Sec}[e + f x]^4 (-a b^4 \operatorname{Sin}[2 e] - 2 b^5 \operatorname{Sin}[2 e] + a b^4 \operatorname{Sin}[2 f x]) \right) / \\
 & \left( 8 a^5 (a+b) f (a + b \operatorname{Sec}[e + f x]^2)^2 (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e]) \right) + \\
 & \left( (3 a - 4 b) \operatorname{Cos}[4 e] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}[4 f x] \right) / \\
 & \left( 256 a^3 f (a + b \operatorname{Sec}[e + f x]^2)^2 \right) + \\
 & \frac{\operatorname{Cos}[6 e] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}[6 f x]}{768 a^2 f (a + b \operatorname{Sec}[e + f x]^2)^2}
 \end{aligned}$$

**Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^5}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}}\right]}{8 \sqrt{a} (a+b)^{5/2} f} + \frac{\operatorname{Sin}[e+fx]}{4 (a+b) f (a+b-a \operatorname{Sin}[e+fx]^2)^2} + \frac{3 \operatorname{Sin}[e+fx]}{8 (a+b)^2 f (a+b-a \operatorname{Sin}[e+fx]^2)}$$

Result (type 3, 2171 leaves):

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \left( \left( 3 \operatorname{ArcTan}\left[ \frac{-i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3e] + i b \operatorname{Cos}[3e] + a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + a \operatorname{Sin}[3e] + b \operatorname{Sin}[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx]} \right)}{a \operatorname{Cos}[e] + 3 b \operatorname{Cos}[e] + a \operatorname{Cos}[3e] + b \operatorname{Cos}[3e] + a \operatorname{Cos}[e+2fx] + a \operatorname{Cos}[3e+2fx] - 3 i a \operatorname{Sin}[e] - i b \operatorname{Sin}[e] - i a \operatorname{Sin}[3e] - i b \operatorname{Sin}[3e] - i a \operatorname{Sin}[e+2fx] + i a \operatorname{Sin}[3e+2fx]} \right) \right] \operatorname{Cos}[e] \right) / \left( 128 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \right) + \left( 3 \operatorname{ArcTan}\left[ \frac{-i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3e] + i b \operatorname{Cos}[3e] + a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + a \operatorname{Sin}[3e] + b \operatorname{Sin}[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx]} \right)}{a \operatorname{Cos}[e] + 3 b \operatorname{Cos}[e] + a \operatorname{Cos}[3e] + b \operatorname{Cos}[3e] + a \operatorname{Cos}[e+2fx] + a \operatorname{Cos}[3e+2fx] - 3 i a \operatorname{Sin}[e] - i b \operatorname{Sin}[e] - i a \operatorname{Sin}[3e] - i b \operatorname{Sin}[3e] - i a \operatorname{Sin}[e+2fx] + i a \operatorname{Sin}[3e+2fx]} \right) \right] \operatorname{Sin}[e] \right) / \left( 128 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \right) \right) + \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \left( - \left( \left( 3 \operatorname{ArcTanh}\left[ \frac{2(a+b) \operatorname{Sin}[e]}{\sqrt{-2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx]} \right]} \right) \operatorname{Cos}[e] \right) / \left( 128 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \right) \right) + \left( 3 \operatorname{ArcTanh}\left[ \frac{2(a+b) \operatorname{Sin}[e]}{\sqrt{-2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx]} \right]} \right) \operatorname{Sin}[e] \right) / \left( 128 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \right) \right) \right) + \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (a+2b+a \operatorname{Cos}[2e+2fx])^3$$

$$\begin{aligned}
 & \text{Sec}[e + f x]^6 \\
 & \left( \left( 3 \operatorname{Cos}[e] \operatorname{Log}\left[ a + 2 a \operatorname{Cos}[2 e] + 2 b \operatorname{Cos}[2 e] - a \operatorname{Cos}[2 e + 2 f x] - 2 i a \operatorname{Sin}[2 e] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 i b \operatorname{Sin}[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[f x] + \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[2 e + f x] \right] \right) \right) / \\
 & \left( 256 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) - \\
 & \left( 3 i \operatorname{Log}\left[ a + 2 a \operatorname{Cos}[2 e] + 2 b \operatorname{Cos}[2 e] - a \operatorname{Cos}[2 e + 2 f x] - 2 i a \operatorname{Sin}[2 e] - \right. \right. \\
 & \quad \left. \left. \left. 2 i b \operatorname{Sin}[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[f x] + \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[2 e + f x] \right] \operatorname{Sin}[e] \right) \right) / \\
 & \left( 256 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) \Bigg) + \\
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e + f x]^2)^3} (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \\
 & \text{Sec}[e + f x]^6 \\
 & \left( - \left( \left( 3 \operatorname{Cos}[e] \operatorname{Log}\left[ -a - 2 a \operatorname{Cos}[2 e] - 2 b \operatorname{Cos}[2 e] + a \operatorname{Cos}[2 e + 2 f x] + 2 i a \operatorname{Sin}[2 e] + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 2 i b \operatorname{Sin}[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[f x] + \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[2 e + f x] \right] \right) \right) / \\
 & \left( 256 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) \Bigg) + \\
 & \left( 3 i \operatorname{Log}\left[ -a - 2 a \operatorname{Cos}[2 e] - 2 b \operatorname{Cos}[2 e] + a \operatorname{Cos}[2 e + 2 f x] + 2 i a \operatorname{Sin}[2 e] + \right. \right. \\
 & \quad \left. \left. \left. 2 i b \operatorname{Sin}[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[f x] + \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[2 e + f x] \right] \operatorname{Sin}[e] \right) \right) / \\
 & \left( 256 \sqrt{a} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) \Bigg) + \\
 & \frac{(a + 2 b + a \operatorname{Cos}[2 e + 2 f x]) \operatorname{Sec}[e + f x]^5 \operatorname{Tan}[e + f x]}{8 (a+b) f (a+b \operatorname{Sec}[e + f x]^2)^3} + \\
 & \frac{3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^5 \operatorname{Tan}[e + f x]}{32 (a+b)^2 f (a+b \operatorname{Sec}[e + f x]^2)^3}
 \end{aligned}$$

**Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{(4 a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e + f x]}{\sqrt{a+b}}\right]}{8 a^{3/2} (a+b)^{5/2} f} - \frac{b \operatorname{Sin}[e + f x]}{4 a (a+b) f (a+b - a \operatorname{Sin}[e + f x]^2)^2} + \frac{(4 a + b) \operatorname{Sin}[e + f x]}{8 a (a+b)^2 f (a+b - a \operatorname{Sin}[e + f x]^2)}$$

Result (type 3, 2214 leaves):

$$\begin{aligned}
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (4a+b) (a+2b+a \cos[2e+2fx])^3 \\
 & \operatorname{Sec}[e+fx]^6 \left( \left( i \operatorname{ArcTan} \left[ \left( -i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + \right. \right. \right. \right. \\
 & \quad a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \\
 & \quad \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + \\
 & \quad b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - \\
 & \quad 2 i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + \\
 & \quad \left. \left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) / (a \cos[e] + 3 b \cos[e] + \right. \\
 & \quad a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3 i a \sin[e] - \\
 & \quad \left. \left. \left. \left. i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e+2fx] + i a \sin[3e+2fx] \right) \right] \right) \\
 & \quad \left. \left. \left. \left. \cos[e] \right) \right] \right) / \left( 128 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\
 & \left( \operatorname{ArcTan} \left[ \left( -i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + \right. \right. \right. \\
 & \quad b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \\
 & \quad \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + \\
 & \quad b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - \\
 & \quad 2 i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + \\
 & \quad \left. \left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) / (a \cos[e] + 3 b \cos[e] + \right. \\
 & \quad a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3 i a \sin[e] - \\
 & \quad \left. \left. \left. \left. i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e+2fx] + i a \sin[3e+2fx] \right) \right] \right) \\
 & \quad \left. \left. \left. \left. \sin[e] \right) \right] \right) / \left( 128 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (-4a-b) (a+2b+a \cos[2e+2fx])^3 \\
 & \operatorname{Sec}[e+fx]^6 \\
 & \left( \left( \operatorname{ArcTanh} \left[ \left( 2(a+b) \sin[e] \right) \right] / \left( -2 i a \cos[e] - 2 i b \cos[e] - \right. \right. \right. \\
 & \quad \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \\
 & \quad \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + \\
 & \quad \left. \left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) \cos[e] \right) / \\
 & \quad \left( 128 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \left( i \operatorname{ArcTanh} \left[ \left( 2(a+b) \sin[e] \right) \right] / \right. \\
 & \quad \left( -2 i a \cos[e] - 2 i b \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \right. \\
 & \quad \left. \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \right. \\
 & \quad \left. \left. \left. \left. \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) \\
 & \quad \left. \left. \left. \left. \sin[e] \right) \right] \right) / \left( 128 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (4a+b) (a+2b+a \cos[2e+2fx])^3 \\
 & \operatorname{Sec}[e+fx]^6 \\
 & \left( \left( \cos[e] \operatorname{Log} \left[ a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - \right. \right. \right. \\
 & \quad 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \\
 & \quad \left. \left. \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right] \right) \right) / \\
 & \quad \left( 256 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \quad \left( i \operatorname{Log} \left[ a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - \right. \right. \right.
 \end{aligned}$$

$$\frac{\begin{aligned} & \left( \frac{2 i b \sin[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2 e] - i \sin[2 e]} \sin[f x] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2 e] - i \sin[2 e]} \sin[2 e + f x]}{256 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2 e] - i \sin[2 e]}} \right) + \\ & \frac{1}{(a+b)^2 (a+b \sec[e+f x]^2)^3} (-4 a-b) (a+2 b+a \cos[2 e+2 f x])^3 \sec[e+f x]^6 \\ & \left( \left( \cos[e] \log[-a-2 a \cos[2 e]-2 b \cos[2 e]+a \cos[2 e+2 f x]+2 i a \sin[2 e] + \right. \right. \\ & \quad \left. \left. \frac{2 i b \sin[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2 e] - i \sin[2 e]} \sin[f x] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2 e] - i \sin[2 e]} \sin[2 e + f x]}{256 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2 e] - i \sin[2 e]}} \right) - \right. \\ & \quad \left. \left( i \log[-a-2 a \cos[2 e]-2 b \cos[2 e]+a \cos[2 e+2 f x]+2 i a \sin[2 e] + \right. \right. \\ & \quad \left. \left. \frac{2 i b \sin[2 e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2 e] - i \sin[2 e]} \sin[f x] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2 e] - i \sin[2 e]} \sin[2 e + f x]}{256 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2 e] - i \sin[2 e]}} \right) \right) + \\ & \left( (a+2 b+a \cos[2 e+2 f x])^2 \sec[e+f x]^6 (4 a \sin[e+f x] + b \sin[e+f x]) \right) / \left( 32 a (a+b)^2 f (a+b \sec[e+f x]^2)^3 \right) - \\ & \frac{b (a+2 b+a \cos[2 e+2 f x]) \sec[e+f x]^5 \tan[e+f x]}{8 a (a+b) f (a+b \sec[e+f x]^2)^3} \end{aligned}$$

**Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+f x]}{(a+b \sec[e+f x]^2)^3} dx$$

Optimal (type 3, 144 leaves, 4 steps):

$$\frac{(8 a^2 + 8 a b + 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[e+f x]}{\sqrt{a+b}}\right]}{8 a^{5/2} (a+b)^{5/2} f} - \frac{b \cos[e+f x]^2 \sin[e+f x]}{4 a (a+b) f (a+b-a \sin[e+f x]^2)^2} - \frac{3 b (2 a+b) \sin[e+f x]}{8 a^2 (a+b)^2 f (a+b-a \sin[e+f x]^2)}$$

Result (type 3, 2256 leaves):

$$\frac{1}{(a+b)^2 (a+b \sec[e+f x]^2)^3} (8 a^2 + 8 a b + 3 b^2) (a+2 b+a \cos[2 e+2 f x])^3 \sec[e+f x]^6 \left( i \operatorname{ArcTan}\left[ \frac{-i a \cos[e] - i b \cos[e] + i a \cos[3 e] + i b \cos[3 e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-f x] \sqrt{\cos[2 e] - i \sin[2 e]}}{256 a^{3/2} \sqrt{a+b} f \sqrt{\cos[2 e] - i \sin[2 e]}} \right] + \right.$$



$$\begin{aligned}
 & \left( \frac{\sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx]}{(a \cos[e] + 3b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3ia \sin[e] - ib \sin[e] - ia \sin[3e] - ib \sin[3e] - ia \sin[e+2fx] + ia \sin[3e+2fx])} \right) \\
 & \cos[e] \left/ \left( 128 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \right. \\
 & \left( \text{ArcTan} \left[ \left( -ia \cos[e] - ib \cos[e] + ia \cos[3e] + ib \cos[3e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right/ \\
 & \left( a \cos[e] + 3b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3ia \sin[e] - ib \sin[e] - ia \sin[3e] - ib \sin[3e] - ia \sin[e+2fx] + ia \sin[3e+2fx] \right) \sin[e] \left/ \right. \\
 & \left. \left( 128 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} \\
 & (-8a^2 - 8ab - 3b^2) (a+2b+a \cos[2e+2fx])^3 \\
 & \text{Sec}[e+fx]^6 \\
 & \left( \left( \text{ArcTanh} \left[ \left( 2(a+b) \sin[e] \right) \right] \right/ \left( -2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) \cos[e] \left/ \right. \\
 & \left( 128 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \left( i \text{ArcTanh} \left[ \left( 2(a+b) \sin[e] \right) \right] \right/ \\
 & \left( -2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \left. \right] \\
 & \sin[e] \left/ \left( 128 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
 & \frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} (8a^2 + 8ab + 3b^2) \\
 & (a+2b+a \cos[2e+2fx])^3 \\
 & \text{Sec}[e+fx]^6 \\
 & \left( \left( \cos[e] \log[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2ia \sin[2e] - 2ib \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right) \left/ \right. \\
 & \left( 256 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \left( i \log[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2ia \sin[2e] - 2ib \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \left. \right] \sin[e] \left/ \right.
 \end{aligned}$$

$$\begin{aligned} & \left( 256 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\ & \frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} \left( -8 a^2 - 8 a b - \right. \\ & \quad \left. 3 b^2 \right) \\ & \frac{1}{(a+2b+a \cos[2e+2fx])^3} \\ & \frac{1}{\sec[e+fx]^6} \\ & \left( \left( \cos[e] \log[-a-2a \cos[2e]-2b \cos[2e]+a \cos[2e+2fx]+2i a \sin[2e]+ \right. \right. \\ & \quad \left. \left. 2i b \sin[2e]+2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i \sin[2e]}\sin[fx]+ \right. \right. \\ & \quad \left. \left. 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i \sin[2e]}\sin[2e+fx] \right) \right) / \\ & \left( 256 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\ & \left( i \log[-a-2a \cos[2e]-2b \cos[2e]+a \cos[2e+2fx]+2i a \sin[2e]+ \right. \\ & \quad \left. 2i b \sin[2e]+2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i \sin[2e]}\sin[fx]+ \right. \\ & \quad \left. 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i \sin[2e]}\sin[2e+fx] \right) \sin[e] / \\ & \left( 256 a^{5/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\ & \left( (a+2b+a \cos[2e+2fx])^2 \sec[e+fx]^6 \right. \\ & \quad \left. (-8 a b \sin[e+fx] - 5 b^2 \sin[e+fx]) \right) / (32 \\ & \quad a^2 \\ & \quad (a+b)^2 \\ & \quad f \\ & \quad (a+b \sec[e+fx])^3) + \\ & \frac{b^2 (a+2b+a \cos[2e+2fx]) \sec[e+fx]^5 \tan[e+fx]}{8 a^2 (a+b) f (a+b \sec[e+fx])^3} \end{aligned}$$

**Problem 209: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]}{(a+b \sec[e+fx])^3} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$\begin{aligned} & - \frac{3 b \left( 4 (a+b)^2 + (2 a+b)^2 \right) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right]}{8 a^{7/2} (a+b)^{5/2} f} + \frac{\sin[e+fx]}{a^3 f} - \\ & \frac{b^3 \sin[e+fx]}{4 a^3 (a+b) f (a+b-a \sin[e+fx])^2} + \frac{3 b^2 (4 a+3 b) \sin[e+fx]}{8 a^3 (a+b)^2 f (a+b-a \sin[e+fx])^2} \end{aligned}$$

Result (type 3, 2382 leaves):

$$\begin{aligned} & \frac{\cos[fx] (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6 \sin[e]}{8 a^3 f (a+b \sec[e+fx])^3} + \\ & \frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} (8 a^2 b + 12 a b^2 + 5 b^3) (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6 \end{aligned}$$

$$\begin{aligned}
 & \left( - \left( \left( 3 \operatorname{ArcTan} \left[ \left( -i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + b \sin[e] - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) / \\
 & \quad \left( a \cos[e] + 3 b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3 i a \sin[e] - i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e+2fx] + \right. \\
 & \quad \left. i a \sin[3e+2fx] \right) \cos[e] \Big/ \left( 128 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \left( 3 \operatorname{ArcTan} \left[ \left( -i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + \right. \right. \right. \\
 & \quad \left. \left. \left. b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \right) / \\
 & \quad \left( a \cos[e] + 3 b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3 i a \sin[e] - i b \sin[e] - i a \sin[3e] - \right. \\
 & \quad \left. i b \sin[3e] - i a \sin[e+2fx] + i a \sin[3e+2fx] \right) \sin[e] \Big/ \\
 & \quad \left( 128 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} \left( 8 a^2 \right. \\
 & \quad \left. b + 12 a \right. \\
 & \quad \left. b^2 + 5 b^3 \right) \\
 & \left( a + 2 b + a \cos[2e+2fx] \right)^3 \\
 & \operatorname{Sec}[e+fx]^6 \\
 & \left( \left( 3 \operatorname{ArcTanh} \left[ \left( 2 (a+b) \sin[e] \right) \right] \right) / \left( -2 i a \cos[e] - 2 i b \cos[e] - \right. \right. \\
 & \quad \left. \left. \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + \right. \right. \\
 & \quad \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right) \cos[e] \Big/ \\
 & \quad \left( 128 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \left( 3 \operatorname{ArcTanh} \left[ \left( 2 (a+b) \sin[e] \right) \right] \right) / \\
 & \quad \left( -2 i a \cos[e] - 2 i b \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \right. \\
 & \quad \left. \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \right. \\
 & \quad \left. \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \Big/ \\
 & \quad \left( 128 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\
 & \quad \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} \left( 8 a^2 b + 12 a b^2 + \right. \\
 & \quad \left. 5 b^3 \right) \\
 & \left( a + 2 b + a \cos[2e+2fx] \right)^3 \\
 & \operatorname{Sec}[e+fx]^6 \\
 & \left( - \left( \left( 3 \cos[e] \log[a + 2 a \cos[2e] + 2 b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx]}{(256 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]})} \right) + \\
 & \left( \frac{3 i \operatorname{Log}[a + 2 a \cos[2e] + 2 b \cos[2e] - a \cos[2e + 2fx] - 2 i a \sin[2e] - 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx]}{(256 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]})} \right) \sin[e] \Big/ \\
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (8 a^2 b + 12 a b^2 + 5 b^3) \\
 & (a + 2 b + a \cos[2e + 2fx])^3 \operatorname{Sec}[e+fx]^6 \\
 & \left( \frac{3 \cos[e] \operatorname{Log}[-a - 2 a \cos[2e] - 2 b \cos[2e] + a \cos[2e + 2fx] + 2 i a \sin[2e] + 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx]}{(256 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]})} - \right. \\
 & \left. \frac{3 i \operatorname{Log}[-a - 2 a \cos[2e] - 2 b \cos[2e] + a \cos[2e + 2fx] + 2 i a \sin[2e] + 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx]}{(256 a^{7/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]})} \right) \sin[e] \Big/ \\
 & \frac{\cos[e] (a + 2 b + a \cos[2e + 2fx])^3 \operatorname{Sec}[e+fx]^6 \sin[fx]}{8 a^3 f (a+b \operatorname{Sec}[e+fx])^3} + \\
 & \left( \frac{3 (a + 2 b + a \cos[2e + 2fx])^2 \operatorname{Sec}[e+fx]^6 (4 a b^2 \sin[e+fx] + 3 b^3 \sin[e+fx])}{a^3 (a+b)^2 f (a+b \operatorname{Sec}[e+fx])^3} - \right. \\
 & \left. \frac{b^3 (a + 2 b + a \cos[2e + 2fx]) \operatorname{Sec}[e+fx]^5 \tan[e+fx]}{8 a^3 (a+b) f (a+b \operatorname{Sec}[e+fx])^3} \right)
 \end{aligned}$$

**Problem 211: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^5}{(a+b \operatorname{Sec}[e+fx])^3} dx$$

Optimal (type 3, 214 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{b^3 (80 a^2 + 140 a b + 63 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}}\right]}{8 a^{11/2} (a+b)^{5/2} f} + \\
 & \frac{(a^2 - 3 a b + 6 b^2) \operatorname{Sin}[e+fx]}{a^5 f} - \frac{(2 a - 3 b) \operatorname{Sin}[e+fx]^3}{3 a^4 f} + \frac{\operatorname{Sin}[e+fx]^5}{5 a^3 f} - \\
 & \frac{b^5 \operatorname{Sin}[e+fx]}{4 a^5 (a+b) f (a+b - a \operatorname{Sin}[e+fx]^2)^2} + \frac{b^4 (20 a + 17 b) \operatorname{Sin}[e+fx]}{8 a^5 (a+b)^2 f (a+b - a \operatorname{Sin}[e+fx]^2)}
 \end{aligned}$$

Result (type 3, 2670 leaves):

$$\begin{aligned}
 & \left( (5 a^2 - 18 a b + 48 b^2) \operatorname{Cos}[fx] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e+fx]^6 \operatorname{Sin}[e] \right) / \\
 & \left( 64 a^5 f (a+b \operatorname{Sec}[e+fx]^2)^3 \right) + \\
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (-80 a^2 b^3 - 140 a b^4 - 63 b^5) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \\
 & \operatorname{Sec}[e+fx]^6 \left( \left( i \operatorname{ArcTan}\left[ \left( -i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3 e] + i b \operatorname{Cos}[3 e] + \right. \right. \right. \right. \\
 & \left. \left. \left. a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \right. \right. \right. \\
 & \left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3 e+fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + a \operatorname{Sin}[3 e] + b \operatorname{Sin}[3 e] - i \sqrt{a} \right. \right. \right. \\
 & \left. \left. \left. \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sin}[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+fx] \right) \right) / (a \operatorname{Cos}[e] + 3 b \right. \\
 & \left. \operatorname{Cos}[e] + a \operatorname{Cos}[3 e] + b \operatorname{Cos}[3 e] + a \operatorname{Cos}[e+2 f x] + a \operatorname{Cos}[3 e+2 f x] - 3 i a \operatorname{Sin}[e] - \right. \\
 & \left. i b \operatorname{Sin}[e] - i a \operatorname{Sin}[3 e] - i b \operatorname{Sin}[3 e] - i a \operatorname{Sin}[e+2 f x] + i a \operatorname{Sin}[3 e+2 f x]) \right] \\
 & \operatorname{Cos}[e] \left) / \left( 128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) + \\
 & \left( \operatorname{ArcTan}\left[ \left( -i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3 e] + i b \operatorname{Cos}[3 e] + a \operatorname{Sin}[e] + \right. \right. \right. \\
 & \left. \left. \left. b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \right. \right. \right. \\
 & \left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3 e+fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + a \operatorname{Sin}[3 e] + b \operatorname{Sin}[3 e] - i \sqrt{a} \right. \right. \right. \\
 & \left. \left. \left. \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sin}[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+fx] \right) \right) / (a \operatorname{Cos}[e] + 3 b \right. \\
 & \left. \operatorname{Cos}[e] + a \operatorname{Cos}[3 e] + b \operatorname{Cos}[3 e] + a \operatorname{Cos}[e+2 f x] + a \operatorname{Cos}[3 e+2 f x] - 3 i a \operatorname{Sin}[e] - \right. \\
 & \left. i b \operatorname{Sin}[e] - i a \operatorname{Sin}[3 e] - i b \operatorname{Sin}[3 e] - i a \operatorname{Sin}[e+2 f x] + i a \operatorname{Sin}[3 e+2 f x]) \right] \\
 & \operatorname{Sin}[e] \left) / \left( 128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) + \\
 & \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (80 a^2 b^3 + 140 a b^4 + \\
 & 63 b^5) \\
 & (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \\
 & \operatorname{Sec}[e+fx]^6 \\
 & \left( \left( \operatorname{ArcTanh}\left[ (2 (a+b) \operatorname{Sin}[e]) \right] / \right. \right. \\
 & \left. \left( -2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \sqrt{a} \right. \right. \\
 & \left. \left. \sqrt{a+b} \operatorname{Cos}[3 e+fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right. \right. \\
 & \left. \left. \operatorname{Sin}[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+fx] \right) \right) \operatorname{Cos}[e] \left) / \\
 & \left( 128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) - \left( i \operatorname{ArcTanh}\left[ (2 (a+b) \operatorname{Sin}[e]) \right] / \right. \\
 & \left. \left( -2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \sqrt{a} \right. \right. \\
 & \left. \left. \sqrt{a+b} \operatorname{Cos}[3 e+fx] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right. \right. \\
 & \left. \left. \operatorname{Sin}[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+fx] \right) \right) \operatorname{Sin}[e] \left) / \left( 128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{\sin[e - fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + fx]}{\left( 128 a^{11/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right)} \right) \sin[e] \Big/ \\
 & \left( \frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} \right) + \\
 & \frac{(-80 a^2 b^3 - 140 a b^4 - 63 b^5) (a + 2b + a \cos[2e + 2fx])^3}{\sec[e+fx]^6} \\
 & \left( \left( \cos[e] \log[a + 2a \cos[2e] + 2b \cos[2e] - a \cos[2e + 2fx] - 2i a \sin[2e] - \right. \right. \\
 & \quad \left. \left. 2i b \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \right. \\
 & \quad \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right) \right) \Big/ \\
 & \left( 256 a^{11/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \left( i \log[a + 2a \cos[2e] + 2b \cos[2e] - a \cos[2e + 2fx] - 2i a \sin[2e] - \right. \\
 & \quad \left. 2i b \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \\
 & \quad \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right) \sin[e] \Big/ \\
 & \left( 256 a^{11/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\
 & \frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} (80 a^2 b^3 + 140 a b^4 + 63 b^5) \\
 & \frac{1}{(a + 2b + a \cos[2e + 2fx])^3} \\
 & \frac{\sec[e+fx]^6}{\left( \cos[e] \log[-a - 2a \cos[2e] - 2b \cos[2e] + a \cos[2e + 2fx] + 2i a \sin[2e] + 2i b \sin[2e] + \right. \\
 & \quad \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \right. \\
 & \quad \left. \sin[2e + fx] \right) \Big/ \left( 256 a^{11/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
 & \left( i \log[-a - 2a \cos[2e] - 2b \cos[2e] + a \cos[2e + 2fx] + 2i a \sin[2e] + \right. \\
 & \quad \left. 2i b \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \\
 & \quad \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right) \sin[e] \Big/ \\
 & \left( 256 a^{11/2} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\
 & \frac{1}{\left( (5a - 12b) \cos[3fx] (a + 2b + a \cos[2e + 2fx])^3 \right)} \\
 & \frac{\sec[e+fx]^6}{\sin[3e]} \Big/ \left( 384 \right. \\
 & \quad \left. a^4 \right. \\
 & \quad \left. f \right. \\
 & \quad \left. (a + b \sec[e+fx])^3 \right) + \\
 & \frac{\cos[5fx] (a + 2b + a \cos[2e + 2fx])^3 \sec[e+fx]^6 \sin[5e]}{640 a^3 f (a + b \sec[e+fx])^3} + \\
 & \left( (5a^2 - 18ab + 48b^2) \right. \\
 & \quad \cos[e] \\
 & \quad (a + 2b + a \cos[2e + 2fx])^3 \\
 & \quad \sec[e+fx]^6 \\
 & \quad \sin[fx] \Big/ \left( 64 \right. \\
 & \quad \left. a^5 \right. \\
 & \quad \left. f \right. \\
 & \quad \left. (a + b \sec[e+fx])^3 \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( (5a - 12b) \cos[3e] (a + 2b + a \cos[2e + 2fx])^3 \right. \\
 & \quad \left. \sec[e + fx]^6 \sin[3fx] \right) / \left( 384 \right. \\
 & \quad \left. a^4 \right. \\
 & \quad \left. f \right. \\
 & \quad \left. (a + b \sec[e + fx]^2)^3 \right) + \\
 & \frac{\cos[5e] (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \sin[5fx]}{640 a^3 f (a + b \sec[e + fx]^2)^3} + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^2 \right. \\
 & \quad \left. \sec[e + fx]^6 \right. \\
 & \quad \left. (20 a b^4 \sin[e + fx] + 17 b^5 \sin[e + fx]) \right) / \\
 & \left( 32 a^5 (a + b)^2 f (a + b \sec[e + fx]^2)^3 \right) - \\
 & \frac{b^5 (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^5 \tan[e + fx]}{8 a^5 (a + b) f (a + b \sec[e + fx]^2)^3}
 \end{aligned}$$

**Problem 213: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^4}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 123 leaves, 4 steps):

$$\frac{(a + 4b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a + b}}\right]}{8 b^{3/2} (a + b)^{5/2} f} - \\
 \frac{a \tan[e + fx]}{4 b (a + b) f (a + b + b \tan[e + fx]^2)^2} + \frac{(a + 4b) \tan[e + fx]}{8 b (a + b)^2 f (a + b + b \tan[e + fx]^2)}$$

Result (type 3, 539 leaves):

$$\left( (-a - 4b) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \left( \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right] \right) \right. \right. \\ \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\ \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \cos[2e] \right) \right) / \\ \left( 64b \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right] \right. \\ \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \\ \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) / \\ \left( 64b \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \Bigg) / \\ \left( (a+b)^2 (a+b \sec[e+fx]^2)^3 \right) + \left( (a+2b+a \cos[2e+2fx]) \sec[e+fx]^6 \right. \\ \left. (-a \sin[2e] - 2b \sin[2e] + a \sin[2fx]) \right) / \left( 16 \right. \\ \left. a \right. \\ \left. (a+b) \right. \\ \left. f \right. \\ \left. (a+b \sec[e+fx]^2)^3 \right. \\ \left. (\cos[e] - \sin[e]) \right. \\ \left. (\cos[e] + \sin[e]) \right) + \\ \left( (a+2b+a \cos[2e+2fx])^2 \sec[e+fx]^6 \right. \\ \left. (a \sin[2e] + 4b \sin[2e] - a \sin[2fx] + 2b \sin[2fx]) \right) / \\ \left( 64b (a+b)^2 f (a+b \sec[e+fx]^2)^3 (\cos[e] - \sin[e]) \right. \\ \left. (\cos[e] + \sin[e]) \right)$$

**Problem 214: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^2}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 106 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}} \right]}{8 \sqrt{b} (a+b)^{5/2} f} + \frac{\operatorname{Tan}[e+fx]}{4 (a+b) f (a+b + b \operatorname{Tan}[e+fx]^2)^2} + \frac{3 \operatorname{Tan}[e+fx]}{8 (a+b)^2 f (a+b + b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 265 leaves):



$$\begin{aligned}
 & \left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx] \right)^6 \\
 & \left( - \left( \left( 3 \operatorname{ArcTan} \left[ \frac{\sec[fx] (\cos[2e] - i \sin[2e])}{\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (- (a + 2b) \sin[fx] + a \sin[2e + fx]) \right) \right) \right. \\
 & \quad \left. \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (a + 2b + a \cos[2(e + fx)])^2 \right. \\
 & \quad \left. (\cos[2e] - i \sin[2e]) \right) \left( \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \\
 & \quad \frac{4b(a+b) \sec[2e] ((a+2b) \sin[2e] - a \sin[2fx])}{a^2} + \frac{1}{a^2} (a + 2b + a \cos[2(e + fx)]) \\
 & \quad \left. \left. \left. \sec[2e] \left( - (5a^2 + 16ab + 8b^2) \sin[2e] + a(5a + 2b) \sin[2fx] \right) \right) \right) \right) / \\
 & \left( 64(a+b)^2 f (a+b \sec[e + fx]^2)^3 \right)
 \end{aligned}$$

**Problem 215: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\begin{aligned}
 & \frac{x}{a^3} - \frac{\sqrt{b} (15a^2 + 20ab + 8b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{8a^3 (a+b)^{5/2} f} - \\
 & \frac{b \tan[e + fx]}{4a(a+b) f (a+b + b \tan[e + fx]^2)^2} - \frac{b(7a + 4b) \tan[e + fx]}{8a^2 (a+b)^2 f (a+b + b \tan[e + fx]^2)}
 \end{aligned}$$

Result (type 3, 627 leaves):

$$\begin{aligned}
 & \frac{x (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{8a^3 (a + b \sec[e + fx]^2)^3} + \\
 & \left( (15a^2 + 20ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \left( b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b}} - \frac{i \sin[2e]}{2\sqrt{a+b}} \right] \right. \right. \right. \\
 & \left. \left. \left. \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
 & \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) \right) / \\
 & \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b}} - \frac{i \sin[2e]}{2\sqrt{a+b}} \right] \right. \\
 & \left. \left. \left. \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
 & \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) \right) / \\
 & \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \Bigg) / \left( (a+b)^2 (a+b \sec[e + fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6 (9a^2 b \sin[2e] + 28a b^2 \sin[2e] + \right. \\
 & \left. 16b^3 \sin[2e] - 9a^2 b \sin[2fx] - 6a b^2 \sin[2fx]) \right) / \\
 & \left( 64a^3 (a+b)^2 f (a+b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) \right. \\
 & \left. (\cos[e] + \sin[e]) \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^6 (-a b^2 \sin[2e] - 2b^3 \sin[2e] + a b^2 \sin[2fx]) \right) / \\
 & \left( 16a^3 (a+b) f (a+b \sec[e + fx]^2)^3 \right. \\
 & \left. (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right)
 \end{aligned}$$

**Problem 217: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + fx]^4}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 269 leaves, 8 steps):

$$\begin{aligned}
 & \frac{3(a^2 - 4ab + 16b^2)x}{8a^5} - \frac{3b^{5/2}(21a^2 + 36ab + 16b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{8a^5 (a+b)^{5/2} f} + \\
 & \frac{(3a - 8b) \cos[e + fx] \sin[e + fx]}{8a^2 f (a+b + b \tan[e + fx]^2)^2} + \frac{\cos[e + fx]^3 \sin[e + fx]}{4af (a+b + b \tan[e + fx]^2)^2} + \\
 & \frac{b(3a^2 - 7ab - 12b^2) \tan[e + fx]}{8a^3 (a+b) f (a+b + b \tan[e + fx]^2)^2} + \frac{3b(a+2b)(a^2 - 4ab - 4b^2) \tan[e + fx]}{8a^4 (a+b)^2 f (a+b + b \tan[e + fx]^2)^2}
 \end{aligned}$$

Result (type 3, 1430 leaves):

$$\begin{aligned}
 & \left( (21 a^2 + 36 a b + 16 b^2) (a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \left( \left( 3 b^3 \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sec[f x] \left( \frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x]) \right] \cos[2 e] \right) \right) \right) / \\
 & \left( 64 a^5 \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) - \left( 3 i b^3 \operatorname{ArcTan} \left[ \right. \right. \\
 & \quad \left. \left. \left. \left. \sec[f x] \left( \frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x]) \right] \sin[2 e] \right) \right) \right) / \\
 & \left( 64 a^5 \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) \left. \right) \left. \right) \left. \right) / \\
 & \left( (a+b)^2 (a+b \sec[e + f x]^2)^3 \right) + \frac{1}{2048 a^5 (a+b)^2 f (a+b \sec[e + f x]^2)^3} \\
 & (a + 2 b + a \cos[2 e + 2 f x]) \\
 & \sec[2 e] \sec[e + f x]^6 \\
 & (144 a^6 f x \cos[2 e] + 96 a^5 b f x \cos[2 e] + 912 a^4 b^2 f x \cos[2 e] + \\
 & 6720 a^3 b^3 f x \cos[2 e] + 16512 a^2 b^4 f x \cos[2 e] + 16896 a b^5 f x \cos[2 e] + \\
 & 6144 b^6 f x \cos[2 e] + 96 a^6 f x \cos[2 f x] + 480 a^4 b^2 f x \cos[2 f x] + \\
 & 4416 a^3 b^3 f x \cos[2 f x] + 6912 a^2 b^4 f x \cos[2 f x] + \\
 & 3072 a b^5 f x \cos[2 f x] + 96 a^6 f x \cos[4 e + 2 f x] + \\
 & 480 a^4 b^2 f x \cos[4 e + 2 f x] + 4416 a^3 b^3 f x \cos[4 e + 2 f x] + \\
 & 6912 a^2 b^4 f x \cos[4 e + 2 f x] + 3072 a b^5 f x \cos[4 e + 2 f x] + \\
 & 24 a^6 f x \cos[2 e + 4 f x] - 48 a^5 b f x \cos[2 e + 4 f x] + 216 a^4 b^2 f x \cos[2 e + 4 f x] + \\
 & 672 a^3 b^3 f x \cos[2 e + 4 f x] + 384 a^2 b^4 f x \cos[2 e + 4 f x] + \\
 & 24 a^6 f x \cos[6 e + 4 f x] - 48 a^5 b f x \cos[6 e + 4 f x] + 216 a^4 b^2 f x \cos[6 e + 4 f x] + \\
 & 672 a^3 b^3 f x \cos[6 e + 4 f x] + 384 a^2 b^4 f x \cos[6 e + 4 f x] + 816 a^3 b^3 \sin[2 e] + \\
 & 2848 a^2 b^4 \sin[2 e] + 3968 a b^5 \sin[2 e] + 1792 b^6 \sin[2 e] + 44 a^6 \sin[2 f x] + \\
 & 104 a^5 b \sin[2 f x] - 180 a^4 b^2 \sin[2 f x] - 1696 a^3 b^3 \sin[2 f x] - 3264 a^2 b^4 \sin[2 f x] - \\
 & 1664 a b^5 \sin[2 f x] + 44 a^6 \sin[4 e + 2 f x] + 104 a^5 b \sin[4 e + 2 f x] - \\
 & 180 a^4 b^2 \sin[4 e + 2 f x] - 608 a^3 b^3 \sin[4 e + 2 f x] - 192 a^2 b^4 \sin[4 e + 2 f x] + \\
 & 128 a b^5 \sin[4 e + 2 f x] + 38 a^6 \sin[2 e + 4 f x] + 60 a^5 b \sin[2 e + 4 f x] - \\
 & 170 a^4 b^2 \sin[2 e + 4 f x] - 640 a^3 b^3 \sin[2 e + 4 f x] - 400 a^2 b^4 \sin[2 e + 4 f x] + \\
 & 38 a^6 \sin[6 e + 4 f x] + 60 a^5 b \sin[6 e + 4 f x] - 170 a^4 b^2 \sin[6 e + 4 f x] - \\
 & 368 a^3 b^3 \sin[6 e + 4 f x] - 176 a^2 b^4 \sin[6 e + 4 f x] + 12 a^6 \sin[4 e + 6 f x] + \\
 & 8 a^5 b \sin[4 e + 6 f x] - 20 a^4 b^2 \sin[4 e + 6 f x] - 16 a^3 b^3 \sin[4 e + 6 f x] + \\
 & 12 a^6 \sin[8 e + 6 f x] + 8 a^5 b \sin[8 e + 6 f x] - 20 a^4 b^2 \sin[8 e + 6 f x] - \\
 & 16 a^3 b^3 \sin[8 e + 6 f x] + a^6 \sin[6 e + 8 f x] + 2 a^5 b \sin[6 e + 8 f x] + a^4 b^2 \sin[6 e + 8 f x] + \\
 & a^6 \sin[10 e + 8 f x] + 2 a^5 b \sin[10 e + 8 f x] + a^4 b^2 \sin[10 e + 8 f x])
 \end{aligned}$$

**Problem 218: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 352 leaves, 9 steps):

$$\begin{aligned} & \frac{(5 a^3 - 18 a^2 b + 48 a b^2 - 160 b^3) x}{16 a^6} + \frac{b^{7/2} (99 a^2 + 176 a b + 80 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 a^6 (a + b)^{5/2} f} + \\ & \frac{(15 a^2 - 34 a b + 80 b^2) \text{Cos}[e + f x] \text{Sin}[e + f x]}{48 a^3 f (a + b + b \text{Tan}[e + f x]^2)^2} + \frac{5 (a - 2 b) \text{Cos}[e + f x]^3 \text{Sin}[e + f x]}{24 a^2 f (a + b + b \text{Tan}[e + f x]^2)^2} + \\ & \frac{\text{Cos}[e + f x]^5 \text{Sin}[e + f x]}{6 a f (a + b + b \text{Tan}[e + f x]^2)^2} + \frac{b (15 a^3 - 29 a^2 b + 64 a b^2 + 120 b^3) \text{Tan}[e + f x]}{48 a^4 (a + b) f (a + b + b \text{Tan}[e + f x]^2)^2} + \\ & \frac{b (5 a^4 - 8 a^3 b + 17 a^2 b^2 + 116 a b^3 + 80 b^4) \text{Tan}[e + f x]}{16 a^5 (a + b)^2 f (a + b + b \text{Tan}[e + f x]^2)} \end{aligned}$$

Result (type 3, 1770 leaves):

$$\begin{aligned} & \left( (99 a^2 + 176 a b + 80 b^2) (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \right. \\ & \quad \left. \text{Sec}[e + f x]^6 \left( - \left( \left( b^4 \text{ArcTan}[\text{Sec}[f x]] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right)} \right) \right. \right. \right. \\ & \quad \left. \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right] \text{Cos}[2 e] \right) \right) / \right. \\ & \quad \left. \left( 64 a^6 \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) \right) + \left( i b^4 \text{ArcTan} \left[ \right. \right. \\ & \quad \left. \left. \text{Sec}[f x] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right. \right. \\ & \quad \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right] \text{Sin}[2 e] \right) \right) / \left. \right. \\ & \quad \left. \left( 64 a^6 \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) \right) \right) / \\ & \left( (a + b)^2 (a + b \text{Sec}[e + f x]^2)^3 \right) + \frac{1}{12288 a^6 (a + b)^2 f (a + b \text{Sec}[e + f x]^2)^3} \\ & (a + 2 b + a \text{Cos}[2 e + 2 f x]) \text{Sec}[2 e] \\ & \text{Sec}[e + f x]^6 (720 a^7 f x \text{Cos}[2 e] + 768 a^6 b f x \text{Cos}[2 e] + \\ & 1296 a^5 b^2 f x \text{Cos}[2 e] - 8352 a^4 b^3 f x \text{Cos}[2 e] - \\ & 64128 a^3 b^4 f x \text{Cos}[2 e] - 158976 a^2 b^5 f x \text{Cos}[2 e] - \end{aligned}$$

$$\begin{aligned}
 & 165\,888 a b^6 f x \operatorname{Cos}[2 e] - 61\,440 b^7 f x \operatorname{Cos}[2 e] + \\
 & 480 a^7 f x \operatorname{Cos}[2 f x] + 192 a^6 b f x \operatorname{Cos}[2 f x] + 96 a^5 b^2 f x \operatorname{Cos}[2 f x] - \\
 & 4608 a^4 b^3 f x \operatorname{Cos}[2 f x] - 41\,856 a^3 b^4 f x \operatorname{Cos}[2 f x] - \\
 & 67\,584 a^2 b^5 f x \operatorname{Cos}[2 f x] - 30\,720 a b^6 f x \operatorname{Cos}[2 f x] + \\
 & 480 a^7 f x \operatorname{Cos}[4 e + 2 f x] + 192 a^6 b f x \operatorname{Cos}[4 e + 2 f x] + \\
 & 96 a^5 b^2 f x \operatorname{Cos}[4 e + 2 f x] - 4608 a^4 b^3 f x \operatorname{Cos}[4 e + 2 f x] - \\
 & 41\,856 a^3 b^4 f x \operatorname{Cos}[4 e + 2 f x] - 67\,584 a^2 b^5 f x \operatorname{Cos}[4 e + 2 f x] - \\
 & 30\,720 a b^6 f x \operatorname{Cos}[4 e + 2 f x] + 120 a^7 f x \operatorname{Cos}[2 e + 4 f x] - \\
 & 192 a^6 b f x \operatorname{Cos}[2 e + 4 f x] + 408 a^5 b^2 f x \operatorname{Cos}[2 e + 4 f x] - \\
 & 1968 a^4 b^3 f x \operatorname{Cos}[2 e + 4 f x] - 6528 a^3 b^4 f x \operatorname{Cos}[2 e + 4 f x] - \\
 & 3840 a^2 b^5 f x \operatorname{Cos}[2 e + 4 f x] + 120 a^7 f x \operatorname{Cos}[6 e + 4 f x] - \\
 & 192 a^6 b f x \operatorname{Cos}[6 e + 4 f x] + 408 a^5 b^2 f x \operatorname{Cos}[6 e + 4 f x] - \\
 & 1968 a^4 b^3 f x \operatorname{Cos}[6 e + 4 f x] - 6528 a^3 b^4 f x \operatorname{Cos}[6 e + 4 f x] - \\
 & 3840 a^2 b^5 f x \operatorname{Cos}[6 e + 4 f x] - 6048 a^3 b^4 \operatorname{Sin}[2 e] - 21\,312 a^2 b^5 \operatorname{Sin}[2 e] - \\
 & 29\,952 a b^6 \operatorname{Sin}[2 e] - 13\,824 b^7 \operatorname{Sin}[2 e] + 262 a^7 \operatorname{Sin}[2 f x] + 524 a^6 b \operatorname{Sin}[2 f x] - \\
 & 26 a^5 b^2 \operatorname{Sin}[2 f x] + 1728 a^4 b^3 \operatorname{Sin}[2 f x] + 14\,976 a^3 b^4 \operatorname{Sin}[2 f x] + \\
 & 28\,416 a^2 b^5 \operatorname{Sin}[2 f x] + 14\,592 a b^6 \operatorname{Sin}[2 f x] + 262 a^7 \operatorname{Sin}[4 e + 2 f x] + \\
 & 524 a^6 b \operatorname{Sin}[4 e + 2 f x] - 26 a^5 b^2 \operatorname{Sin}[4 e + 2 f x] + 1728 a^4 b^3 \operatorname{Sin}[4 e + 2 f x] + \\
 & 6912 a^3 b^4 \operatorname{Sin}[4 e + 2 f x] + 5376 a^2 b^5 \operatorname{Sin}[4 e + 2 f x] + 768 a b^6 \operatorname{Sin}[4 e + 2 f x] + \\
 & 238 a^7 \operatorname{Sin}[2 e + 4 f x] + 304 a^6 b \operatorname{Sin}[2 e + 4 f x] - 250 a^5 b^2 \operatorname{Sin}[2 e + 4 f x] + \\
 & 1556 a^4 b^3 \operatorname{Sin}[2 e + 4 f x] + 5904 a^3 b^4 \operatorname{Sin}[2 e + 4 f x] + 3744 a^2 b^5 \operatorname{Sin}[2 e + 4 f x] + \\
 & 238 a^7 \operatorname{Sin}[6 e + 4 f x] + 304 a^6 b \operatorname{Sin}[6 e + 4 f x] - 250 a^5 b^2 \operatorname{Sin}[6 e + 4 f x] + \\
 & 1556 a^4 b^3 \operatorname{Sin}[6 e + 4 f x] + 3888 a^3 b^4 \operatorname{Sin}[6 e + 4 f x] + 2016 a^2 b^5 \operatorname{Sin}[6 e + 4 f x] + \\
 & 87 a^7 \operatorname{Sin}[4 e + 6 f x] + 46 a^6 b \operatorname{Sin}[4 e + 6 f x] - 9 a^5 b^2 \operatorname{Sin}[4 e + 6 f x] + \\
 & 192 a^4 b^3 \operatorname{Sin}[4 e + 6 f x] + 160 a^3 b^4 \operatorname{Sin}[4 e + 6 f x] + 87 a^7 \operatorname{Sin}[8 e + 6 f x] + \\
 & 46 a^6 b \operatorname{Sin}[8 e + 6 f x] - 9 a^5 b^2 \operatorname{Sin}[8 e + 6 f x] + 192 a^4 b^3 \operatorname{Sin}[8 e + 6 f x] + \\
 & 160 a^3 b^4 \operatorname{Sin}[8 e + 6 f x] + 13 a^7 \operatorname{Sin}[6 e + 8 f x] + 16 a^6 b \operatorname{Sin}[6 e + 8 f x] - \\
 & 7 a^5 b^2 \operatorname{Sin}[6 e + 8 f x] - 10 a^4 b^3 \operatorname{Sin}[6 e + 8 f x] + 13 a^7 \operatorname{Sin}[10 e + 8 f x] + \\
 & 16 a^6 b \operatorname{Sin}[10 e + 8 f x] - 7 a^5 b^2 \operatorname{Sin}[10 e + 8 f x] - 10 a^4 b^3 \operatorname{Sin}[10 e + 8 f x] + \\
 & a^7 \operatorname{Sin}[8 e + 10 f x] + 2 a^6 b \operatorname{Sin}[8 e + 10 f x] + a^5 b^2 \operatorname{Sin}[8 e + 10 f x] + \\
 & a^7 \operatorname{Sin}[12 e + 10 f x] + 2 a^6 b \operatorname{Sin}[12 e + 10 f x] + a^5 b^2 \operatorname{Sin}[12 e + 10 f x]
 \end{aligned}$$

**Problem 219: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x])^4} dx$$

Optimal (type 3, 204 leaves, 7 steps):

$$\begin{aligned}
 & \frac{x}{a^4} - \frac{\sqrt{b} (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[c + d x]}{\sqrt{a + b}}\right]}{16 a^4 (a + b)^{7/2} d} - \\
 & \frac{b \operatorname{Tan}[c + d x]}{6 a (a + b) d (a + b + b \operatorname{Tan}[c + d x])^3} - \frac{b (11 a + 6 b) \operatorname{Tan}[c + d x]}{24 a^2 (a + b)^2 d (a + b + b \operatorname{Tan}[c + d x])^2} - \\
 & \frac{b (19 a^2 + 22 a b + 8 b^2) \operatorname{Tan}[c + d x]}{16 a^3 (a + b)^3 d (a + b + b \operatorname{Tan}[c + d x])^2}
 \end{aligned}$$

Result (type 3, 1411 leaves):

$$\left( (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) (a + 2 b + a \operatorname{Cos}[2 c + 2 d x])^4 \operatorname{Sec}[c + d x]^8 \left( \left( b \operatorname{ArcTan} \left[ \operatorname{Sec}[d x] \left( \frac{\operatorname{Cos}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} - \frac{i \operatorname{Sin}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} \right) \right. \right. \right. \right. \\ \left. \left. \left. \left. (-a \operatorname{Sin}[d x] - 2 b \operatorname{Sin}[d x] + a \operatorname{Sin}[2 c + d x]) \right] \operatorname{Cos}[2 c] \right) \right) / \right. \\ \left. \left( 256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]} \right) - \left( i b \operatorname{ArcTan} \left[ \operatorname{Sec}[d x] \left( \frac{\operatorname{Cos}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} - \frac{i \operatorname{Sin}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} \right) \right. \right. \right. \\ \left. \left. \left. \left. (-a \operatorname{Sin}[d x] - 2 b \operatorname{Sin}[d x] + a \operatorname{Sin}[2 c + d x]) \right] \operatorname{Sin}[2 c] \right) \right) / \right. \\ \left. \left( 256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]} \right) \right) \right) / \\ \left( (a+b)^3 (a+b \operatorname{Sec}[c+d x]^2)^4 \right) + \frac{1}{3072 a^4 (a+b)^3 d (a+b \operatorname{Sec}[c+d x]^2)^4} \\ (a + 2 b + a \operatorname{Cos}[2 c + 2 d x]) \\ \operatorname{Sec}[2 c] \operatorname{Sec}[c + d x]^8 \\ (480 a^6 d x \operatorname{Cos}[2 c] + 3168 a^5 b d x \operatorname{Cos}[2 c] + 8928 a^4 b^2 d x \operatorname{Cos}[2 c] + \\ 14112 a^3 b^3 d x \operatorname{Cos}[2 c] + 13248 a^2 b^4 d x \operatorname{Cos}[2 c] + 6912 a b^5 d x \operatorname{Cos}[2 c] + \\ 1536 b^6 d x \operatorname{Cos}[2 c] + 360 a^6 d x \operatorname{Cos}[2 d x] + 2232 a^5 b d x \operatorname{Cos}[2 d x] + \\ 5688 a^4 b^2 d x \operatorname{Cos}[2 d x] + 7272 a^3 b^3 d x \operatorname{Cos}[2 d x] + \\ 4608 a^2 b^4 d x \operatorname{Cos}[2 d x] + 1152 a b^5 d x \operatorname{Cos}[2 d x] + 360 a^6 d x \operatorname{Cos}[4 c + 2 d x] + \\ 2232 a^5 b d x \operatorname{Cos}[4 c + 2 d x] + 5688 a^4 b^2 d x \operatorname{Cos}[4 c + 2 d x] + \\ 7272 a^3 b^3 d x \operatorname{Cos}[4 c + 2 d x] + 4608 a^2 b^4 d x \operatorname{Cos}[4 c + 2 d x] + \\ 1152 a b^5 d x \operatorname{Cos}[4 c + 2 d x] + 144 a^6 d x \operatorname{Cos}[2 c + 4 d x] + 720 a^5 b d x \operatorname{Cos}[2 c + 4 d x] + \\ 1296 a^4 b^2 d x \operatorname{Cos}[2 c + 4 d x] + 1008 a^3 b^3 d x \operatorname{Cos}[2 c + 4 d x] + \\ 288 a^2 b^4 d x \operatorname{Cos}[2 c + 4 d x] + 144 a^6 d x \operatorname{Cos}[6 c + 4 d x] + \\ 720 a^5 b d x \operatorname{Cos}[6 c + 4 d x] + 1296 a^4 b^2 d x \operatorname{Cos}[6 c + 4 d x] + \\ 1008 a^3 b^3 d x \operatorname{Cos}[6 c + 4 d x] + 288 a^2 b^4 d x \operatorname{Cos}[6 c + 4 d x] + 24 a^6 d x \operatorname{Cos}[4 c + 6 d x] + \\ 72 a^5 b d x \operatorname{Cos}[4 c + 6 d x] + 72 a^4 b^2 d x \operatorname{Cos}[4 c + 6 d x] + 24 a^3 b^3 d x \operatorname{Cos}[4 c + 6 d x] + \\ 24 a^6 d x \operatorname{Cos}[8 c + 6 d x] + 72 a^5 b d x \operatorname{Cos}[8 c + 6 d x] + 72 a^4 b^2 d x \operatorname{Cos}[8 c + 6 d x] + \\ 24 a^3 b^3 d x \operatorname{Cos}[8 c + 6 d x] + 870 a^5 b \operatorname{Sin}[2 c] + 4292 a^4 b^2 \operatorname{Sin}[2 c] + \\ 8792 a^3 b^3 \operatorname{Sin}[2 c] + 9936 a^2 b^4 \operatorname{Sin}[2 c] + 5824 a b^5 \operatorname{Sin}[2 c] + 1408 b^6 \operatorname{Sin}[2 c] - \\ 870 a^5 b \operatorname{Sin}[2 d x] - 3792 a^4 b^2 \operatorname{Sin}[2 d x] - 6432 a^3 b^3 \operatorname{Sin}[2 d x] - \\ 4608 a^2 b^4 \operatorname{Sin}[2 d x] - 1248 a b^5 \operatorname{Sin}[2 d x] + 435 a^5 b \operatorname{Sin}[4 c + 2 d x] + \\ 2124 a^4 b^2 \operatorname{Sin}[4 c + 2 d x] + 3972 a^3 b^3 \operatorname{Sin}[4 c + 2 d x] + 3072 a^2 b^4 \operatorname{Sin}[4 c + 2 d x] + \\ 864 a b^5 \operatorname{Sin}[4 c + 2 d x] - 435 a^5 b \operatorname{Sin}[2 c + 4 d x] - 1374 a^4 b^2 \operatorname{Sin}[2 c + 4 d x] - \\ 1248 a^3 b^3 \operatorname{Sin}[2 c + 4 d x] - 384 a^2 b^4 \operatorname{Sin}[2 c + 4 d x] + 87 a^5 b \operatorname{Sin}[6 c + 4 d x] + \\ 366 a^4 b^2 \operatorname{Sin}[6 c + 4 d x] + 408 a^3 b^3 \operatorname{Sin}[6 c + 4 d x] + 144 a^2 b^4 \operatorname{Sin}[6 c + 4 d x] - \\ 87 a^5 b \operatorname{Sin}[4 c + 6 d x] - 116 a^4 b^2 \operatorname{Sin}[4 c + 6 d x] - 44 a^3 b^3 \operatorname{Sin}[4 c + 6 d x])$$

### Problem 228: Unable to integrate problem.

$$\int \text{Sec}[e + f x]^5 \sqrt{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 4, 471 leaves, 11 steps):

$$\begin{aligned} & - \left( \left( (2 a^2 - 3 a b - 8 b^2) \sqrt{a + b \text{Sec}[e + f x]^2} \text{Sin}[e + f x] \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) / \right. \\ & \quad \left. (15 b^2 f \sqrt{b + a \text{Cos}[e + f x]^2}) \right) + \\ & \left( (2 a^2 - 3 a b - 8 b^2) \sqrt{\text{Cos}[e + f x]^2} \text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a + b}] \sqrt{a + b \text{Sec}[e + f x]^2} \right. \\ & \quad \left. \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) / \left( 15 b^2 f \sqrt{b + a \text{Cos}[e + f x]^2} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a + b}} \right) - \\ & \left( (a - 8 b) (a + b) \sqrt{\text{Cos}[e + f x]^2} \text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a + b}] \sqrt{a + b \text{Sec}[e + f x]^2} \right. \\ & \quad \left. \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a + b}} \right) / \left( 15 b f \sqrt{b + a \text{Cos}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) + \\ & \left( (a + 4 b) \text{Sec}[e + f x] \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \text{Tan}[e + f x] \right) / \\ & \quad \left( 15 b f \sqrt{b + a \text{Cos}[e + f x]^2} \right) + \\ & \left( \text{Sec}[e + f x]^3 \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \text{Tan}[e + f x] \right) / \\ & \quad \left( 5 f \sqrt{b + a \text{Cos}[e + f x]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \text{Sec}[e + f x]^5 \sqrt{a + b \text{Sec}[e + f x]^2} dx$$

### Problem 229: Unable to integrate problem.

$$\int \text{Sec}[e + f x]^3 \sqrt{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 4, 364 leaves, 10 steps):

$$\frac{(a+2b)\sqrt{a+b\sec[e+fx]^2}\sin[e+fx]\sqrt{a+b-a\sin[e+fx]^2}}{3bf\sqrt{b+a\cos[e+fx]^2}} -$$

$$\left( (a+2b)\sqrt{\cos[e+fx]^2}\operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{a+b\sec[e+fx]^2}\right.$$

$$\left. \sqrt{a+b-a\sin[e+fx]^2} \right) / \left( 3bf\sqrt{b+a\cos[e+fx]^2}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}} \right) +$$

$$\left( 2(a+b)\sqrt{\cos[e+fx]^2}\operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{a+b\sec[e+fx]^2}\right.$$

$$\left. \sqrt{1-\frac{a\sin[e+fx]^2}{a+b}} \right) / \left( 3f\sqrt{b+a\cos[e+fx]^2}\sqrt{a+b-a\sin[e+fx]^2} \right) +$$

$$\left( \sec[e+fx]\sqrt{a+b\sec[e+fx]^2}\sqrt{a+b-a\sin[e+fx]^2}\tan[e+fx] \right) /$$

$$\left( 3f\sqrt{b+a\cos[e+fx]^2} \right)$$

Result (type 8, 27 leaves):

$$\int \sec[e+fx]^3 \sqrt{a+b\sec[e+fx]^2} dx$$

### Problem 230: Unable to integrate problem.

$$\int \sec[e+fx]\sqrt{a+b\sec[e+fx]^2} dx$$

Optimal (type 4, 271 leaves, 10 steps):

$$\frac{\sqrt{a+b\sec[e+fx]^2}\sin[e+fx]\sqrt{a+b-a\sin[e+fx]^2}}{f\sqrt{b+a\cos[e+fx]^2}} -$$

$$\left( \sqrt{\cos[e+fx]^2}\operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{a+b\sec[e+fx]^2}\right.$$

$$\left. \sqrt{a+b-a\sin[e+fx]^2} \right) / \left( f\sqrt{b+a\cos[e+fx]^2}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}} \right) +$$

$$\left( (a+b)\sqrt{\cos[e+fx]^2}\operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{a+b\sec[e+fx]^2}\right.$$

$$\left. \sqrt{1-\frac{a\sin[e+fx]^2}{a+b}} \right) / \left( f\sqrt{b+a\cos[e+fx]^2}\sqrt{a+b-a\sin[e+fx]^2} \right)$$



Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \, dx$$

Problem 232: Unable to integrate problem.

$$\int \operatorname{Cos}[e + f x]^3 \sqrt{a + b \operatorname{Sec}[e + f x]^2} \, dx$$

Optimal (type 4, 299 leaves, 9 steps):

$$\begin{aligned} & \left( \operatorname{Cos}[e + f x]^2 \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Sin}[e + f x] \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right) / \\ & \left( 3 f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \right) + \\ & \left( (2 a + b) \sqrt{\operatorname{Cos}[e + f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \right. \\ & \left. \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right) / \left( 3 a f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) - \\ & \left( b (a + b) \sqrt{\operatorname{Cos}[e + f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \right. \\ & \left. \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) / \left( 3 a f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Cos}[e + f x]^3 \sqrt{a + b \operatorname{Sec}[e + f x]^2} \, dx$$

Problem 233: Unable to integrate problem.

$$\int \operatorname{Cos}[e + f x]^5 \sqrt{a + b \operatorname{Sec}[e + f x]^2} \, dx$$

Optimal (type 4, 400 leaves, 10 steps):

$$\begin{aligned} & \left( 2 (2 a - b) \cos [e + f x]^2 \sqrt{a + b \sec [e + f x]^2} \sin [e + f x] \sqrt{a + b - a \sin [e + f x]^2} \right) / \\ & \left( 15 a f \sqrt{b + a \cos [e + f x]^2} \right) + \\ & \left( \cos [e + f x]^2 \sqrt{a + b \sec [e + f x]^2} \sin [e + f x] (a + b - a \sin [e + f x]^2)^{3/2} \right) / \\ & \left( 5 a f \sqrt{b + a \cos [e + f x]^2} \right) + \\ & \left( (8 a^2 + 3 a b - 2 b^2) \sqrt{\cos [e + f x]^2} \operatorname{EllipticE} \left[ \operatorname{ArcSin} [\sin [e + f x]], \frac{a}{a + b} \right] \sqrt{a + b \sec [e + f x]^2} \right. \\ & \left. \sqrt{a + b - a \sin [e + f x]^2} \right) / \left( 15 a^2 f \sqrt{b + a \cos [e + f x]^2} \sqrt{1 - \frac{a \sin [e + f x]^2}{a + b}} \right) - \\ & \left( 2 (2 a - b) b (a + b) \sqrt{\cos [e + f x]^2} \operatorname{EllipticF} \left[ \operatorname{ArcSin} [\sin [e + f x]], \frac{a}{a + b} \right] \sqrt{a + b \sec [e + f x]^2} \right. \\ & \left. \sqrt{1 - \frac{a \sin [e + f x]^2}{a + b}} \right) / \left( 15 a^2 f \sqrt{b + a \cos [e + f x]^2} \sqrt{a + b - a \sin [e + f x]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \cos [e + f x]^5 \sqrt{a + b \sec [e + f x]^2} dx$$

**Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [e + f x]^6 \sqrt{a + b \sec [e + f x]^2} dx$$

Optimal (type 3, 186 leaves, 6 steps):

$$\begin{aligned} & \frac{(a + b) (a^2 - 2 a b + 5 b^2) \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \tan [e + f x]}{\sqrt{a + b \tan [e + f x]^2}} \right]}{16 b^{5/2} f} + \\ & \frac{(a^2 - 2 a b + 5 b^2) \tan [e + f x] \sqrt{a + b \tan [e + f x]^2}}{16 b^2 f} - \\ & \frac{(3 a - 5 b) \tan [e + f x] (a + b \tan [e + f x]^2)^{3/2}}{24 b^2 f} + \\ & \frac{\sec [e + f x]^2 \tan [e + f x] (a + b \tan [e + f x]^2)^{3/2}}{6 b f} \end{aligned}$$

Result (type 3, 407 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx] \left( -\frac{1}{(1+e^{2i(e+fx)})^6} i \sqrt{b} (-1+e^{2i(e+fx)}) \right) \right. \\ \left. \left( -3a^2 (1+e^{2i(e+fx)})^4 + 4ab (1+e^{2i(e+fx)})^2 (1+4e^{2i(e+fx)}+e^{4i(e+fx)}) + \right. \right. \\ \left. \left. b^2 (15+100e^{2i(e+fx)}+298e^{4i(e+fx)}+100e^{6i(e+fx)}+15e^{8i(e+fx)}) \right) - \right. \\ \left. \left( 3(a^3-a^2b+3ab^2+5b^3) \operatorname{Log}\left[\frac{1}{1+e^{2i(e+fx)}} \left( -4\sqrt{b} (-1+e^{2i(e+fx)}) f + \right. \right. \right. \right. \\ \left. \left. \left. 4i \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} f \right) \right] \right) \right) / \left( \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \\ \left. \sqrt{a+b \operatorname{Sec}[e+fx]^2} \right) / \left( 24\sqrt{2} b^{5/2} f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \right)$$

**Problem 235: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{(a-3b)(a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8b^{3/2}f} - \frac{(a-3b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8bf} + \frac{\operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4bf}$$

Result (type 3, 322 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx] \left( -\frac{1}{(1+e^{2i(e+fx)})^4} i \sqrt{b} (-1+e^{2i(e+fx)}) \left( a(1+e^{2i(e+fx)})^2 + b(3+14e^{2i(e+fx)}+3e^{4i(e+fx)}) \right) \right) + \right. \\ \left( (a^2-2ab-3b^2) \operatorname{Log}\left[\frac{1}{1+e^{2i(e+fx)}} \left( -4\sqrt{b} (-1+e^{2i(e+fx)}) f + \right. \right. \right. \right. \\ \left. \left. \left. 4i \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} f \right) \right] \right) \right) / \left( \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \\ \left. \sqrt{a+b \operatorname{Sec}[e+fx]^2} \right) / \left( 4\sqrt{2} b^{3/2} f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \right)$$

**Problem 236: Result unnecessarily involves complex numbers and more than**

twice size of optimal antiderivative.

$$\int \text{Sec}[e + f x]^2 \sqrt{a + b \text{Sec}[e + f x]^2} \, dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$\frac{(a + b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{2 \sqrt{b} f} + \frac{\text{Tan}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{2 f}$$

Result (type 3, 257 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \text{Cos}[e + f x] \right. \\ \left. - \frac{i(-1 + e^{2i(e+fx)})}{(1 + e^{2i(e+fx)})^2} - \frac{(a + b) \text{Log}\left[\frac{-4\sqrt{b}(-1 + e^{2i(e+fx)}) f + 4i\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} f}{1 + e^{2i(e+fx)}}\right]}{\sqrt{b} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}} \right) \\ \left. \sqrt{a + b \text{Sec}[e + f x]^2} \right) / \left( \sqrt{2} f \sqrt{a + 2b + a \text{Cos}[2e + 2fx]} \right)$$

Problem 237: Unable to integrate problem.

$$\int \sqrt{a + b \text{Sec}[e + f x]^2} \, dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{f} + \frac{\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{f}$$

Result (type 8, 18 leaves):

$$\int \sqrt{a + b \text{Sec}[e + f x]^2} \, dx$$

Problem 238: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Cos}[e + f x]^2 \sqrt{a + b \text{Sec}[e + f x]^2} \, dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$\frac{(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2\sqrt{a}f} + \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 322 leaves):

$$\left( e^{-i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \right. \\ \left. \operatorname{Cos}[e+fx] \left( -i(-1+e^{2i(e+fx)}) + \left( 2(a+b) e^{2i(e+fx)} \right. \right. \right. \\ \left. \left. \left( 2fx - i \operatorname{Log}[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] + \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] \right) \right) \right) / \\ \left( \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \left( \sqrt{a+b \operatorname{Sec}[e+fx]^2} \right) / \\ \left( 4\sqrt{2} f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \right)$$

**Problem 239: Unable to integrate problem.**

$$\int \operatorname{Cos}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{(3a-b)(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8a^{3/2}f} + \\ \frac{(3a-b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8af} + \\ \frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4af}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Cos}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

**Problem 240: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [e+f x]^6 \sqrt{a+b \operatorname{Sec}[e+f x]^2} \, dx$$

Optimal (type 3, 196 leaves, 7 steps):

$$\frac{(a+b) (5 a^2-2 a b+b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{16 a^{5/2} f} +$$

$$\frac{(3 a-b) (5 a+3 b) \cos [e+f x] \operatorname{Sin}[e+f x] \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}}{48 a^2 f} +$$

$$\frac{(5 a+b) \cos [e+f x]^3 \operatorname{Sin}[e+f x] \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}}{24 a f} +$$

$$\frac{\cos [e+f x]^5 \operatorname{Sin}[e+f x] \sqrt{a+b+b \operatorname{Tan}[e+f x]^2}}{6 f}$$

Result (type 6, 1902 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] \right. \\ \left. \cos [e+f x]^{10} \sqrt{a+2 b+a \cos [2(e+f x)]} \sqrt{a+b \operatorname{Sec}[e+f x]^2} \operatorname{Sin}[e+f x]\right) /$$

$$\left( f \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] - \right. \right. \\ \left. \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] + \right. \right. \\ \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] \right) \operatorname{Sin}[e+f x]^2 \right)$$

$$\left( \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] \cos [e+f x]^5 \right. \right. \\ \left. \left. \sqrt{a+2 b+a \cos [2(e+f x)]} \right) / \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}[e+f x]^2, \right. \right. \right. \\ \left. \left. \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] - \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] + \right. \right. \\ \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] \right) \operatorname{Sin}[e+f x]^2 \right) -$$

$$\begin{aligned}
 & \left( 12 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. \cos[e+fx]^3 \sqrt{a+2b+a \cos[2(e+fx)]} \sin[e+fx]^2 \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \\
 & \quad \left( a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \Big) + \\
 & \left( 3 (a+b) \cos[e+fx]^4 \sqrt{a+2b+a \cos[2(e+fx)]} \sin[e+fx] \left( -\frac{1}{3(a+b)} a f \operatorname{AppellF1} \left[ \right. \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \\
 & \quad \left. \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) \right) / \\
 & \left( f \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \quad \left( a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \Big) - \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \right. \\
 & \quad \left. \sqrt{a+2b+a \cos[2(e+fx)]} \sin[e+fx] \right. \\
 & \quad \left( -2 f \left( a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \right. \right. \\
 & \quad \left. \left. -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \cos[e+fx] \sin[e+fx] + 3 (a+b) \right. \\
 & \quad \left( -\frac{1}{3(a+b)} a f \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \right. \\
 & \quad \left. \left. \sin[e+fx] - \frac{4}{3} f \operatorname{AppellF1} \left[ \frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \left. \left( \cos[e+fx] \sin[e+fx] \right) - \sin[e+fx]^2 \right\} a \left( \frac{1}{5(a+b)} {}_3F_2 \operatorname{AppellF1} \left[ \frac{5}{2}, -2, \frac{3}{2}, \right. \right. \\ & \left. \left. \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{12}{5} f \operatorname{AppellF1} \left[ \right. \right. \\ & \left. \left. \frac{5}{2}, -1, \frac{1}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) + \\ & 4(a+b) \left( -\frac{1}{5(a+b)} {}_3F_2 \operatorname{AppellF1} \left[ \frac{5}{2}, -1, \frac{1}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right. \\ & \left. \cos[e+fx] \sin[e+fx] - \frac{6}{5} f \cos[e+fx] \sin[e+fx] \left( 1 - \frac{a \sin[e+fx]^2}{a+b} \right)^{3/2} \right. \\ & \left. \left( \frac{5}{6 \left( 1 - \frac{a \sin[e+fx]^2}{a+b} \right)} + \left( 5(a+b)^3 \csc[e+fx]^6 \left( -\frac{2a \sin[e+fx]^2}{a+b} - \right. \right. \right. \right. \\ & \left. \left. \left. \frac{4a^2 \sin[e+fx]^4}{3(a+b)^2} + \frac{2\sqrt{a} \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}} \right] \sin[e+fx]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} \right) \right) \right) \left/ \left( 32 \right. \right. \\ & \left. \left. a^3 \left( 1 - \frac{a \sin[e+fx]^2}{a+b} \right) \right) \right) \left/ \right. \\ & \left( f \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\ & \left. \left( a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + 4(a+b) \right. \right. \\ & \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) - \\ & \left( 3a(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \right. \\ & \left. \sin[e+fx] \sin[2(e+fx)] \right) \left/ \left( \sqrt{a+2b+a \cos[2(e+fx)]} \right) \right. \\ & \left. \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \end{aligned}$$



$$\left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx], \frac{a \sin[e+fx]^2}{a+b}\right] + 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right)$$

### Problem 241: Unable to integrate problem.

$$\int \sec[e+fx]^5 (a+b \sec[e+fx]^2)^{3/2} dx$$

Optimal (type 4, 572 leaves, 12 steps):

$$\begin{aligned} & - \left( \left( 2(a+2b)(a^2-4ab-4b^2) \sqrt{a+b \sec[e+fx]^2} \sin[e+fx] \sqrt{a+b-a \sin[e+fx]^2} \right) / \right. \\ & \quad \left. \left( 35b^2 f \sqrt{b+a \cos[e+fx]^2} \right) \right) + \\ & \left( 2(a+2b)(a^2-4ab-4b^2) \sqrt{\cos[e+fx]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] \right. \\ & \quad \left. \sqrt{a+b \sec[e+fx]^2} \sqrt{a+b-a \sin[e+fx]^2} \right) / \\ & \left( 35b^2 f \sqrt{b+a \cos[e+fx]^2} \sqrt{1-\frac{a \sin[e+fx]^2}{a+b}} \right) - \\ & \left( (a+b)(a^2-16ab-16b^2) \sqrt{\cos[e+fx]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] \right. \\ & \quad \left. \sqrt{a+b \sec[e+fx]^2} \sqrt{1-\frac{a \sin[e+fx]^2}{a+b}} \right) / \\ & \left( 35bf \sqrt{b+a \cos[e+fx]^2} \sqrt{a+b-a \sin[e+fx]^2} \right) + \\ & \left( (a^2+11ab+8b^2) \sec[e+fx] \sqrt{a+b \sec[e+fx]^2} \sqrt{a+b-a \sin[e+fx]^2} \tan[e+fx] \right) / \\ & \left( 35bf \sqrt{b+a \cos[e+fx]^2} \right) + \\ & \left( 2(4a+3b) \sec[e+fx]^3 \sqrt{a+b \sec[e+fx]^2} \sqrt{a+b-a \sin[e+fx]^2} \tan[e+fx] \right) / \\ & \left( 35f \sqrt{b+a \cos[e+fx]^2} \right) + \\ & \left( b \sec[e+fx]^5 \sqrt{a+b \sec[e+fx]^2} \sqrt{a+b-a \sin[e+fx]^2} \tan[e+fx] \right) / \\ & \left( 7f \sqrt{b+a \cos[e+fx]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \text{Sec}[e + f x]^5 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Problem 242: Unable to integrate problem.

$$\int \text{Sec}[e + f x]^3 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 470 leaves, 11 steps):

$$\begin{aligned} & \left( (3a^2 + 13ab + 8b^2) \sqrt{a + b \text{Sec}[e + f x]^2} \text{Sin}[e + f x] \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) / \\ & \left( 15bf \sqrt{b + a \text{Cos}[e + f x]^2} \right) - \\ & \left( (3a^2 + 13ab + 8b^2) \sqrt{\text{Cos}[e + f x]^2} \text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]]], \frac{a}{a+b} \right) \sqrt{a + b \text{Sec}[e + f x]^2} \\ & \sqrt{a + b - a \text{Sin}[e + f x]^2} \Big/ \left( 15bf \sqrt{b + a \text{Cos}[e + f x]^2} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}} \right) + \\ & \left( (a+b) (9a + 8b) \sqrt{\text{Cos}[e + f x]^2} \text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]]], \frac{a}{a+b} \right) \sqrt{a + b \text{Sec}[e + f x]^2} \\ & \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}} \Big/ \left( 15f \sqrt{b + a \text{Cos}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) + \\ & \left( 2(3a + 2b) \text{Sec}[e + f x] \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \text{Tan}[e + f x] \right) / \\ & \left( 15f \sqrt{b + a \text{Cos}[e + f x]^2} \right) + \\ & \left( b \text{Sec}[e + f x]^3 \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \text{Tan}[e + f x] \right) / \\ & \left( 5f \sqrt{b + a \text{Cos}[e + f x]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \text{Sec}[e + f x]^3 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Problem 243: Unable to integrate problem.

$$\int \text{Sec}[e + f x] (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 366 leaves, 10 steps):

$$\frac{2 (2 a + b) \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Sin}[e + f x] \sqrt{a + b - a \operatorname{Sin}[e + f x]^2}}{3 f \sqrt{b + a \operatorname{Cos}[e + f x]^2}} -$$

$$\left( 2 (2 a + b) \sqrt{\operatorname{Cos}[e + f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \right.$$

$$\left. \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right) / \left( 3 f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) +$$

$$\left( (a + b) (3 a + 2 b) \sqrt{\operatorname{Cos}[e + f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \right.$$

$$\left. \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) / \left( 3 f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right) +$$

$$\left( b \operatorname{Sec}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \operatorname{Tan}[e + f x] \right) /$$

$$\left( 3 f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \right)$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

**Problem 244: Unable to integrate problem.**

$$\int \operatorname{Cos}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 277 leaves, 9 steps):

$$\frac{b \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Sin}[e + f x] \sqrt{a + b - a \operatorname{Sin}[e + f x]^2}}{f \sqrt{b + a \operatorname{Cos}[e + f x]^2}} +$$

$$\left( (a - b) \sqrt{\operatorname{Cos}[e + f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \right.$$

$$\left. \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right) / \left( f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) +$$

$$\left( b (a + b) \sqrt{\operatorname{Cos}[e + f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[e + f x]^2} \right.$$

$$\left. \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) / \left( f \sqrt{b + a \operatorname{Cos}[e + f x]^2} \sqrt{a + b - a \operatorname{Sin}[e + f x]^2} \right)$$

Result (type 8, 25 leaves):

$$\int \cos [e + f x] (a + b \operatorname{Sec} [e + f x]^2)^{3/2} dx$$

Problem 246: Unable to integrate problem.

$$\int \cos [e + f x]^5 (a + b \operatorname{Sec} [e + f x]^2)^{3/2} dx$$

Optimal (type 4, 395 leaves, 10 steps):

$$\begin{aligned} & - \left( \left( 2 (a - 3 (a + b)) \cos [e + f x]^2 \sqrt{a + b \operatorname{Sec} [e + f x]^2} \sin [e + f x] \sqrt{a + b - a \sin [e + f x]^2} \right) / \right. \\ & \quad \left. \left( 15 f \sqrt{b + a \cos [e + f x]^2} \right) \right) + \\ & \left( a \cos [e + f x]^4 \sqrt{a + b \operatorname{Sec} [e + f x]^2} \sin [e + f x] \sqrt{a + b - a \sin [e + f x]^2} \right) / \\ & \left( 5 f \sqrt{b + a \cos [e + f x]^2} \right) + \\ & \left( 8 a^2 + 13 a b + 3 b^2 \right) \sqrt{\cos [e + f x]^2} \operatorname{EllipticE} \left[ \operatorname{ArcSin} [\sin [e + f x]], \frac{a}{a + b} \right] \sqrt{a + b \operatorname{Sec} [e + f x]^2} \\ & \quad \sqrt{a + b - a \sin [e + f x]^2} \left/ \left( 15 a f \sqrt{b + a \cos [e + f x]^2} \sqrt{1 - \frac{a \sin [e + f x]^2}{a + b}} \right) \right. - \\ & \left( b (a + b) (4 a + 3 b) \sqrt{\cos [e + f x]^2} \operatorname{EllipticF} \left[ \operatorname{ArcSin} [\sin [e + f x]], \frac{a}{a + b} \right] \sqrt{a + b \operatorname{Sec} [e + f x]^2} \right. \\ & \quad \left. \sqrt{1 - \frac{a \sin [e + f x]^2}{a + b}} \right) \left/ \left( 15 a f \sqrt{b + a \cos [e + f x]^2} \sqrt{a + b - a \sin [e + f x]^2} \right) \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \cos [e + f x]^5 (a + b \operatorname{Sec} [e + f x]^2)^{3/2} dx$$

Problem 247: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sec [e + f x]^6 (a + b \operatorname{Sec} [e + f x]^2)^{3/2} dx$$

Optimal (type 3, 243 leaves, 7 steps):

$$\frac{(a+b)^2 (3a^2 - 10ab + 35b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{128 b^{5/2} f} +$$

$$\frac{(a+b) (3a^2 - 10ab + 35b^2) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{128 b^2 f} +$$

$$\frac{(3a^2 - 10ab + 35b^2) \operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{192 b^2 f} -$$

$$\frac{(3a-7b) \operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{48 b^2 f} +$$

$$\frac{\operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{8 b f}$$

Result (type 3, 512 leaves):

$$\frac{1}{96 \sqrt{2} b^{5/2} f (a+2b+a \operatorname{Cos}[2e+2fx])^{3/2}}$$

$$e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \operatorname{Cos}[e+fx]^3$$

$$\left( -\frac{1}{(1+e^{2i(e+fx)})^8} i \sqrt{b} (-1+e^{2i(e+fx)}) \left( -9a^3 (1+e^{2i(e+fx)})^6 + 3a^2 b (1+e^{2i(e+fx)})^4 \right. \right.$$

$$\left. \left( 5+18e^{2i(e+fx)} + 5e^{4i(e+fx)} \right) + a b^2 (1+e^{2i(e+fx)})^2 (145+948e^{2i(e+fx)} + \right.$$

$$\left. 2758e^{4i(e+fx)} + 948e^{6i(e+fx)} + 145e^{8i(e+fx)} \right) + b^3 (105+910e^{2i(e+fx)} + \right.$$

$$\left. 3591e^{4i(e+fx)} + 8644e^{6i(e+fx)} + 3591e^{8i(e+fx)} + 910e^{10i(e+fx)} + 105e^{12i(e+fx)} \right) \left. \right) -$$

$$\left( 3(a+b)^2 (3a^2 - 10ab + 35b^2) \operatorname{Log}\left[ \frac{1}{1+e^{2i(e+fx)}} \left( -4\sqrt{b} (-1+e^{2i(e+fx)}) f + \right. \right. \right.$$

$$\left. \left. 4i \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} f \right) \right] \right) /$$

$$\left( \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) (a+b \operatorname{Sec}[e+fx]^2)^{3/2}$$

**Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^4 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(a-5b)(a+b)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16 b^{3/2} f} \\
 & - \frac{(a-5b)(a+b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16 b f} \\
 & + \frac{(a-5b) \operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{24 b f} + \frac{\operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{6 b f}
 \end{aligned}$$

Result (type 3, 400 leaves):

$$\begin{aligned}
 & \left( e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \operatorname{Cos}[e+fx]^3 \left( -\frac{1}{(1+e^{2i(e+fx)})^6} i \sqrt{b} (-1+e^{2i(e+fx)}) \right. \right. \\
 & \quad \left. \left( 3 a^2 (1+e^{2i(e+fx)})^4 + 2 a b (1+e^{2i(e+fx)})^2 (11+50 e^{2i(e+fx)} + 11 e^{4i(e+fx)}) + \right. \right. \\
 & \quad \left. \left. b^2 (15+100 e^{2i(e+fx)} + 298 e^{4i(e+fx)} + 100 e^{6i(e+fx)} + 15 e^{8i(e+fx)}) \right) + \right. \\
 & \quad \left. \left( 3 (a-5b)(a+b)^2 \operatorname{Log}\left[\frac{1}{1+e^{2i(e+fx)}} \left( -4 \sqrt{b} (-1+e^{2i(e+fx)}) f + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 4 i \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2} f\right)\right] \right) \right) / \left( \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2} \right) \\
 & \left. (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / \left( 12 \sqrt{2} b^{3/2} f (a+2 b+a \operatorname{Cos}[2 e+2 f x])^{3/2} \right)
 \end{aligned}$$

**Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^2 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 111 leaves, 5 steps):

$$\begin{aligned}
 & \frac{3(a+b)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8 \sqrt{b} f} + \\
 & \frac{3(a+b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8 f} + \frac{\operatorname{Tan}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4 f}
 \end{aligned}$$

Result (type 3, 313 leaves):

$$\begin{aligned}
 & \left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\
 & \left( -\frac{1}{(1 + e^{2i(e+fx)})^4} i (-1 + e^{2i(e+fx)}) (5a (1 + e^{2i(e+fx)})^2 + b (3 + 14 e^{2i(e+fx)} + 3 e^{4i(e+fx)})) - \right. \\
 & \left. \left( 3(a+b)^2 \operatorname{Log}\left[ \frac{1}{1 + e^{2i(e+fx)}} \left( -4\sqrt{b} (-1 + e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) \right] \right) \right) / \left( \sqrt{b} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \\
 & \left. (a + b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / \left( 2\sqrt{2} f (a + 2b + a \cos[2e + 2fx])^{3/2} \right)
 \end{aligned}$$

**Problem 250: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

Optimal (type 3, 118 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} (3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves):

$$\frac{1}{f (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}} \sqrt{2} e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2}$$

$$\operatorname{Cos}[e + f x]^3 \left( -\frac{i b (-1 + e^{2 i (e+f x)})}{(1 + e^{2 i (e+f x)})^2} + \frac{1}{\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}} \right)$$

$$\left( 2 a^{3/2} f x - i a^{3/2} \operatorname{Log}[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] + \right.$$

$$i a^{3/2} \operatorname{Log}[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] -$$

$$3 a \sqrt{b} \operatorname{Log}\left[ -2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f \right] /$$

$$(b (3 a + b) (1 + e^{2 i (e+f x)})) -$$

$$b^{3/2} \operatorname{Log}\left[ -2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f \right] /$$

$$(b (3 a + b) (1 + e^{2 i (e+f x)})) \left. \right] (a + b \operatorname{Sec}[e + f x]^2)^{3/2}$$

**Problem 251: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + f x]^2 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 124 leaves, 7 steps):

$$\frac{\sqrt{a} (a + 3 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{2 f} +$$

$$\frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{f} + \frac{a \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{2 f}$$

Result (type 3, 466 leaves):



$$\frac{1}{2\sqrt{2} f (a + 2b + a \operatorname{Cos}[2e + 2fx])^{3/2}}$$

$$e^{-i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \operatorname{Cos}[e + fx]^3 \left( -i a (-1 + e^{2i(e+fx)}) + \right.$$

$$\frac{1}{\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} 2 e^{2i(e+fx)} \left( 2 a^{3/2} f x + 6 \sqrt{a} b f x - i \sqrt{a} (a + 3b) \operatorname{Log} \left[ \right. \right.$$

$$e^{-2ie} \left( a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) + i \sqrt{a} (a + 3b)$$

$$\left. \operatorname{Log} \left[ e^{-2ie} \left( a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \right.$$

$$4b^{3/2} \operatorname{Log} \left[ - \left( \left( e^{3ie} \left( \sqrt{b} (-1 + e^{2i(e+fx)}) - i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) f \right) / \right.$$

$$\left. \left. \left. \left. \left. \left. (2b^2 (1 + e^{2i(e+fx)})) \right) \right) \right) \right) \right) \right) \right) (a + b \operatorname{Sec}[e + fx]^2)^{3/2}$$

**Problem 252: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + fx]^4 (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

Optimal (type 3, 125 leaves, 5 steps):

$$\frac{3(a+b)^2 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{8\sqrt{a}f} + \frac{3(a+b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} +$$

$$\frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4f}$$

Result (type 3, 369 leaves):

$$\left( e^{-3i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\ \left. \left( -i(-1 + e^{2i(e+fx)}) (10b e^{2i(e+fx)} + a(1 + 8e^{2i(e+fx)} + e^{4i(e+fx)})) + \left( 12(a+b)^2 e^{4i(e+fx)} \right. \right. \right. \\ \left. \left. \left( 2fx - i \log[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}] + \right. \right. \right. \\ \left. \left. \left. i \log[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}] \right) \right) \right) / \\ \left( \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right) \left( a + b \sec[e+fx]^2 \right)^{3/2} / \\ \left( 16\sqrt{2} f (a + 2b + a \cos[2e + 2fx])^{3/2} \right)$$

**Problem 253: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^6 (a + b \sec[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 193 leaves, 6 steps):

$$\frac{(5a - b)(a + b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b+b \tan[e+fx]^2}}\right]}{16 a^{3/2} f} + \\ \frac{(5a - b)(a + b) \cos[e+fx] \sin[e+fx] \sqrt{a + b + b \tan[e+fx]^2}}{16 a f} + \\ \frac{(5a - b) \cos[e+fx]^3 \sin[e+fx] (a + b + b \tan[e+fx]^2)^{3/2}}{24 a f} + \\ \frac{\cos[e+fx]^5 \sin[e+fx] (a + b + b \tan[e+fx]^2)^{5/2}}{6 a f}$$

Result (type 3, 453 leaves):

$$\begin{aligned}
 & \frac{1}{96 \sqrt{2} a^{3/2} f (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}} \\
 & e^{-5 i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} \operatorname{Cos}[e + f x]^3 \\
 & \left( -i \sqrt{a} (-1 + e^{2 i (e+f x)}) (6 b^2 e^{4 i (e+f x)} + a b e^{2 i (e+f x)} (7 + 58 e^{2 i (e+f x)} + 7 e^{4 i (e+f x)} + \right. \\
 & \quad \left. a^2 (1 + 9 e^{2 i (e+f x)} + 46 e^{4 i (e+f x)} + 9 e^{6 i (e+f x)} + e^{8 i (e+f x)})) + (12 (5 a - b) (a + b)^2 \right. \\
 & \quad \left. e^{6 i (e+f x)} \left( 2 f x - i \operatorname{Log}[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] + \right. \right. \\
 & \quad \left. \left. i \operatorname{Log}[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] \right) \right) / \\
 & \left( \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) (a + b \operatorname{Sec}[e + f x]^2)^{3/2}
 \end{aligned}$$

**Problem 254: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x]^2)^{5/2} dx$$

Optimal (type 3, 166 leaves, 8 steps):

$$\begin{aligned}
 & \frac{a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+b \operatorname{Tan}[c+d x]^2}}\right]}{d} + \frac{\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[c+d x]}{\sqrt{a+b \operatorname{Tan}[c+d x]^2}}\right]}{8 d} + \\
 & \frac{b (7 a + 3 b) \operatorname{Tan}[c + d x] \sqrt{a + b \operatorname{Tan}[c + d x]^2}}{8 d} + \frac{b \operatorname{Tan}[c + d x] (a + b + b \operatorname{Tan}[c + d x]^2)^{3/2}}{4 d}
 \end{aligned}$$

Result (type 3, 706 leaves):

$$\frac{1}{\sqrt{2} d (a + 2 b + a \operatorname{Cos}[2 c + 2 d x])^{5/2}} e^{i(c+dx)} \sqrt{4 b + a e^{-2 i(c+dx)} (1 + e^{2 i(c+dx)})^2} \operatorname{Cos}[c + d x]^5$$

$$\left( -\frac{1}{(1 + e^{2 i(c+dx)})^4} i b (-1 + e^{2 i(c+dx)}) (9 a (1 + e^{2 i(c+dx)})^2 + b (3 + 14 e^{2 i(c+dx)} + 3 e^{4 i(c+dx)})) \right) +$$

$$\frac{1}{\sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}}$$

$$\left( 8 a^{5/2} d x - 4 i a^{5/2} \operatorname{Log}[a + 2 b + a e^{2 i(c+dx)} + \sqrt{a} \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}] + \right.$$

$$4 i a^{5/2} \operatorname{Log}[a + a e^{2 i(c+dx)} + 2 b e^{2 i(c+dx)} + \sqrt{a} \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}] -$$

$$15 a^2 \sqrt{b} \operatorname{Log}\left[ \left( -4 \sqrt{b} d (-1 + e^{2 i(c+dx)}) + 4 i d \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2} \right) / \right.$$

$$\left. \left. \left( b (15 a^2 + 10 a b + 3 b^2) (1 + e^{2 i(c+dx)}) \right) \right] - 10 a b^{3/2} \operatorname{Log}\left[ \left( -4 \sqrt{b} d (-1 + e^{2 i(c+dx)}) + \right. \right.$$

$$\left. \left. 4 i d \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2} \right) / \left( b (15 a^2 + 10 a b + 3 b^2) (1 + e^{2 i(c+dx)}) \right) \right] -$$

$$3 b^{5/2} \operatorname{Log}\left[ \left( -4 \sqrt{b} d (-1 + e^{2 i(c+dx)}) + 4 i d \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2} \right) / \right.$$

$$\left. \left. \left( b (15 a^2 + 10 a b + 3 b^2) (1 + e^{2 i(c+dx)}) \right) \right] \right] (a + b \operatorname{Sec}[c + d x]^2)^{5/2}$$

**Problem 255: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (1 + \operatorname{Sec}[x]^2)^{3/2} dx$$

Optimal (type 3, 42 leaves, 6 steps):

$$2 \operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{2}}\right] + \operatorname{ArcTan}\left[\frac{\operatorname{Tan}[x]}{\sqrt{2 + \operatorname{Tan}[x]^2}}\right] + \frac{1}{2} \operatorname{Tan}[x] \sqrt{2 + \operatorname{Tan}[x]^2}$$

Result (type 3, 109 leaves):

$$\frac{1}{(3 + \operatorname{Cos}[2 x])^{3/2}} (1 + \operatorname{Cos}[x]^2) \operatorname{Sec}[x] \sqrt{1 + \operatorname{Sec}[x]^2} \left( 4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \operatorname{Sin}[x]}{\sqrt{3 + \operatorname{Cos}[2 x]}}\right] \operatorname{Cos}[x]^2 - \right.$$

$$\left. 2 i \sqrt{2} \operatorname{Cos}[x]^2 \operatorname{Log}\left[\sqrt{3 + \operatorname{Cos}[2 x]} + i \sqrt{2} \operatorname{Sin}[x]\right] + \sqrt{3 + \operatorname{Cos}[2 x]} \operatorname{Sin}[x] \right)$$

### Problem 256: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 + \sec[x]^2} \, dx$$

Optimal (type 3, 24 leaves, 5 steps):

$$\text{ArcSinh}\left[\frac{\text{Tan}[x]}{\sqrt{2}}\right] + \text{ArcTan}\left[\frac{\text{Tan}[x]}{\sqrt{2 + \text{Tan}[x]^2}}\right]$$

Result (type 3, 57 leaves):

$$\frac{\sqrt{2} \left( \text{ArcSin}\left[\frac{\text{Sin}[x]}{\sqrt{2}}\right] + \text{ArcTanh}\left[\frac{\sqrt{2} \text{Sin}[x]}{\sqrt{3 + \text{Cos}[2x]}}\right] \right) \text{Cos}[x] \sqrt{1 + \text{Sec}[x]^2}}{\sqrt{3 + \text{Cos}[2x]}}$$

### Problem 257: Unable to integrate problem.

$$\int \frac{\text{Sec}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} \, dx$$

Optimal (type 4, 380 leaves, 10 steps):

$$\begin{aligned} & \left( 2 (a - b) \sqrt{b + a \text{Cos}[e + f x]^2} \text{EllipticE}\left[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) / \\ & \left( 3 b^2 f \sqrt{\text{Cos}[e + f x]^2} \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a + b}} \right) - \\ & \left( (a - 2b) \sqrt{b + a \text{Cos}[e + f x]^2} \text{EllipticF}\left[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a + b}} \right) / \\ & \left( 3 b f \sqrt{\text{Cos}[e + f x]^2} \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \right) - \\ & \left( 2 (a - b) \sqrt{b + a \text{Cos}[e + f x]^2} \text{Sec}[e + f x] \sqrt{a + b - a \text{Sin}[e + f x]^2} \text{Tan}[e + f x] \right) / \\ & \left( 3 b^2 f \sqrt{a + b \text{Sec}[e + f x]^2} \right) + \\ & \left( \sqrt{b + a \text{Cos}[e + f x]^2} \text{Sec}[e + f x]^3 \sqrt{a + b - a \text{Sin}[e + f x]^2} \text{Tan}[e + f x] \right) / \\ & \left( 3 b f \sqrt{a + b \text{Sec}[e + f x]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Sec}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} \, dx$$

**Problem 258: Unable to integrate problem.**

$$\int \frac{\text{Sec}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 4, 202 leaves, 7 steps):

$$- \left( \left( \sqrt{a} \sqrt{a+b} \sqrt{b+a \text{Cos}[e+fx]^2} \right. \right. \\ \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a} \text{Sin}[e+fx]}{\sqrt{a+b}}\right], \frac{a+b}{a}\right] \sqrt{1 - \frac{a \text{Sin}[e+fx]^2}{a+b}} \right] \right. \\ \left. \left( b f \sqrt{\text{Cos}[e+fx]^2} \sqrt{a+b \text{Sec}[e+fx]^2} \sqrt{a+b-a \text{Sin}[e+fx]^2} \right) \right) + \\ \left( \sqrt{b+a \text{Cos}[e+fx]^2} \text{Sec}[e+fx] \sqrt{a+b-a \text{Sin}[e+fx]^2} \text{Tan}[e+fx] \right) / \\ \left( b f \sqrt{a+b \text{Sec}[e+fx]^2} \right)$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Sec}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

**Problem 260: Unable to integrate problem.**

$$\int \frac{\text{Cos}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$\left( \sqrt{a+b} \sqrt{b+a \text{Cos}[e+fx]^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a} \text{Sin}[e+fx]}{\sqrt{a+b}}\right], \frac{a+b}{a}\right] \sqrt{1 - \frac{a \text{Sin}[e+fx]^2}{a+b}} \right) / \\ \left( \sqrt{a} f \sqrt{\text{Cos}[e+fx]^2} \sqrt{a+b \text{Sec}[e+fx]^2} \sqrt{a+b-a \text{Sin}[e+fx]^2} \right)$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cos}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

### Problem 261: Unable to integrate problem.

$$\int \frac{\cos [e+f x]^3}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}} dx$$

Optimal (type 4, 296 leaves, 9 steps):

$$\frac{\sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2}}{3 a f \sqrt{a+b \operatorname{Sec}[e+f x]^2}} +$$

$$\left( 2 (a-b) \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) /$$

$$\left( 3 a^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \operatorname{Sec}[e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) -$$

$$\left( (a-2 b) b \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) /$$

$$\left( 3 a^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \operatorname{Sec}[e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [e+f x]^3}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}} dx$$

### Problem 262: Unable to integrate problem.

$$\int \frac{\cos [e+f x]^5}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}} dx$$

Optimal (type 4, 395 leaves, 10 steps):

$$\frac{4(a-b)\sqrt{b+a\cos[e+fx]^2}\sin[e+fx]\sqrt{a+b-a\sin[e+fx]^2}}{15a^2f\sqrt{a+b\sec[e+fx]^2}} +$$

$$\left(\cos[e+fx]^2\sqrt{b+a\cos[e+fx]^2}\sin[e+fx]\sqrt{a+b-a\sin[e+fx]^2}\right) /$$

$$\left(5af\sqrt{a+b\sec[e+fx]^2}\right) + \left((8a^2-7ab+8b^2)\sqrt{b+a\cos[e+fx]^2}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{a+b-a\sin[e+fx]^2}\right) /$$

$$\left(15a^3f\sqrt{\cos[e+fx]^2}\sqrt{a+b\sec[e+fx]^2}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}\right) -$$

$$\left(b(4a^2-3ab+8b^2)\sqrt{b+a\cos[e+fx]^2}\text{EllipticF}\left[\text{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\right.$$

$$\left.\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}\right) / \left(15a^3f\sqrt{\cos[e+fx]^2}\sqrt{a+b\sec[e+fx]^2}\sqrt{a+b-a\sin[e+fx]^2}\right)$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e+fx]^5}{\sqrt{a+b\sec[e+fx]^2}} dx$$

**Problem 263: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]^6}{\sqrt{a+b\sec[e+fx]^2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{(3a^2-2ab+3b^2)\text{ArcTanh}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{8b^{5/2}f} -$$

$$\frac{3(a-b)\tan[e+fx]\sqrt{a+b+b\tan[e+fx]^2}}{8b^2f} + \frac{\sec[e+fx]^2\tan[e+fx]\sqrt{a+b+b\tan[e+fx]^2}}{4bf}$$

Result (type 3, 326 leaves):



$$\left( e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \sqrt{a+2b+a \cos[2e+2fx]} \left( -\frac{1}{(1+e^{2i(e+fx)})^4} i \sqrt{b} (-1+e^{2i(e+fx)}) \right. \right. \\ \left. \left. (-3a(1+e^{2i(e+fx)})^2 + b(3+14e^{2i(e+fx)} + 3e^{4i(e+fx)})) - \left( (3a^2 - 2ab + 3b^2) \log \left[ \frac{1}{1+e^{2i(e+fx)}} \left( -4\sqrt{b}(-1+e^{2i(e+fx)})f + 4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f \right) \right] \right) \right) \right) / \\ \left( \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \sec[e+fx] \Big/ \left( 8\sqrt{2} b^{5/2} f \sqrt{a+b \sec[e+fx]^2} \right)$$

**Problem 264: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]^4}{\sqrt{a+b \sec[e+fx]^2}} dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$-\frac{(a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{2b^{3/2}f} + \frac{\tan[e+fx] \sqrt{a+b \tan[e+fx]^2}}{2bf}$$

Result (type 3, 266 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \sqrt{a+2b+a \cos[2e+2fx]} \right. \\ \left. \left( -\frac{i \sqrt{b} (-1+e^{2i(e+fx)})}{(1+e^{2i(e+fx)})^2} + \frac{(a-b) \log\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f + 4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \right) \\ \left. \sec[e+fx] \right) \Big/ \left( 2\sqrt{2} b^{3/2} f \sqrt{a+b \sec[e+fx]^2} \right)$$

### Problem 265: Unable to integrate problem.

$$\int \frac{\text{Sec}[e + f x]^2}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{\sqrt{b} f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Sec}[e + f x]^2}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

### Problem 266: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{\sqrt{a} f}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

### Problem 267: Unable to integrate problem.

$$\int \frac{\text{Cos}[e + f x]^2}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 87 leaves, 4 steps):

$$\frac{(a - b) \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{2 a^{3/2} f} + \frac{\text{Cos}[e + f x] \text{Sin}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{2 a f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cos}[e + f x]^2}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Problem 268: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos[e + f x]^4}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 143 leaves, 6 steps):

$$\frac{(3 a^2 - 2 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{8 a^{5/2} f} +$$

$$\frac{3 (a - b) \cos[e + f x] \sin[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{8 a^2 f} +$$

$$\frac{\cos[e + f x]^3 \sin[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{4 a f}$$

Result (type 6, 1840 leaves):

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x]^8 \sin[e + f x] \right) /$$

$$\left( f \sqrt{a + 2 b + a \cos[2 (e + f x)]} \sqrt{a + b \sec[e + f x]^2} \right.$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] + \right.$$

$$\left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] - \right.$$

$$\left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \right) \sin[e + f x]^2 \Bigg)$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x]^5 \right) /$$

$$\left( \sqrt{a + 2 b + a \cos[2 (e + f x)]} \left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \right. \right. \right.$$

$$\left. \left. \frac{a \sin[e + f x]^2}{a + b} \right] + \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] - \right. \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] \right) \sin[e + f x]^2 \right) \Bigg) -$$

$$\left( 12 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x]^3 \right)$$

$$\begin{aligned}
 & \left. \left( \frac{\sin[e+fx]^2}{\sqrt{a+2b+a\cos[2(e+fx)]}} \right) \right) + \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right) + \\
 & \left( 3(a+b) \cos[e+fx]^4 \sin[e+fx] \left( \frac{1}{3(a+b)} a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \left. \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \right) / \\
 & \left( f \sqrt{a+2b+a\cos[2(e+fx)]} \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \left. \left. \left. \frac{a\sin[e+fx]^2}{a+b}\right] + \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] - \right. \right. \right. \\
 & \left. \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right) - \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \right) \\
 & \sin[e+fx] \left( 2f \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \right) \right) \\
 & \cos[e+fx] \sin[e+fx] + 3(a+b) \left( \frac{1}{3(a+b)} a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \right. \right. \\
 & \left. \left. \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \\
 & \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \\
 & \sin[e+fx]^2 \left( a \left( \frac{1}{5(a+b)} 9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \cos [e+f x] \sin [e+f x]-\frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2},-1, \frac{3}{2}, \frac{7}{2}, \sin [e+f x]^2,\right. \\
 & \left.\frac{a \sin [e+f x]^2}{a+b}\right] \cos [e+f x] \sin [e+f x]-4(a+b)\left(\frac{1}{5(a+b)} 3 a f \operatorname{AppellF1}\left[\frac{5}{2},-1, \frac{3}{2}, \frac{7}{2}, \sin [e+f x]^2,\right.\right. \\
 & \left.\left.\frac{a \sin [e+f x]^2}{a+b}\right] \cos [e+f x] \sin [e+f x]-\frac{1}{8 a^3}\right. \\
 & \left.9(a+b)^3 f \cot [e+f x] \csc [e+f x]^4 \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}}\left(-\frac{2 a \sin [e+f x]^2}{a+b}\right.\right. \\
 & \left.\left.\frac{4 a^2 \sin [e+f x]^4}{3(a+b)^2}+\frac{2 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [e+f x]}{\sqrt{a+b}}\right] \sin [e+f x]}{\sqrt{a+b} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}}}\right)\right) \\
 & \left(f \sqrt{a+2 b+a \cos [2(e+f x)]}\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-2, \frac{1}{2}, \frac{3}{2},\right.\right.\right. \\
 & \left.\left.\sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right]+ \right. \\
 & \left.\left(a \operatorname{AppellF1}\left[\frac{3}{2},-2, \frac{3}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right]-4(a+b)\right.\right. \\
 & \left.\left.\operatorname{AppellF1}\left[\frac{3}{2},-1, \frac{1}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right]\right) \sin [e+f x]^2\right)^2\right) + \\
 & \left(3 a(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-2, \frac{1}{2}, \frac{3}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right] \cos [e+f x]^4\right. \\
 & \left.\sin [e+f x] \sin [2(e+f x)]\right) / \left((a+2 b+a \cos [2(e+f x)])^{3 / 2}\right. \\
 & \left.\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-2, \frac{1}{2}, \frac{3}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right]+ \right.\right. \\
 & \left.\left(a \operatorname{AppellF1}\left[\frac{3}{2},-2, \frac{3}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right]- \right.\right. \\
 & \left.\left.4(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-1, \frac{1}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b}\right]\right) \sin [e+f x]^2\right) \\
 & \left.\right)
 \end{aligned}$$

Problem 269: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{\cos [e+f x]^6}{\sqrt{a+b \sec [e+f x]^2}} d x$$

Optimal (type 3, 204 leaves, 7 steps):

$$\frac{(a-b)\left(5 a^2+2 a b+5 b^2\right) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [e+f x]}{\sqrt{a+b \tan [e+f x]^2}}\right]}{16 a^{7 / 2} f}+\frac{1}{48 a^3 f}$$

$$\frac{\left(15 a^2-14 a b+15 b^2\right) \cos [e+f x] \sin [e+f x] \sqrt{a+b \tan [e+f x]^2}+5(a-b) \cos [e+f x]^3 \sin [e+f x] \sqrt{a+b \tan [e+f x]^2}}{24 a^2 f}+$$

$$\frac{\cos [e+f x]^5 \sin [e+f x] \sqrt{a+b \tan [e+f x]^2}}{6 a f}$$

Result (type 6, 1739 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-3,\frac{1}{2},\frac{3}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right] \cos [e+f x]^{12} \sin [e+f x]\right) /$$

$$\left(f \sqrt{a+2 b+a \cos [2(e+f x)]} \sqrt{a+b \sec [e+f x]^2}\right.$$

$$\left.\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-3,\frac{1}{2},\frac{3}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]+(a \operatorname{AppellF1}\left[\frac{3}{2},-3,\frac{3}{2},\frac{5}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]-6(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-2,\frac{1}{2},\frac{5}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]) \sin [e+f x]^2\right)\right)$$

$$\left(\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-3,\frac{1}{2},\frac{3}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right] \cos [e+f x]^7\right) / \left(\sqrt{a+2 b+a \cos [2(e+f x)]}\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-3,\frac{1}{2},\frac{3}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]+(a \operatorname{AppellF1}\left[\frac{3}{2},-3,\frac{3}{2},\frac{5}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]-6(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-2,\frac{1}{2},\frac{5}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]) \sin [e+f x]^2\right)\right)\right)-$$

$$\left(18(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-3,\frac{1}{2},\frac{3}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right] \cos [e+f x]^5 \sin [e+f x]^2\right) / \left(\sqrt{a+2 b+a \cos [2(e+f x)]}\right.$$

$$\left.\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-3,\frac{1}{2},\frac{3}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]+(a \operatorname{AppellF1}\left[\frac{3}{2},-3,\frac{3}{2},\frac{5}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]-6(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-2,\frac{1}{2},\frac{5}{2}, \sin [e+f x]^2,\frac{a \sin [e+f x]^2}{a+b}\right]) \sin [e+f x]^2\right)\right)$$

$$\begin{aligned}
 & 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \Bigg) + \\
 & \left( 3 (a+b) \cos[e+fx]^6 \sin[e+fx] \left( \frac{1}{3(a+b)} a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \right. \right. \\
 & \quad \left. \left. 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]^2 \right) \right) / \\
 & \left( f \sqrt{a+2b+a \cos[2(e+fx)]} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] + \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) - \right. \\
 & \quad \left. \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^6 \right. \right. \\
 & \quad \left. \left. \sin[e+fx] \left( 2 f \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right) \right. \\
 & \quad \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - 2 f \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) + \\
 & \quad \sin[e+fx]^2 \left( a \left( \frac{1}{5(a+b)} 9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \right. \\
 & \quad \left. \left. \cos[e+fx] \sin[e+fx] - \frac{18}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) - 6 (a+b) \left( \frac{1}{5(a+b)} \right. \right. \\
 & \quad \left. \left. 3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \right. \right. \\
 & \quad \left. \left. \sin[e+fx] - \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{1}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \right. \\
 & \quad \left. \left. \cos[e+fx] \sin[e+fx] \right) \right) \Bigg) \Bigg) / \left( f \sqrt{a+2b+a \cos[2(e+fx)]} \right) \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 6 (a+b) \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \left. \left. \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2\right)^2\right) + \right. \right. \\ & \left. \left. \left( 3 a (a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^6 \right. \right. \right. \\ & \left. \left. \left. \sin[e+fx] \sin[2(e+fx)] \right) / \left( (a+2b+a \cos[2(e+fx)])^{3/2} \right) \right. \right. \\ & \left. \left. \left( 3 (a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \right. \\ & \left. \left. \left. \left( a \text{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right. \\ & \left. \left. \left. \left. 6 (a+b) \text{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) \right) \right) \right) \end{aligned}$$

**Problem 270: Unable to integrate problem.**

$$\int \frac{\text{Sec}[e+fx]^5}{(a+b \text{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 4, 367 leaves, 10 steps):

$$\begin{aligned} & \frac{a (2 a+b) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{b^2 (a+b) f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} - \\ & \left( (2 a+b) \sqrt{b+a \cos [e+f x]^2} \text{EllipticE}\left[\text{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\ & \left( b^2 (a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) + \\ & \left( \sqrt{b+a \cos [e+f x]^2} \text{EllipticF}\left[\text{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \\ & \left( b f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right) + \\ & \frac{\sqrt{b+a \cos [e+f x]^2} \sec [e+f x] \tan [e+f x]}{b f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Sec}[e+fx]^5}{(a+b \text{Sec}[e+fx]^2)^{3/2}} dx$$



### Problem 272: Unable to integrate problem.

$$\int \frac{\text{Sec}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 284 leaves, 9 steps):

$$\frac{\sqrt{b + a \text{Cos}[e + f x]^2} \text{Sin}[e + f x]}{(a + b) f \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2}} -$$

$$\left( \sqrt{b + a \text{Cos}[e + f x]^2} \text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]]], \frac{a}{a + b} \right] \sqrt{a + b - a \text{Sin}[e + f x]^2} \Big/$$

$$\left( a (a + b) f \sqrt{\text{Cos}[e + f x]^2} \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a + b}} \right) +$$

$$\left( \sqrt{b + a \text{Cos}[e + f x]^2} \text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]]], \frac{a}{a + b} \right] \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a + b}} \Big/$$

$$\left( a f \sqrt{\text{Cos}[e + f x]^2} \sqrt{a + b \text{Sec}[e + f x]^2} \sqrt{a + b - a \text{Sin}[e + f x]^2} \right)$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Sec}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

### Problem 273: Unable to integrate problem.

$$\int \frac{\text{Cos}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 295 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{b \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{a(a+b) f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} + \\
 & \left( (a+2 b) \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\
 & \left( a^2(a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) - \\
 & \left( 2 b \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \\
 & \left( a^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\cos [e+f x]}{(a+b \sec [e+f x]^2)^{3/2}} dx$$

Problem 274: Unable to integrate problem.

$$\int \frac{\cos [e+f x]^3}{(a+b \sec [e+f x]^2)^{3/2}} dx$$

Optimal (type 4, 399 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{b \cos [e+f x]^2 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{a(a+b) f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} + \\
 & \frac{(a+4 b) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2}}{3 a^2(a+b) f \sqrt{a+b \sec [e+f x]^2}} + \\
 & \left( (2 a^2-3 a b-8 b^2) \sqrt{b+a \cos [e+f x]^2} \right. \\
 & \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\
 & \left( 3 a^3(a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) - \\
 & \left( (a-8 b) b \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \\
 & \left( 3 a^3 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [e+f x]^3}{(a+b \sec [e+f x]^2)^{3/2}} dx$$

Problem 275: Unable to integrate problem.

$$\int \frac{\cos [e+f x]^5}{(a+b \sec [e+f x]^2)^{3/2}} dx$$

Optimal (type 4, 509 leaves, 11 steps):

$$\begin{aligned} & -\frac{b \cos [e+f x]^4 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{a(a+b) f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} + \\ & \left( (4 a^2-5 a b-24 b^2) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\ & \left( 15 a^3(a+b) f \sqrt{a+b \sec [e+f x]^2} \right) + \\ & \left( (a+6 b) \cos [e+f x]^2 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\ & \left( 5 a^2(a+b) f \sqrt{a+b \sec [e+f x]^2} \right) + \left( (8 a^3-9 a^2 b+16 a b^2+48 b^3) \sqrt{b+a \cos [e+f x]^2} \right. \\ & \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\ & \left( 15 a^4(a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) - \\ & \left( 4 b\left(a^2-2 a b+12 b^2\right) \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \right. \\ & \quad \left. \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \left( 15 a^4 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right) \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [e+f x]^5}{(a+b \sec [e+f x]^2)^{3/2}} dx$$

Problem 276: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sec [e+f x]^6}{(a+b \sec [e+f x]^2)^{3/2}} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$-\frac{(3a - b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2 b^{5/2} f} - \frac{a \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]}{b(a+b) f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} + \frac{(3a+b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2 b^2 (a+b) f}$$

Result (type 3, 375 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right. \\ \left. (a + 2b + a \operatorname{Cos}[2e + 2fx])^{3/2} \left( - \left( i \sqrt{b} (-1 + e^{2i(e+fx)}) \right. \right. \right. \\ \left. \left. \left( 4b^2 e^{2i(e+fx)} + 3a^2 (1 + e^{2i(e+fx)})^2 + ab(1 + 6e^{2i(e+fx)} + e^{4i(e+fx)}) \right) \right) \right) / \\ \left( (a+b) (1 + e^{2i(e+fx)})^2 (4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2) \right) + \left( (3a-b) \operatorname{Log}\left[ \right. \right. \\ \left. \left. \frac{1}{1 + e^{2i(e+fx)}} \left( -4\sqrt{b} (-1 + e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} f \right) \right] \right) / \\ \left( \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right) \operatorname{Sec}[e+fx]^3 \left. \right) / \\ (4\sqrt{2} b^{5/2} f (a+b \operatorname{Sec}[e+fx]^2)^{3/2})$$

**Problem 277: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^4}{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{b^{3/2} f} - \frac{a \operatorname{Tan}[e+fx]}{b(a+b) f \sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 3, 289 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right.$$

$$(a + 2b + a \cos[2e + 2fx])^{3/2} \left( \frac{i a \sqrt{b} (-1 + e^{2i(e+fx)})}{(a+b) (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)} - \right.$$

$$\left. \frac{\text{Log} \left[ \frac{-4\sqrt{b} (-1 + e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f}{1 + e^{2i(e+fx)}} \right]}{\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} \right)$$

$$\left. \text{Sec}[e + fx]^3 \right) / \left( 2\sqrt{2} b^{3/2} f (a + b \text{Sec}[e + fx]^2)^{3/2} \right)$$

**Problem 279: Unable to integrate problem.**

$$\int \frac{1}{(a + b \text{Sec}[e + fx]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\text{ArcTan} \left[ \frac{\sqrt{a} \text{Tan}[e+fx]}{\sqrt{a+b \text{Tan}[e+fx]^2}} \right]}{a^{3/2} f} - \frac{b \text{Tan}[e + fx]}{a (a+b) f \sqrt{a+b \text{Tan}[e + fx]^2}}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{(a + b \text{Sec}[e + fx]^2)^{3/2}} dx$$

**Problem 280: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[e + fx]^2}{(a + b \text{Sec}[e + fx]^2)^{3/2}} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$\frac{(a - 3b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2 a^{5/2} f} + \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{2 a f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} + \frac{b(a+3b) \operatorname{Tan}[e+fx]}{2 a^2 (a+b) f \sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 6, 2059 leaves):

$$\begin{aligned} & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^6 \operatorname{Sin}[e+fx] \right) / \\ & \left( 2 f \sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b \operatorname{Sec}[e+fx]^2)^{3/2} (a+b-a \operatorname{Sin}[e+fx]^2) \right. \\ & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \right. \\ & \left. \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right. \right. \\ & \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right) \\ & \left( \left( 3 a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^5 \right. \right. \\ & \left. \left. \operatorname{Sin}[e+fx]^2 \right) / \left( \sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b-a \operatorname{Sin}[e+fx]^2)^2 \right) \right. \\ & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \right. \\ & \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right. \\ & \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right) \right) + \\ & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^5 \right) / \\ & \left( 2 \sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b-a \operatorname{Sin}[e+fx]^2) \right. \\ & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \right. \\ & \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right. \\ & \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right) \right) - \\ & \left( 6(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^3 \right. \\ & \left. \operatorname{Sin}[e+fx]^2 \right) / \left( \sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b-a \operatorname{Sin}[e+fx]^2) \right) \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) + \\
 & \left( 3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left( \frac{1}{a+b} a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \right) / \\
 & \left( 2 f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \right. \\
 & \left. \sin[e+fx] \left( 2 f \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 4 (a+b) \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \cos[e+fx] \sin[e+fx] + \right. \right. \\
 & \left. \left. 3 (a+b) \left( \frac{1}{a+b} a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \right. \right. \\
 & \left. \left. \left. \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \right) + \\
 & \sin[e+fx]^2 \left( 3 a \left( \frac{1}{a+b} 3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \right. \\
 & \left. \left. \cos[e+fx] \sin[e+fx] - \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \left. \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) - 4 (a+b) \left( \frac{1}{5 (a+b)} 9 a f \operatorname{AppellF1}\left[ \right. \right. \right. \\
 & \left. \left. \left. \frac{5}{2}, -1, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{6}{5} f \right. \right. \\
 & \left. \left. \left. \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx] \right) \right) \right) \right) /
 \end{aligned}$$

$$\left( 2 f \sqrt{a + 2 b + a \cos [2 (e + f x)]} (a + b - a \sin [e + f x]^2) \right. \\ \left( 3 (a + b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] + \right. \\ \left( 3 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] - 4 (a + b) \right. \\ \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] \right) \sin [e + f x]^2 \right)^2 \Bigg) + \\ \left( 3 a (a + b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] \cos [e + f x]^4 \right. \\ \left. \sin [e + f x] \sin [2 (e + f x)] \right) \Bigg) / \\ \left( 2 (a + 2 b + a \cos [2 (e + f x)])^{3/2} (a + b - a \sin [e + f x]^2) \right. \\ \left( 3 (a + b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] + \right. \\ \left( 3 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] - \right. \\ \left. \left. 4 (a + b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] \right) \sin [e + f x]^2 \right) \Bigg) \Bigg) \Bigg)$$

**Problem 281: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [e + f x]^4}{(a + b \sec [e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$\frac{3 (a^2 - 2 a b + 5 b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + b \tan [e + f x]^2}} \right]}{8 a^{7/2} f} + \frac{(3 a - 5 b) \cos [e + f x] \sin [e + f x]}{8 a^2 f \sqrt{a + b \tan [e + f x]^2}} + \\ \frac{\cos [e + f x]^3 \sin [e + f x]}{4 a f \sqrt{a + b \tan [e + f x]^2}} + \frac{(a - 3 b) b (3 a + 5 b) \tan [e + f x]}{8 a^3 (a + b) f \sqrt{a + b \tan [e + f x]^2}}$$

Result (type 6, 2046 leaves):

$$\left( (a + b) \operatorname{AppellF1} \left[ \frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] \cos [e + f x]^{10} \sin [e + f x] \right) / \\ \left( 2 f \sqrt{a + 2 b + a \cos [2 (e + f x)]} (a + b \sec [e + f x]^2)^{3/2} (a + b - a \sin [e + f x]^2) \right. \\ \left. \left( (a + b) \operatorname{AppellF1} \left[ \frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin [e + f x]^2, \frac{a \sin [e + f x]^2}{a + b} \right] + \right. \right)$$



$$\begin{aligned}
 & \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \\
 & \left( \left( a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^7 \sin[e+fx]^2 \right) / \right. \\
 & \quad \left( \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2)^2 \right. \\
 & \quad \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \quad \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \Bigg) + \\
 & \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^7 \right) / \\
 & \left( 2 \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \quad \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \quad \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \Bigg) - \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^5 \right. \\
 & \quad \left. \sin[e+fx]^2 \right) / \left( \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \quad \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \quad \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \Bigg) + \\
 & \left( (a+b) \cos[e+fx]^6 \sin[e+fx] \left( \frac{1}{a+b} a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \right. \\
 & \quad \left. 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \Bigg) / \\
 & \left( 2 f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \left. \left. 2(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) - \\
 & \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^6 \right. \\
 & \sin[e+fx] \left( 2f \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 2(a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \cos[e+fx] \sin[e+fx] + \right. \\
 & (a+b) \left( \frac{1}{a+b} a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \\
 & \cos[e+fx] \sin[e+fx] - 2f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \right. \\
 & \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \sin[e+fx]^2 \\
 & \left( a \left( \frac{1}{a+b} 3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \right. \right. \\
 & \sin[e+fx] - \frac{18}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \\
 & \cos[e+fx] \sin[e+fx] \left. \right) - 2(a+b) \left( \frac{1}{5(a+b)} 9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \right. \right. \\
 & \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \left. \right] \cos[e+fx] \sin[e+fx] - \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, \right. \\
 & \left. \left. -1, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \left. \right) \left. \right) \left. \right) / \\
 & \left( 2f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 2(a+b) \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \left. \right) + \\
 & \left( a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^6 \right. \\
 & \left. \sin[e+fx] \sin[2(e+fx)] \right) / \\
 & \left( 2(a+2b+a \cos[2(e+fx)])^{3/2} (a+b-a \sin[e+fx]^2) \right)
 \end{aligned}$$

$$\left( (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\ \left. \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\ \left. \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right)$$

**Problem 282: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^6}{(a+b \sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 271 leaves, 8 steps):

$$\frac{(5a^3 - 9a^2b + 15ab^2 - 35b^3) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{16a^{9/2}f} + \\ \frac{(15a^2 - 22ab + 35b^2) \cos[e+fx] \sin[e+fx]}{48a^3f \sqrt{a+b \tan[e+fx]^2}} + \frac{(5a - 7b) \cos[e+fx]^3 \sin[e+fx]}{24a^2f \sqrt{a+b \tan[e+fx]^2}} + \\ \frac{\cos[e+fx]^5 \sin[e+fx]}{6af \sqrt{a+b \tan[e+fx]^2}} + \frac{b(15a^3 - 17a^2b + 25ab^2 + 105b^3) \tan[e+fx]}{48a^4(a+b)f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 6, 2068 leaves):

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^{14} \sin[e+fx] \right) / \\ \left( 2f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b \sec[e+fx]^2)^{3/2} (a+b - a \sin[e+fx]^2) \right. \\ \left. \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \\ \left. \left( 3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\ \left. \left. 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \\ \left( \left( 3a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^9 \right. \right. \\ \left. \left. \sin[e+fx]^2 \right) / \left( \sqrt{a+2b+a \cos[2(e+fx)]} (a+b - a \sin[e+fx]^2) \right)^2 \right. \\ \left. \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \\ \left. \left( 3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\ \left. \left. 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right)$$

$$\begin{aligned}
 & \left. \left( 8 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) + \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^9 \right) / \\
 & \left( 2 \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. 8 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) - \\
 & \left( 12 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^7 \right. \\
 & \left. \sin[e+fx]^2 \right) / \left( \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. 8 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) + \\
 & \left( 3 (a+b) \cos[e+fx]^8 \sin[e+fx] \left( \frac{1}{a+b} a f \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{8}{3} f \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
 & \left( 2 f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. 8 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^8 \right. \\
 & \left. \sin[e+fx] \left( 2 f \left( 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 8 (a+b) \right. \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \cos[e+fx] \sin[e+fx] + \right.
 \end{aligned}$$

$$\begin{aligned}
& 3 (a+b) \left( \frac{1}{a+b} a f \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \right. \\
& \quad \left. \cos [e+f x] \sin [e+f x] - \frac{8}{3} f \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \right. \\
& \quad \left. \frac{\cos [e+f x] \sin [e+f x]}{a+b} \right) + \sin [e+f x]^2 \\
& \left( 3 a \left( \frac{1}{a+b} 3 a f \operatorname{AppellF1} \left[ \frac{5}{2}, -4, \frac{7}{2}, \frac{7}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \cos [e+f x] \right. \right. \\
& \quad \left. \sin [e+f x] - \frac{24}{5} f \operatorname{AppellF1} \left[ \frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \right. \\
& \quad \left. \cos [e+f x] \sin [e+f x] \right) - 8 (a+b) \left( \frac{1}{5(a+b)} 9 a f \operatorname{AppellF1} \left[ \frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \cos [e+f x] \sin [e+f x] - \frac{18}{5} f \operatorname{AppellF1} \left[ \frac{5}{2}, \right. \right. \\
& \quad \left. \left. -2, \frac{3}{2}, \frac{7}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \cos [e+f x] \sin [e+f x] \right) \left. \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 2 f \sqrt{a+2 b+a \cos [2(e+f x)]} (a+b-a \sin [e+f x]^2) \right. \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] + \right. \\
& \left( 3 a \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] - 8 (a+b) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \right) \sin [e+f x]^2 \left. \right)^2 \Bigg) + \\
& \left( 3 a (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \cos [e+f x]^8 \right. \\
& \quad \left. \sin [e+f x] \sin [2(e+f x)] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 2 (a+2 b+a \cos [2(e+f x)])^{3/2} (a+b-a \sin [e+f x]^2) \right. \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] + \right. \\
& \left( 3 a \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] - \right. \\
& \quad \left. 8 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin [e+f x]^2, \frac{a \sin [e+f x]^2}{a+b} \right] \right) \sin [e+f x]^2 \left. \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

**Problem 284: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+f x]^3}{(a+b \operatorname{Sec}[e+f x]^2)^{5/2}} dx$$

Optimal (type 4, 381 leaves, 10 steps):

$$\frac{\sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3(a+b) f \sqrt{a+b \sec [e+f x]^2} (a+b-a \sin [e+f x]^2)^{3/2}} -$$

$$\frac{(a-b) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 b(a+b)^2 f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} +$$

$$\left( (a-b) \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) /$$

$$\left( 3 a b(a+b)^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) +$$

$$\left( \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) /$$

$$\left( 3 a(a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)$$

Result (type 8, 27 leaves):

$$\int \frac{\sec [e+f x]^3}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

Problem 285: Unable to integrate problem.

$$\int \frac{\sec [e+f x]}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

Optimal (type 4, 389 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{b \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a (a+b) f \sqrt{a+b \sec [e+f x]^2} (a+b-a \sin [e+f x]^2)^{3/2}} + \\
 & \frac{2 (2 a+b) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a (a+b)^2 f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} - \left( 2 (2 a+b) \right. \\
 & \quad \left. \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\
 & \left( 3 a^2 (a+b)^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) + \\
 & \left( (3 a+2 b) \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \\
 & \left( 3 a^2 (a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\sec [e+f x]}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

**Problem 286: Unable to integrate problem.**

$$\int \frac{\cos [e+f x]}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

Optimal (type 4, 411 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{b \cos [e+f x]^2 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a (a+b) f \sqrt{a+b \sec [e+f x]^2} (a+b-a \sin [e+f x]^2)^{3/2}} - \\
 & \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} + \\
 & \left( (3 a^2+13 a b+8 b^2) \sqrt{b+a \cos [e+f x]^2} \right. \\
 & \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\
 & \left( 3 a^3 (a+b)^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) - \left( b (9 a+8 b) \right. \\
 & \quad \left. \sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \\
 & \left( 3 a^3 (a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\cos [e+f x]}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

**Problem 287: Unable to integrate problem.**

$$\int \frac{\cos [e+f x]^3}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

Optimal (type 4, 512 leaves, 11 steps):



$$\begin{aligned}
 & - \frac{b \cos [e+f x]^4 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a (a+b) f \sqrt{a+b \sec [e+f x]^2} (a+b-a \sin [e+f x]^2)^{3/2}} - \\
 & \frac{2 b (4 a+3 b) \cos [e+f x]^2 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} + \\
 & \left( (a^2+11 a b+8 b^2) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\
 & \left( 3 a^3 (a+b)^2 f \sqrt{a+b \sec [e+f x]^2} \right) + \left( 2 (a+2 b) (a^2-4 a b-4 b^2) \sqrt{b+a \cos [e+f x]^2} \right. \\
 & \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2} \right) / \\
 & \left( 3 a^4 (a+b)^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) - \\
 & \left( b (a^2-16 a b-16 b^2) \sqrt{b+a \cos [e+f x]^2} \right. \\
 & \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}} \right) / \\
 & \left( 3 a^4 (a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2} \right)
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [e+f x]^3}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

**Problem 288: Unable to integrate problem.**

$$\int \frac{\cos [e+f x]^5}{(a+b \sec [e+f x]^2)^{5/2}} dx$$

Optimal (type 4, 639 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{b \cos [e+f x]^6 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a(a+b) f \sqrt{a+b \sec [e+f x]^2} (a+b-a \sin [e+f x]^2)^{3 / 2}} - \\
 & \frac{2 b(5 a+4 b) \cos [e+f x]^4 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x]}{3 a^2(a+b)^2 f \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}} + \\
 & \left(2\left(2 a^3-3 a^2 b-42 a b^2-32 b^3\right) \sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2}\right) / \\
 & \left(15 a^4(a+b)^2 f \sqrt{a+b \sec [e+f x]^2}\right) + \\
 & \left(\left(3 a^2+61 a b+48 b^2\right) \cos [e+f x]^2 \sqrt{b+a \cos [e+f x]^2} \sin [e+f x] \sqrt{a+b-a \sin [e+f x]^2}\right) / \\
 & \left(15 a^3(a+b)^2 f \sqrt{a+b \sec [e+f x]^2}\right) + \left(\left(8 a^4-11 a^3 b+27 a^2 b^2+184 a b^3+128 b^4\right)\right. \\
 & \left.\sqrt{b+a \cos [e+f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{a+b-a \sin [e+f x]^2}\right) / \\
 & \left(15 a^5(a+b)^2 f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}}\right) - \\
 & \left(b\left(4 a^3-9 a^2 b+120 a b^2+128 b^3\right) \sqrt{b+a \cos [e+f x]^2}\right. \\
 & \left.\operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin [e+f x]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \sin [e+f x]^2}{a+b}}\right) / \\
 & \left(15 a^5(a+b) f \sqrt{\cos [e+f x]^2} \sqrt{a+b \sec [e+f x]^2} \sqrt{a+b-a \sin [e+f x]^2}\right)
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [e+f x]^5}{(a+b \sec [e+f x]^2)^{5 / 2}} dx$$

**Problem 289: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [e+f x]^6}{(a+b \sec [e+f x]^2)^{5 / 2}} dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{b^{5/2} f} - \frac{a \sec[e+fx]^2 \tan[e+fx]}{3 b (a+b) f (a+b+b \tan[e+fx]^2)^{3/2}} - \frac{a (3a+5b) \tan[e+fx]}{3 b^2 (a+b)^2 f \sqrt{a+b+b \tan[e+fx]^2}}$$

Result (type 3, 357 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 (a+2b+a \cos[2e+2fx])^{5/2} \right. \\ \left. \left( i a \sqrt{b} (-1+e^{2i(e+fx)}) (24b^2 e^{2i(e+fx)} + 3a^2 (1+e^{2i(e+fx)})^2 + \right. \right. \\ \left. \left. a b (5+26 e^{2i(e+fx)} + 5 e^{4i(e+fx)})) \right) / \left( (a+b)^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^2 \right) - \right. \\ \left. \frac{3 \log\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f + 4i\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}} \right) \sec[e+fx]^5 / \\ (12\sqrt{2} b^{5/2} f (a+b \sec[e+fx]^2)^{5/2})$$

**Problem 292: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \tan[e+fx]}{3 a (a+b) f (a+b+b \tan[e+fx]^2)^{3/2}} - \frac{b (5a+3b) \tan[e+fx]}{3 a^2 (a+b)^2 f \sqrt{a+b+b \tan[e+fx]^2}}$$

Result (type 6, 1927 leaves):

$$\left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \sin[e+fx] \right) /$$

$$\left( 4 \sqrt{2} f (a + b \operatorname{Sec}[e + f x]^2)^{5/2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right.$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right.$$

$$\left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right]\right) \operatorname{Sin}[e + f x]^2 \right)$$

$$\left( \left( 15 a (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right. \right.$$

$$\left. \left. \operatorname{Cos}[e + f x]^5 \operatorname{Sin}[e + f x]^2 \right) \right) / \left( 4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{7/2} \right.$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right.$$

$$\left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right]\right) \operatorname{Sin}[e + f x]^2 \right) \right) +$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^5 \right) /$$

$$\left( 4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \right. \right. \right.$$

$$\left. \left. \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right]\right) \operatorname{Sin}[e + f x]^2 \right) \right) -$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^3 \right.$$

$$\left. \operatorname{Sin}[e + f x]^2 \right) / \left( \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right.$$

$$\left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right.$$

$$\left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right]\right) \operatorname{Sin}[e + f x]^2 \right) \right) +$$

$$\begin{aligned}
 & \left( 3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left( \frac{1}{3(a+b)} 5af \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
 & \left( 4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{a \sin[e+fx]^2}{a+b} \right] + \left( 5a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \right. \\
 & \quad \left. \sin[e+fx] \left( 2f \left( 5a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right) \right. \\
 & \quad \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} 5af \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) + \\
 & \quad \left( \sin[e+fx]^2 \left( 5a \left( \frac{1}{5(a+b)} 21af \operatorname{AppellF1} \left[ \frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[e+fx]^2, \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{12}{5} f \operatorname{AppellF1} \left[ \frac{5}{2}, -1, \frac{7}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) - 4 (a+b) \right) \right)
 \end{aligned}$$

$$\left( \frac{1}{a+b} 3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \right. \\ \left. \sin[e+fx] - \left( 6 (a+b)^3 f \cot[e+fx] \operatorname{Csc}[e+fx]^4 \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2 \right. \right. \\ \left. \left. \left( \frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right] \sin[e+fx]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} + \frac{a^2 \sin[e+fx]^4}{3 (a+b)^2 \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2} + \right. \right. \right. \\ \left. \left. \left. \frac{a \sin[e+fx]^2}{(a+b) \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)} \right) \right) \right) \left( a^3 \left( 1 - \frac{a \sin[e+fx]^2}{a+b} \right)^{3/2} \right) \right) \left. \right) \\ \left( 4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \right. \right. \right. \\ \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \right. \\ \left. \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \right. \right. \\ \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \sin[e+fx]^2 \right)^2 \right) \right) \right) \right)$$

**Problem 293: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\frac{(a-5b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2 a^{7/2} f} + \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{2 a f (a+b+b \operatorname{Tan}[e+fx]^2)^{3/2}} +$$

$$\frac{b(3a+5b) \operatorname{Tan}[e+fx]}{6 a^2 (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)^{3/2}} + \frac{b(3a^2+22ab+15b^2) \operatorname{Tan}[e+fx]}{6 a^3 (a+b)^2 f \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 6, 1775 leaves):

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^8 \operatorname{Sin}[e+fx]^2 \right) /$$

$$\left( 4 \sqrt{2} f (a+b \operatorname{Sec}[e+fx]^2)^{5/2} (a+b-a \operatorname{Sin}[e+fx]^2)^{5/2} \right.$$

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \right.$$

$$\left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right.$$

$$\left. \left. 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right)$$

$$\left( \left( 15 a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right. \right.$$

$$\left. \left. \operatorname{Cos}[e+fx]^7 \operatorname{Sin}[e+fx]^2 \right) \right) / \left( 4 \sqrt{2} (a+b-a \operatorname{Sin}[e+fx]^2)^{7/2} \right.$$

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \right.$$

$$\left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right.$$

$$\left. \left. 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right) \right) +$$

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^7 \right) /$$

$$\left( 4 \sqrt{2} (a+b-a \operatorname{Sin}[e+fx]^2)^{5/2} \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \right. \right. \right.$$

$$\left. \left. \frac{a \operatorname{Sin}[e+fx]^2}{a+b} \right] + \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right. \right.$$

$$\left. \left. 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right) \right) -$$

$$\left( 9(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^5 \right.$$

$$\left. \operatorname{Sin}[e+fx]^2 \right) / \left( 2 \sqrt{2} (a+b-a \operatorname{Sin}[e+fx]^2)^{5/2} \right.$$

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \right.$$

$$\begin{aligned}
 & \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] - \right. \\
 & \quad \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \sin[e+f x]^2 \right) + \\
 & \left( 3 (a+b) \cos[e+f x]^6 \sin[e+f x] \left( \frac{1}{3 (a+b)} 5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] - \right. \\
 & \quad \left. \left. 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) \right) / \\
 & \left( 4 \sqrt{2} f (a+b-a \sin[e+f x]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+f x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+f x]^2}{a+b}\right] + \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] - \right. \right. \\
 & \quad \left. \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \sin[e+f x]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x]^6 \right. \\
 & \quad \left. \sin[e+f x] \left( 2 f \left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] - \right. \right. \right. \\
 & \quad \left. \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \right) \right. \\
 & \quad \left. \cos[e+f x] \sin[e+f x] + 3 (a+b) \left( \frac{1}{3 (a+b)} 5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] - 2 f \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) \right) + \\
 & \sin[e+f x]^2 \left( 5 a \left( \frac{1}{5 (a+b)} 21 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{9}{2}, \frac{7}{2}, \sin[e+f x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] - \frac{18}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{7}{2}, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) - 6 (a+b) \\
 & \left( \frac{1}{a+b} 3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{7}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \right. \\
 & \quad \left. \sin[e+f x] - \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{5}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \right. \\
 & \quad \left. \left. \cos[e+f x] \sin[e+f x] \right) \right) \right) / \left( 4 \sqrt{2} f (a+b-a \sin[e+f x]^2)^{5/2} \right) \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] + \right.
 \end{aligned}$$



$$\left( 5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \right)^2 \right)$$

**Problem 294: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^4}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 261 leaves, 8 steps):

$$\frac{(3a^2 - 10ab + 35b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right] + 8a^{9/2} f}{8a^2 f (a+b \tan[e+fx]^2)^{3/2}} + \frac{\cos[e+fx]^3 \sin[e+fx]}{4af (a+b \tan[e+fx]^2)^{3/2}} + \frac{b(9a^2 - 18ab - 35b^2) \tan[e+fx]}{24a^3 (a+b) f (a+b \tan[e+fx]^2)^{3/2}} + \frac{b(9a^3 - 15a^2b - 145ab^2 - 105b^3) \tan[e+fx]}{24a^4 (a+b)^2 f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 6, 1777 leaves):

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^{12} \sin[e+fx] \right) / \left( 4\sqrt{2} f (a+b \sec[e+fx]^2)^{5/2} (a+b - a \sin[e+fx]^2)^{5/2} \right. \\ \left. \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left( 5a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right. \\ \left. \left( \left( 15a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^9 \sin[e+fx]^2 \right) / \left( 4\sqrt{2} (a+b - a \sin[e+fx]^2)^{7/2} \right) \right. \right. \\ \left. \left. \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left( 5a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right) \right)$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^9 \right) / \\
 & \left( 4 \sqrt{2} (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \right. \\
 & \quad \left. \left. \frac{5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right]}{a+b} \right] - \right. \\
 & \quad \left. 8 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 \sqrt{2} (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. \cos[e+fx]^7 \sin[e+fx]^2 \right) / \left( (a+b - a \sin[e+fx]^2)^{5/2} \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \\
 & \quad \left. 8 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \left. \right) \right) + \\
 & \left( 3 (a+b) \cos[e+fx]^8 \sin[e+fx] \left( \frac{1}{3 (a+b)} 5 a f \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{8}{3} f \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
 & \left( 4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \right. \\
 & \quad \left. \left. \frac{5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right]}{a+b} \right] - \right. \\
 & \quad \left. 8 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^8 \right. \\
 & \quad \left. \sin[e+fx] \left( 2 f \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
 & \quad \left. \left. 8 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right. \\
 & \quad \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left( \frac{1}{3 (a+b)} 5 a f \operatorname{AppellF1} \left[ \frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{8}{3} f \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) \right) +
 \end{aligned}$$



$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & 5 \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \\
 & \left( \left( 15 a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \right. \\
 & \quad \left. \left. \cos[e+fx]^{11} \sin[e+fx]^2 \right) / \left( 4 \sqrt{2} (a+b-a \sin[e+fx]^2)^{7/2} \right. \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad 5 \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \Bigg) + \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^{11} \right) / \\
 & \left( 4 \sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] + 5 \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \quad \left. \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \Bigg) - \\
 & \left( 15 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^9 \right. \\
 & \quad \left. \sin[e+fx]^2 \right) / \left( 2 \sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad 5 \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \Bigg) \Bigg) + \\
 & \left( 3 (a+b) \cos[e+fx]^{10} \sin[e+fx] \left( \frac{1}{3 (a+b)} 5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] - \frac{10}{3} f \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
 & \left( 4 \sqrt{2} f (a+b-a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{a \sin[e + f x]^2}{a + b} \Big] + 5 \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] - \right. \\
 & \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \right) \sin[e + f x]^2 \Big) - \\
 & \left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x]^{10} \right. \\
 & \sin[e + f x] \left( 10 f \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] - \right. \right. \\
 & \left. \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \right) \right. \\
 & \cos[e + f x] \sin[e + f x] + 3 (a + b) \left( \frac{1}{3 (a + b)} 5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \right. \right. \\
 & \left. \left. \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x] \sin[e + f x] - \frac{10}{3} f \right. \\
 & \left. \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x] \sin[e + f x] \right) + \\
 & 5 \sin[e + f x]^2 \left( a \left( \frac{1}{5 (a + b)} 21 a f \operatorname{AppellF1}\left[\frac{5}{2}, -5, \frac{9}{2}, \frac{7}{2}, \sin[e + f x]^2, \right. \right. \right. \\
 & \left. \left. \frac{a \sin[e + f x]^2}{a + b} \right] \cos[e + f x] \sin[e + f x] - 6 f \operatorname{AppellF1}\left[\frac{5}{2}, -4, \frac{7}{2}, \frac{7}{2}, \right. \right. \\
 & \left. \left. \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] \cos[e + f x] \sin[e + f x] \right) - 2 (a + b) \\
 & \left( \frac{1}{a + b} 3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -4, \frac{7}{2}, \frac{7}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x] \right. \\
 & \left. \sin[e + f x] - \frac{24}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \right. \\
 & \left. \left. \cos[e + f x] \sin[e + f x] \right) \right) \Big) \Big) \Big) \Big) \Big) / \left( 4 \sqrt{2} f (a + b - a \sin[e + f x]^2)^{5/2} \right) \\
 & \left( 3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] + \right. \\
 & \left. 5 \left( a \operatorname{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] - 2 (a + b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \right) \sin[e + f x]^2 \right) \Big) \Big) \Big) \Big) \Big)
 \end{aligned}$$

**Problem 296:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \sec[c + d x]^2)^{7/2}} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+b \tan[c+dx]^2}}\right]}{a^{7/2} d} - \frac{b \tan[c+dx]}{5 a (a+b) d (a+b+b \tan[c+dx]^2)^{5/2}} - \frac{b (9 a+5 b) \tan[c+dx]}{15 a^2 (a+b)^2 d (a+b+b \tan[c+dx]^2)^{3/2}} - \frac{b (33 a^2+40 a b+15 b^2) \tan[c+dx]}{15 a^3 (a+b)^3 d \sqrt{a+b+b \tan[c+dx]^2}}$$

Result (type 6, 1777 leaves):

$$\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx]^6 \sin[c+dx]\right) / \left(8 \sqrt{2} d (a+b \sec[c+dx]^2)^{7/2} (a+b-a \sin[c+dx]^2)^{7/2}\right. \\ \left. \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] + \left(7 a \text{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - 6(a+b) \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right]\right) \sin[c+dx]^2\right) \right. \\ \left. \left(\left(21 a(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx]^7 \sin[c+dx]^2\right) / \left(8 \sqrt{2} (a+b-a \sin[c+dx]^2)^{9/2}\right) \right. \right. \\ \left. \left. \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] + \left(7 a \text{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - 6(a+b) \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right]\right) \sin[c+dx]^2\right) \right) \right) + \\ \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx]^7\right) / \left(8 \sqrt{2} (a+b-a \sin[c+dx]^2)^{7/2} \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] + \left(7 a \text{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - 6(a+b) \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right]\right) \sin[c+dx]^2\right) \right) - \\ \left(9(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx]^5 \sin[c+dx]^2\right) / \left(4 \sqrt{2} (a+b-a \sin[c+dx]^2)^{7/2} \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] + \left(7 a \text{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - 6(a+b) \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right]\right) \sin[c+dx]^2\right) \right)$$

$$\begin{aligned}
 & \left( 7 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - \right. \\
 & \quad \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \sin[c+dx]^2 \right) + \\
 & \left( 3 (a+b) \cos[c+dx]^6 \sin[c+dx] \left( \frac{1}{3(a+b)} 7 a d \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \sin[c+dx] - \right. \\
 & \quad \left. \left. 2 d \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \sin[c+dx] \right) \right) / \\
 & \left( 8 \sqrt{2} d (a+b - a \sin[c+dx]^2)^{7/2} \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[c+dx]^2}{a+b}\right] + \left( 7 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - \right. \right. \\
 & \quad \left. \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \sin[c+dx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx]^6 \right. \\
 & \quad \left. \sin[c+dx] \left( 2 d \left( 7 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - \right. \right. \right. \\
 & \quad \left. \left. 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \right) \right) \\
 & \quad \cos[c+dx] \sin[c+dx] + 3 (a+b) \left( \frac{1}{3(a+b)} 7 a d \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \sin[c+dx] - 2 d \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \sin[c+dx] \right) \right) + \\
 & \sin[c+dx]^2 \left( 7 a \left( \frac{1}{5(a+b)} 27 a d \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{11}{2}, \frac{7}{2}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \sin[c+dx] - \frac{18}{5} d \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \right. \right. \\
 & \quad \left. \left. \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \sin[c+dx] \right) - 6 (a+b) \left( \frac{1}{5(a+b)} \right. \\
 & \quad \left. 21 a d \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \cos[c+dx] \right. \\
 & \quad \left. \sin[c+dx] - \frac{12}{5} d \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \right. \\
 & \quad \left. \left. \cos[c+dx] \sin[c+dx] \right) \right) \right) / \left( 8 \sqrt{2} d (a+b - a \sin[c+dx]^2)^{7/2} \right) \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] + \right.
 \end{aligned}$$

$$\left( 7 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] - 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b}\right] \sin[c+dx]^2 \right)^2 \right)$$

**Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1 + \sec[x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\operatorname{ArcTan}\left[\frac{\tan[x]}{\sqrt{2 + \tan[x]^2}}\right]$$

Result (type 3, 47 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} \sin[x]}{\sqrt{3 + \cos[2x]}}\right] \sqrt{3 + \cos[2x]} \sec[x]}{\sqrt{2} \sqrt{1 + \sec[x]^2}}$$

**Problem 298: Result more than twice size of optimal antiderivative.**

$$\int (d \sec[e+fx])^m (a+b \sec[e+fx]^2)^p dx$$

Optimal (type 6, 111 leaves, ? steps):

$$\frac{1}{f^m} \operatorname{AppellF1}\left[\frac{m}{2}, \frac{1}{2}, -p, \frac{2+m}{2}, \sec[e+fx]^2, -\frac{b \sec[e+fx]^2}{a}\right] \cot[e+fx] (d \sec[e+fx])^m (a+b \sec[e+fx]^2)^p \left(1 + \frac{b \sec[e+fx]^2}{a}\right)^{-p} \sqrt{-\tan[e+fx]^2}$$

Result (type 6, 2195 leaves):

$$\left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p (d \sec[e+fx])^m (\sec[e+fx]^2)^{-1+\frac{m}{2}+p} (a+b \sec[e+fx]^2)^p \tan[e+fx] \right) / \left( f \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + (a+b) (-2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right)$$



$$\begin{aligned}
 & \left( (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{\frac{m}{2}+p} \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + \right. \\
 & \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + (a + b)(-2 + m) \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right) \tan[e + fx]^2 \right) - \\
 & \left( 6a(a + b)^p \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right. \\
 & \left. (a + 2b + a \cos[2(e + fx)])^{-1+p} (\sec[e + fx]^2)^{-1+\frac{m}{2}+p} \sin[2(e + fx)] \tan[e + fx] \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + \right. \\
 & \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + (a + b)(-2 + m) \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right) \tan[e + fx]^2 \right) + \\
 & \left( 6(a + b) \left(-1 + \frac{m}{2} + p\right) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right. \\
 & \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{-1+\frac{m}{2}+p} \tan[e + fx] \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + \right. \\
 & \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + (a + b)(-2 + m) \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right) \tan[e + fx]^2 \right) + \\
 & \left( 3(a + b) (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{-1+\frac{m}{2}+p} \tan[e + fx] \right. \\
 & \left. \left( \frac{1}{3(a + b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right. \right. \\
 & \left. \left. \sec[e + fx]^2 \tan[e + fx] - \frac{2}{3} \left(1 - \frac{m}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \left. -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \sec[e + fx]^2 \tan[e + fx] \right) \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + \right. \\
 & \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] + (a + b)(-2 + m) \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a + b}\right] \right) \tan[e + fx]^2 \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right. \\
 & \quad (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-1+\frac{m}{2}+p} \operatorname{Tan}[e+fx] \\
 & \quad \left( 2 \left( 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \right. \\
 & \quad \quad (a+b) (-2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \Big) \\
 & \quad \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, \right. \right. \\
 & \quad \quad \left. \left. -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{2}{3} \left( 1 - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \right. \right. \\
 & \quad \quad \left. \left. \frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) + \\
 & \quad \operatorname{Tan}[e+fx]^2 \left( 2 b p \left( -\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 - \frac{m}{2}, 2-p, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \\
 & \quad \quad \left. \left. -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{6}{5} \left( 1 - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2}, 2 - \frac{m}{2}, \right. \right. \\
 & \quad \quad \left. \left. 1-p, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) + \\
 & \quad (a+b) (-2+m) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 2 - \frac{m}{2}, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, \right. \right. \\
 & \quad \quad \left. \left. -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{6}{5} \left( 2 - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2}, 3 - \frac{m}{2}, \right. \right. \\
 & \quad \quad \left. \left. -p, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \Big) \Big) \Big) \Big) \Big) / \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] + (a+b) (-2+m) \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \right) \operatorname{Tan}[e+fx]^2 \right) \Big) \Big) \Big) \Big) \Big)
 \end{aligned}$$

**Problem 299: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 124 leaves, 5 steps):

$$\begin{aligned}
 & \frac{1}{f} \operatorname{AppellF1} \left[ \frac{1}{2}, 3+p, -p, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b} \right] (\operatorname{Cos}[e+fx]^2)^p (b+a \operatorname{Cos}[e+fx]^2)^{-p} \\
 & (a+b \operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[e+fx] (a+b-a \operatorname{Sin}[e+fx]^2)^p \left( 1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b} \right)^{-p}
 \end{aligned}$$

Result (type 6, 3930 leaves):

$$\begin{aligned}
 & \left( (a+b) (a+2b+a \cos[2(e+fx)])^p \sec[e+fx]^5 (\sec[e+fx]^2)^{\frac{1}{2}+p} (a+b \sec[e+fx]^2)^p \right. \\
 & \quad \tan[e+fx] \left( \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right. \\
 & \quad \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Big) + \\
 & \quad \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 \right) / \\
 & \quad \left( 5(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2bp \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Big) \Big) / \\
 & \left. \left( 3f \left( \frac{1}{3} (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{3}{2}+p} \right. \right. \right. \\
 & \quad \left( \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right. \\
 & \quad \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Big) + \\
 & \quad \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 \right) / \\
 & \quad \left( 5(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2bp \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \Big) - \\
 & \quad \frac{2}{3} a (a+b)^p (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{\frac{1}{2}+p} \sin[2(e+fx)] \\
 & \quad \tan[e+fx] \left( \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Bigg) + \\
 & \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 \right) / \\
 & \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + (a+b) \right. \\
 & \quad \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Bigg) + \\
 & \frac{2}{3} (a+b) \left(\frac{1}{2}+p\right) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{1}{2}+p} \tan[e+fx]^2 \\
 & \left( \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Bigg) + \\
 & \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 \right) / \\
 & \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + (a+b) \right. \\
 & \quad \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \Bigg) + \\
 & \frac{1}{3} (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{1}{2}+p} \tan[e+fx] \\
 & \left( \left( 9 \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \right. \right. \\
 & \quad \left. \left. \left. \sec[e+fx]^2 \tan[e+fx] + \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2 + \\
 & \left( 10 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) / \\
 & \quad \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
 & \quad \left. \left( 2 b p \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
 & \left( 5 \operatorname{Tan}[e+f x]^2 \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \\
 & \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
 & \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \\
 & \left( 2 \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Sec}[e+f x]^2 \right. \\
 & \quad \operatorname{Tan}[e+f x] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \operatorname{Tan}[e+f x]^2 \right) \\
 & \left( 2 b p \left( -\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 2-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \right.
 \end{aligned}$$

$$\begin{aligned}
& (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{3}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}, -p, \right. \right. \\
& \quad \left. \left. \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left( 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right)^2 - \\
& \left( 5 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \tan[e+fx]^2 \right. \\
& \quad \left( 2 \left( 2 b p \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \operatorname{Sec}[e+fx]^2 \right. \\
& \quad \left. \tan[e+fx] + 5(a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{3}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \tan[e+fx]^2 \right. \\
& \quad \left. \left( 2 b p \left( -\frac{1}{7(a+b)} 10 b (1-p) \operatorname{AppellF1} \left[ \frac{7}{2}, -\frac{1}{2}, 2-p, \frac{9}{2}, -\tan[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{5}{7} \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{1}{2}, 1-p, \right. \right. \\
& \quad \left. \left. \frac{9}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \right. \\
& \quad \left. (a+b) \left( \frac{1}{7(a+b)} 10 b p \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{1}{2}, 1-p, \frac{9}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{5}{7} \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{3}{2}, -p, \right. \right. \\
& \quad \left. \left. \frac{9}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 5(a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left( 2 b p \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) \right.
\end{aligned}$$

$$\text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Tan}[e+fx]^2 \Bigg)^2 \Bigg) \Bigg) \Bigg) \Bigg)$$

**Problem 300: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e+fx]^3 (a+b \text{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 124 leaves, 5 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2+p, -p, \frac{3}{2}, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] (\text{Cos}[e+fx]^2)^p (b+a \text{Cos}[e+fx]^2)^{-p}$$

$$(a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx] (a+b-a \text{Sin}[e+fx]^2)^p \left(1 - \frac{a \text{Sin}[e+fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 1989 leaves):

$$\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] (a+2b+a \text{Cos}[2(e+fx)])^p\right.$$

$$\left. \text{Sec}[e+fx]^3 (\text{Sec}[e+fx]^2)^{\frac{1}{2}+p} (a+b \text{Sec}[e+fx]^2)^p \text{Tan}[e+fx]\right) /$$

$$\left(f \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] +\right.\right.$$

$$\left.\left(2bp \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] +\right.\right.$$

$$\left.\left.(a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right]\right) \text{Tan}[e+fx]^2\right)$$

$$\left(\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right]\right.\right.$$

$$\left.\left.(a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{\frac{3}{2}+p}\right) /$$

$$\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] +\right.$$

$$\left.\left(2bp \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] +\right.\right.$$

$$\left.\left.(a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right]\right) \text{Tan}[e+fx]^2 -$$

$$\left(6a(a+b)p \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right]\right.$$

$$\left.\left.(a+2b+a \text{Cos}[2(e+fx)])^{-1+p} (\text{Sec}[e+fx]^2)^{\frac{1}{2}+p} \text{Sin}[2(e+fx)] \text{Tan}[e+fx]\right) /$$

$$\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] +\right.$$

$$\left.\left(2bp \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] +\right.\right.$$

$$\begin{aligned}
 & (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 + \\
 & \left(6(a+b) \left(\frac{1}{2}+p\right) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{1}{2}+p} \tan[e+fx]^2\right) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left(2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \right. \\
 & \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right]\right) \tan[e+fx]^2\right) + \\
 & \left(3(a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{1}{2}+p} \tan[e+fx] \right. \\
 & \left. \left(\frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \right. \\
 & \left. \left. \sec[e+fx]^2 \tan[e+fx] + \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx]\right)\right) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left(2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \right. \\
 & \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right]\right) \tan[e+fx]^2\right) - \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{1}{2}+p} \tan[e+fx] \right. \\
 & \left. \left(2 \left(2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \right. \right. \\
 & \left. \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right]\right)\right) \right. \\
 & \left. \sec[e+fx]^2 \tan[e+fx] + 3(a+b) \left(\frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \right. \right. \right. \\
 & \left. \left. \left. \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] + \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
 & \left. \left. \left. \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx]\right)\right) + \\
 & \tan[e+fx]^2 \left(2bp \left(-\frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 2-p, \frac{7}{2}, \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \right. \\
 & \left. \frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \Bigg) + \\
 & (a+b) \left( \frac{1}{5(a+b)} 6bp \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \\
 & \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, -p, \right. \right. \\
 & \left. \left. \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Bigg) \Bigg) \Bigg) / \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + (a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right)^2 \Bigg) \Bigg)
 \end{aligned}$$

### Problem 301: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 124 leaves, 5 steps):

$$\begin{aligned}
 & \frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, 1+p, -p, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] (\cos[e+fx]^2)^p (b+a \cos[e+fx]^2)^{-p} \\
 & (a+b \operatorname{Sec}[e+fx]^2)^p \sin[e+fx] (a+b-a \sin[e+fx]^2)^p \left(1 - \frac{a \sin[e+fx]^2}{a+b}\right)^{-p}
 \end{aligned}$$

Result (type 6, 1995 leaves):

$$\begin{aligned}
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p \right. \\
 & \left. \operatorname{Sec}[e+fx] (\operatorname{Sec}[e+fx]^2)^{-\frac{1}{2}+p} (a+b \operatorname{Sec}[e+fx]^2)^p \tan[e+fx] \right) / \\
 & \left( f \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \right. \\
 & \left. \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \\
 & \left( \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \right. \\
 & \left. \left. (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{\frac{1}{2}+p} \right) \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 6 a (a+b) p \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \quad \left. (a+2 b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-\frac{1}{2}+p} \sin[2(e+fx)] \tan[e+fx] \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 6 (a+b) \left(-\frac{1}{2}+p\right) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \quad \left. (a+2 b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{1}{2}+p} \tan[e+fx]^2 \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 3 (a+b) (a+2 b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{1}{2}+p} \tan[e+fx] \right. \\
 & \quad \left. \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \right. \\
 & \quad \left. \left. \sec[e+fx]^2 \tan[e+fx] - \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right.
 \end{aligned}$$

$$\begin{aligned}
 & (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{-\frac{1}{2}+p} \tan[e + fx] \\
 & \left( 2 \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \right) \right. \\
 & \quad \left. \sec[e + fx]^2 \tan[e + fx] + 3(a+b) \left( \frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] - \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] \right) \right) + \\
 & \tan[e + fx]^2 \left( 2bp \left( -\frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2-p, \frac{7}{2}, -\tan[e + fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] - \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] \right) - \\
 & \quad \left. (a+b) \left( \frac{1}{5(a+b)} 6bp \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1-p, \frac{7}{2}, -\tan[e + fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] - \frac{9}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, -p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] \right) \right) \right) \Bigg) / \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] + \right. \\
 & \quad \left. \left( 2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] - (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \tan[e + fx]^2 \right)^2 \right) \right)
 \end{aligned}$$

**Problem 302: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + fx] (a + b \sec[e + fx]^2)^p dx$$

Optimal (type 6, 122 leaves, 5 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, p, -p, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] (\cos[e + fx]^2)^p (b + a \cos[e + fx]^2)^{-p} \\
 (a + b \sec[e + fx]^2)^p \sin[e + fx] (a + b - a \sin[e + fx]^2)^p \left(1 - \frac{a \sin[e + fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 1983 leaves):

$$\begin{aligned}
 & - \left( \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \\
 & \quad \left. \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{3}{2}+p} (a+b \sec[e+fx]^2)^p \sin[e+fx] \right) \right) / \\
 & \left( f \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
 & \quad \left( -2bp \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. \left. 3 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \\
 & \left( - \left( \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \right. \\
 & \quad \left. \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{1}{2}+p} \right) \right) / \\
 & \quad \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( -2bp \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3 (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \right) + \\
 & \left( 6a (a+b) p \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-\frac{3}{2}+p} \sin[2(e+fx)] \tan[e+fx] \right) / \\
 & \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( -2bp \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3 (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 6 (a+b) \left( -\frac{3}{2} + p \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{3}{2}+p} \tan[e+fx]^2 \right) / \\
 & \left( -3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( -2bp \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3 (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 3 (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{3}{2}+p} \tan[e+fx] \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, \right. \right. \\
 & \quad \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Bigg/ \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( -2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3(a+b) \right. \\
 & \quad \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \Bigg) + \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-\frac{3}{2}+p} \tan[e+fx] \right. \\
 & \quad \left. \left( 2 \left( -2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3 \right. \right. \right. \\
 & \quad \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \right. \\
 & \quad \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - 3(a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \right. \right. \right. \\
 & \quad \quad \left. \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \\
 & \quad \quad \left. \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Bigg) + \\
 & \tan[e+fx]^2 \left( -2 b p \left( -\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}, 2-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
 & \quad \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{9}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{2}, 1-p, \right. \right. \\
 & \quad \quad \left. \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + 3 \\
 & \quad \left( a+b \right) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \\
 & \quad \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - 3 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{2}, -p, \right. \right. \\
 & \quad \quad \left. \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Bigg) \Bigg) \Bigg) \Bigg/ \\
 & \left( -3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left( -2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3(a+b) \right.
 \end{aligned}$$

$$\text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Tan}[e+fx]^2\right)^2\right)$$

**Problem 303: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[e+fx]^3 (a+b \text{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 124 leaves, 5 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, -1+p, -p, \frac{3}{2}, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] (\text{Cos}[e+fx]^2)^p (b+a \text{Cos}[e+fx]^2)^{-p} (a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx] (a+b-a \text{Sin}[e+fx]^2)^p \left(1-\frac{a \text{Sin}[e+fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 1987 leaves):

$$\begin{aligned} & -\left(\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-\frac{7}{2}+p} (a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx]\right)\right) / \\ & \left(f \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left. 5(a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right]\right) \text{Tan}[e+fx]^2\right) \\ & \left(-\left(\left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-\frac{3}{2}+p}\right)\right) / \right. \\ & \quad \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \\ & \quad \left. \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + 5(a+b) \right. \right. \\ & \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right]\right) \text{Tan}[e+fx]^2\right) \right) + \\ & \left(6a(a+b)p \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] (a+2b+a \text{Cos}[2(e+fx)])^{-1+p} (\text{Sec}[e+fx]^2)^{-\frac{5}{2}+p} \text{Sin}[2(e+fx)] \text{Tan}[e+fx]\right) / \\ & \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \\ & \quad \left. \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + 5(a+b) \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Tan}[e+fx]^2 - \\
 & \left(6(a+b) \left(-\frac{5}{2}+p\right) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right. \\
 & \quad \left. (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-\frac{5}{2}+p} \text{Tan}[e+fx]^2\right) / \\
 & \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + 5(a+b) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right) \text{Tan}[e+fx]^2 - \\
 & \left(3(a+b) (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-\frac{5}{2}+p} \text{Tan}[e+fx] \right. \\
 & \quad \left. \left(\frac{1}{3(a+b)} 2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right. \right. \\
 & \quad \left. \left. \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - \frac{5}{3} \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx]\right)\right) / \\
 & \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. (-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + 5(a+b) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right) \text{Tan}[e+fx]^2 + \\
 & \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right. \\
 & \quad \left. (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-\frac{5}{2}+p} \text{Tan}[e+fx] \right. \\
 & \quad \left. \left(2 \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + 5 \right. \right. \right. \\
 & \quad \left. \left. (a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right) \right. \\
 & \quad \left. \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - 3(a+b) \left(\frac{1}{3(a+b)} 2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] - \frac{5}{3} \text{AppellF1}\left[\right. \right. \\
 & \quad \left. \left. \frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx]\right) + \\
 & \quad \left. \text{Tan}[e+fx]^2 \left(-2bp \left(-\frac{1}{5(a+b)} 6b(1-p) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, 2-p, \frac{7}{2}, -\text{Tan}[e+fx]^2, \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned} & -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - 3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}, 1-p, \right. \\ & \left. \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + 5 \\ & (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \\ & \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{21}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}, -p, \right. \right. \\ & \left. \left. \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) \Big/ \\ & \left( -3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\ & \left( -2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + 5(a+b) \right. \\ & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Tan}[e+f x]^2 \right)^2 \right) \right) \Big/ \end{aligned}$$

**Problem 304: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+f x]^5 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 124 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, -2+p, -p, \frac{3}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right] (\operatorname{Cos}[e+f x]^2)^p (b+a \operatorname{Cos}[e+f x]^2)^{-p} \\ & (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] (a+b-a \operatorname{Sin}[e+f x]^2)^p \left(1-\frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right)^{-p} \end{aligned}$$

Result (type 6, 1997 leaves):

$$\begin{aligned} & -\left( \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Cos}[e+f x]^4 \right. \right. \\ & \left. \left. (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-\frac{7}{2}+p} (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \right) \right) \Big/ \\ & \left( f \left( -3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \right. \\ & \left( -2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\ & \left. \left. 7(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2 \right) \\ & \left( -\left( \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right. \right. \right. \\ & \left. \left. (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-\frac{5}{2}+p} \right) \right) \Big/ \end{aligned}$$



$$\begin{aligned}
 & \left( -3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( -2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 7 (a+b) \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \Bigg) + \\
 & \left( 6 a (a+b) p \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \quad \left. (a+2 b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-\frac{7}{2}+p} \sin[2(e+fx)] \tan[e+fx] \right) \Bigg) / \\
 & \left( -3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( -2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 7 (a+b) \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 6 (a+b) \left(-\frac{7}{2}+p\right) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \quad \left. (a+2 b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{7}{2}+p} \tan[e+fx]^2 \right) \Bigg) / \\
 & \left( -3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( -2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 7 (a+b) \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 3 (a+b) (a+2 b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{7}{2}+p} \tan[e+fx] \right. \\
 & \quad \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \quad \quad \left. \sec[e+fx]^2 \tan[e+fx] - \frac{7}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, \right. \right. \\
 & \quad \quad \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] \right) \Bigg) / \\
 & \left( -3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( -2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 7 (a+b) \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right.
 \end{aligned}$$

$$\begin{aligned}
 & (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{-\frac{7}{2}+p} \tan[e + fx] \\
 & \left( 2 \left( -2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] + 7 \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \right) \right. \\
 & \quad \left. \sec[e + fx]^2 \tan[e + fx] - 3(a+b) \left( \frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] - \frac{7}{3} \operatorname{AppellF1}\left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] \right) \right) + \\
 & \tan[e + fx]^2 \left( -2bp \left( -\frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}, 2-p, \frac{7}{2}, -\tan[e + fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] - \frac{21}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] \right) + 7 \right. \\
 & \quad \left. (a+b) \left( \frac{1}{5(a+b)} 6bp \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}, 1-p, \frac{7}{2}, -\tan[e + fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] - \frac{27}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2}, -p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \sec[e + fx]^2 \tan[e + fx] \right) \right) \right) \Big/ \\
 & \left( -3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] + \right. \\
 & \quad \left( -2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] + 7(a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \tan[e + fx]^2 \right) \right) \right) \Big)
 \end{aligned}$$

**Problem 308: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, -\tan[e + fx]^2, -\frac{b \tan[e + fx]^2}{a+b}\right] \tan[e + fx] (a + b + b \tan[e + fx]^2)^p \left(1 + \frac{b \tan[e + fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 2137 leaves):

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p (a+b \sec[e+fx]^2)^p \sin[e+fx] \right) / \\
 & \left( f \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) \\
 & \left( \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \quad \left. \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-1+p} \right) / \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx]^2 \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 6 (a+b) p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx]^2 \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 6 a (a+b) p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx] \right.
 \end{aligned}$$

$$\begin{aligned}
 & (a + 2b + a \cos[2(e + fx)])^{-1+p} (\sec[e + fx]^2)^p \sin[e + fx] \sin[2(e + fx)] \Big/ \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \\
 & \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \Big) + \\
 & \left( 3(a + b) \cos[e + fx] (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p \sin[e + fx] \right. \\
 & \left( \frac{1}{3(a + b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \\
 & \left. \sec[e + fx]^2 \tan[e + fx] - \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, \right. \right. \\
 & \left. \left. -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \sec[e + fx]^2 \tan[e + fx] \right) \Big) \Big/ \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \\
 & \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \Big) - \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \cos[e + fx] \right. \\
 & (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p \sin[e + fx] \\
 & \left( 4 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \right. \\
 & \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \right. \\
 & \left. \sec[e + fx]^2 \tan[e + fx] + 3(a + b) \left( \frac{1}{3(a + b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \sec[e + fx]^2 \tan[e + fx] - \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
 & \left. \left. -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \sec[e + fx]^2 \tan[e + fx] \right) \Big) + \\
 & 2 \tan[e + fx]^2 \left( b p \left( -\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \right. \\
 & \left. \left. \sec[e + fx]^2 \tan[e + fx] - \frac{1}{5(a + b)} 6 b (1 - p) \operatorname{AppellF1}\left[\frac{5}{2}, 2 - p, 1, \right. \right. \right. \\
 & \left. \left. \frac{7}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \sec[e + fx]^2 \tan[e + fx] \right) \Big) -
 \end{aligned}$$

$$\begin{aligned}
 & (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{12}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, -p, 3, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Big) \Big) \Big) \Big) \Big) / \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right)^2 \Big) \Big) \Big) \Big) \Big)
 \end{aligned}$$

**Problem 309: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^2 (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\begin{aligned}
 & \frac{1}{f} \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \\
 & \tan[e+fx] (a+b + b \tan[e+fx]^2)^p \left( 1 + \frac{b \tan[e+fx]^2}{a+b} \right)^{-p}
 \end{aligned}$$

Result (type 6, 1914 leaves):

$$\begin{aligned}
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \cos[e+fx] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-2+p} (a+b \operatorname{Sec}[e+fx]^2)^p \sin[e+fx] \right) \Big) \Big) \Big) \Big) \Big) / \\
 & \left( f \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2(a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \right) \Big) \Big) \Big) \Big) \Big) \\
 & \left( \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \\
 & \quad \left. \left. (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-1+p} \right) \right) \Big) \Big) \Big) \Big) \Big) / \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \quad \left. \left. 2(a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \Big) \Big) \Big) \Big) \Big)
 \end{aligned}$$

$$\begin{aligned}
 & 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \tan[e + f x]^2 - \\
 & \left(6 a (a + b)^p \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right. \\
 & \quad \left. (a + 2 b + a \cos[2(e + f x)])^{-1+p} (\sec[e + f x]^2)^{-2+p} \sin[2(e + f x)] \tan[e + f x]\right) / \\
 & \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + \right. \\
 & \quad \left. 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1 - p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - \right. \right. \\
 & \quad \left. \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \tan[e + f x]^2 + \right. \\
 & \left. \left(6 (a + b) (-2 + p) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right. \right. \\
 & \quad \left. \left. (a + 2 b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-2+p} \tan[e + f x]^2\right) / \right. \\
 & \left. \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + \right. \right. \\
 & \quad \left. \left. 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1 - p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \tan[e + f x]^2 + \right. \right. \\
 & \left. \left. \left(3 (a + b) (a + 2 b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-2+p} \tan[e + f x] \right. \right. \right. \\
 & \quad \left. \left. \left. \left(\frac{1}{3(a + b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1 - p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right. \right. \right. \\
 & \quad \left. \left. \left. \sec[e + f x]^2 \tan[e + f x] - \frac{4}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \sec[e + f x]^2 \tan[e + f x]\right) \right) / \right. \right. \\
 & \left. \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + \right. \right. \\
 & \quad \left. \left. 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1 - p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \tan[e + f x]^2 - \right. \right. \\
 & \left. \left. \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right. \right. \right. \\
 & \quad \left. \left. (a + 2 b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-2+p} \tan[e + f x] \right. \right. \\
 & \quad \left. \left. \left(4 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1 - p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \\
 & \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{4}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
 & \quad \left. \left. 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
 & 2 \operatorname{Tan}[e+f x]^2 \left( b p \left( -\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2, 2-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 3, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) - \\
 & 2 (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 3, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{18}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 4, -p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \left. \right) \left. \right) \left. \right) \left. \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
 & 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] - 2 (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2 \right)^2 \left. \right) \left. \right) \left. \right) \left. \right)
 \end{aligned}$$

**Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+f x]^4 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \\
 \operatorname{Tan}[e+f x] (a+b+b \operatorname{Tan}[e+f x]^2)^p \left( 1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p}$$

Result (type 6, 1912 leaves):

$$\left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Cos}[e+f x]^3 \right. \\
 \left. (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-3+p} (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \right) /$$

$$\begin{aligned}
 & \left( f \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] - \right. \\
 & \quad \quad \left. \left. 3 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \tan [e+f x]^2 \right) \\
 & \left( \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \right. \\
 & \quad \left. (a+2 b+a \cos [2 (e+f x)])^p (\sec [e+f x]^2)^{-2+p} \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] - \right. \\
 & \quad \quad \left. \left. 3 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \tan [e+f x]^2 \right) - \\
 & \left( 6 a (a+b) p \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right. \\
 & \quad \left. (a+2 b+a \cos [2 (e+f x)])^{-1+p} (\sec [e+f x]^2)^{-3+p} \sin [2 (e+f x)] \tan [e+f x] \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] - \right. \\
 & \quad \quad \left. \left. 3 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \tan [e+f x]^2 \right) + \\
 & \left( 6 (a+b) (-3+p) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right. \\
 & \quad \left. (a+2 b+a \cos [2 (e+f x)])^p (\sec [e+f x]^2)^{-3+p} \tan [e+f x]^2 \right) / \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3, -p, \frac{3}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] - \right. \\
 & \quad \quad \left. \left. 3 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \right) \tan [e+f x]^2 \right) + \\
 & \left( 3 (a+b) (a+2 b+a \cos [2 (e+f x)])^p (\sec [e+f x]^2)^{-3+p} \tan [e+f x] \right. \\
 & \quad \left. \left( \frac{1}{3 (a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \sec [e+f x]^2 \right. \right. \\
 & \quad \quad \left. \left. \tan [e+f x] - 2 \operatorname{AppellF1} \left[ \frac{3}{2}, 4, -p, \frac{5}{2}, -\tan [e+f x]^2, -\frac{b \tan [e+f x]^2}{a+b} \right] \sec [e+f x]^2 \right) \right)
 \end{aligned}$$





Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Tan}[e + f x] (a + b + b \text{Tan}[e + f x]^2)^p \left(1 + \frac{b \text{Tan}[e + f x]^2}{a + b}\right)^{-p}$$

Result (type 6, 1914 leaves):

$$\begin{aligned} & \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x]^5 \right. \\ & \quad \left. (a + 2 b + a \text{Cos}[2 (e + f x)])^p (\text{Sec}[e + f x]^2)^{-4+p} (a + b \text{Sec}[e + f x]^2)^p \text{Sin}[e + f x] \right) / \\ & \left( f \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] + \right. \right. \\ & \quad 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 4, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] - \right. \\ & \quad \left. \left. 4 (a + b) \text{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right) \text{Tan}[e + f x]^2 \right) \\ & \left( \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right. \right. \\ & \quad \left. \left. (a + 2 b + a \text{Cos}[2 (e + f x)])^p (\text{Sec}[e + f x]^2)^{-3+p} \right) / \right. \\ & \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] + \right. \\ & \quad 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 4, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] - \right. \\ & \quad \left. \left. 4 (a + b) \text{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right) \text{Tan}[e + f x]^2 \right) - \\ & \left( 6 a (a + b) p \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right. \\ & \quad \left. (a + 2 b + a \text{Cos}[2 (e + f x)])^{-1+p} (\text{Sec}[e + f x]^2)^{-4+p} \text{Sin}[2 (e + f x)] \text{Tan}[e + f x] \right) / \\ & \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] + \right. \\ & \quad 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 4, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] - \right. \\ & \quad \left. \left. 4 (a + b) \text{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right) \text{Tan}[e + f x]^2 \right) + \\ & \left( 6 (a + b) (-4 + p) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right. \\ & \quad \left. (a + 2 b + a \text{Cos}[2 (e + f x)])^p (\text{Sec}[e + f x]^2)^{-4+p} \text{Tan}[e + f x]^2 \right) / \\ & \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] + \right. \end{aligned}$$

$$\begin{aligned}
 & 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \\
 & \quad \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 + \\
 & \left( 3 (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-4+p} \tan[e+fx] \right. \\
 & \quad \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. \sec[e+fx]^2 \tan[e+fx] - \frac{8}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, 5, -p, \frac{5}{2}, \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) \Big/ \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
 & \quad \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-4+p} \tan[e+fx] \right. \\
 & \quad \left. \left( 4 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
 & \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \right. \\
 & \quad \left. \sec[e+fx]^2 \tan[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 4, 1-p, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - \frac{8}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \right. \\
 & \quad \left. \left. 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) + \\
 & \quad 2 \tan[e+fx]^2 \left( b p \left( -\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 4, 2-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - \frac{24}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 5, 1-p, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) - \\
 & \quad 4 (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 5, 1-p, \frac{7}{2}, -\tan[e+fx]^2, \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - 6 \operatorname{AppellF1} \left[ \frac{5}{2}, 6, -p, \right. \right.
 \end{aligned}$$

$$\left. \left. \left. \left. \left. \left. \left. \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) \right) \right) \right) \right) \right) /$$

$$\left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right.$$

$$\left. 2 \left( b^p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - 4(a+b) \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right)^2 \right)$$

**Problem 328: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx] (a+b \sec[e+fx]^2)^2 dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{b(2a+b) \log[\cos[e+fx]]}{f} + \frac{(a+b)^2 \log[\sin[e+fx]]}{f} + \frac{b^2 \sec[e+fx]^2}{2f}$$

Result (type 3, 180 leaves):

$$\frac{1}{4f} \left( 2b^2 + 2ia^2fx + 4iabfx + 2ib^2fx - 4i(a+b)^2 \operatorname{ArcTan}[\tan[e+fx]] \cos[e+fx]^2 - \right.$$

$$4ab \log[\cos[e+fx]] - 2b^2 \log[\cos[e+fx]] + a^2 \log[\sin[e+fx]^2] +$$

$$2ab \log[\sin[e+fx]^2] + b^2 \log[\sin[e+fx]^2] + \cos[2(e+fx)]$$

$$\left. \left. \left. \left. \left. \left. \left. -2b(2a+b) \log[\cos[e+fx]] + (a+b)^2 (2ifx + \log[\sin[e+fx]^2]) \right) \right) \right) \right) \right) \right) \sec[e+fx]^2$$

**Problem 329: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^3 (a+b \sec[e+fx]^2)^2 dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{(a+b)^2 \operatorname{Csc}[e+fx]^2}{2f} - \frac{b^2 \log[\cos[e+fx]]}{f} - \frac{(a^2-b^2) \log[\sin[e+fx]]}{f}$$

Result (type 3, 163 leaves):

$$\frac{1}{4f} \operatorname{Csc}[e+fx]^2 \left( -2a^2 - 4ab - 2b^2 - 2ia^2fx + 2ib^2fx - \right.$$

$$2b^2 \log[\cos[e+fx]] - a^2 \log[\sin[e+fx]^2] + b^2 \log[\sin[e+fx]^2] +$$

$$\cos[2(e+fx)] \left( 2b^2 \log[\cos[e+fx]] + (a^2-b^2) (2ifx + \log[\sin[e+fx]^2]) \right) \left. \right) +$$

$$4i(a^2-b^2) \operatorname{ArcTan}[\tan[e+fx]] \sin[e+fx]^2$$

**Problem 330: Result unnecessarily involves complex numbers and more than**

### twice size of optimal antiderivative.

$$\int \text{Cot}[e + f x]^5 (a + b \text{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$\frac{a(a+b) \text{Csc}[e + f x]^2}{f} - \frac{(a+b)^2 \text{Csc}[e + f x]^4}{4f} + \frac{a^2 \text{Log}[\text{Sin}[e + f x]]}{f}$$

Result (type 3, 132 leaves):

$$\left( (b + a \text{Cos}[e + f x]^2)^2 \right. \\ \left. \left( -4 a^2 \text{ArcTan}[\text{Tan}[e + f x]] \text{Cos}[e + f x]^4 + 4 a (a + b) \text{Cos}[e + f x]^2 \text{Cot}[e + f x]^2 - \right. \right. \\ \left. \left. (a + b)^2 \text{Cot}[e + f x]^4 + 2 a^2 \text{Cos}[e + f x]^4 (2 i f x + \text{Log}[\text{Sin}[e + f x]^2]) \right) \right) \\ \text{Sec}[e + f x]^4 \Big/ \left( f (a + 2 b + a \text{Cos}[2(e + f x)])^2 \right)$$

### Problem 331: Result more than twice size of optimal antiderivative.

$$\int (a + b \text{Sec}[e + f x]^2)^2 \text{Tan}[e + f x]^6 dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$-a^2 x + \frac{a^2 \text{Tan}[e + f x]}{f} - \frac{a^2 \text{Tan}[e + f x]^3}{3f} + \\ \frac{a^2 \text{Tan}[e + f x]^5}{5f} + \frac{b(2a+b) \text{Tan}[e + f x]^7}{7f} + \frac{b^2 \text{Tan}[e + f x]^9}{9f}$$

Result (type 3, 275 leaves):

$$-\frac{1}{315 f (a + 2 b + a \text{Cos}[2(e + f x)])^2} 4 (b + a \text{Cos}[e + f x]^2)^2 \text{Sec}[e + f x]^9 \\ (315 a^2 f x \text{Cos}[e + f x]^9 - 35 b^2 \text{Sec}[e] \text{Sin}[f x] - 5 (18 a - 19 b) b \text{Cos}[e + f x]^2 \text{Sec}[e] \text{Sin}[f x] - \\ 3 (21 a^2 - 90 a b + 25 b^2) \text{Cos}[e + f x]^4 \text{Sec}[e] \text{Sin}[f x] + (231 a^2 - 270 a b + 5 b^2) \\ \text{Cos}[e + f x]^6 \text{Sec}[e] \text{Sin}[f x] - (483 a^2 - 90 a b - 10 b^2) \text{Cos}[e + f x]^8 \text{Sec}[e] \text{Sin}[f x] - \\ 35 b^2 \text{Cos}[e + f x] \text{Tan}[e] - 5 (18 a - 19 b) b \text{Cos}[e + f x]^3 \text{Tan}[e] - \\ 3 (21 a^2 - 90 a b + 25 b^2) \text{Cos}[e + f x]^5 \text{Tan}[e] + (231 a^2 - 270 a b + 5 b^2) \text{Cos}[e + f x]^7 \text{Tan}[e])$$

### Problem 332: Result more than twice size of optimal antiderivative.

$$\int (a + b \text{Sec}[e + f x]^2)^2 \text{Tan}[e + f x]^4 dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$a^2 x - \frac{a^2 \text{Tan}[e + f x]}{f} + \frac{a^2 \text{Tan}[e + f x]^3}{3f} + \frac{b(2a+b) \text{Tan}[e + f x]^5}{5f} + \frac{b^2 \text{Tan}[e + f x]^7}{7f}$$

Result (type 3, 395 leaves):

$$\frac{1}{13440 f} \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^7 \left( 3675 a^2 f x \operatorname{Cos}[f x] + 3675 a^2 f x \operatorname{Cos}[2 e + f x] + 2205 a^2 f x \operatorname{Cos}[2 e + 3 f x] + 2205 a^2 f x \operatorname{Cos}[4 e + 3 f x] + 735 a^2 f x \operatorname{Cos}[4 e + 5 f x] + 735 a^2 f x \operatorname{Cos}[6 e + 5 f x] + 105 a^2 f x \operatorname{Cos}[6 e + 7 f x] + 105 a^2 f x \operatorname{Cos}[8 e + 7 f x] - 5320 a^2 \operatorname{Sin}[f x] + 1680 a b \operatorname{Sin}[f x] + 840 b^2 \operatorname{Sin}[f x] + 4480 a^2 \operatorname{Sin}[2 e + f x] - 1260 a b \operatorname{Sin}[2 e + f x] + 420 b^2 \operatorname{Sin}[2 e + f x] - 3780 a^2 \operatorname{Sin}[2 e + 3 f x] + 924 a b \operatorname{Sin}[2 e + 3 f x] - 168 b^2 \operatorname{Sin}[2 e + 3 f x] + 2100 a^2 \operatorname{Sin}[4 e + 3 f x] - 840 a b \operatorname{Sin}[4 e + 3 f x] - 420 b^2 \operatorname{Sin}[4 e + 3 f x] - 1540 a^2 \operatorname{Sin}[4 e + 5 f x] + 168 a b \operatorname{Sin}[4 e + 5 f x] + 84 b^2 \operatorname{Sin}[4 e + 5 f x] + 420 a^2 \operatorname{Sin}[6 e + 5 f x] - 420 a b \operatorname{Sin}[6 e + 5 f x] - 280 a^2 \operatorname{Sin}[6 e + 7 f x] + 84 a b \operatorname{Sin}[6 e + 7 f x] + 12 b^2 \operatorname{Sin}[6 e + 7 f x] \right)$$

**Problem 333: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 \operatorname{Tan}[e + f x]^2 dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$-a^2 x + \frac{a^2 \operatorname{Tan}[e + f x]}{f} + \frac{b(2a + b) \operatorname{Tan}[e + f x]^3}{3f} + \frac{b^2 \operatorname{Tan}[e + f x]^5}{5f}$$

Result (type 3, 281 leaves):

$$-\frac{1}{480 f} \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^5 \left( 150 a^2 f x \operatorname{Cos}[f x] + 150 a^2 f x \operatorname{Cos}[2 e + f x] + 75 a^2 f x \operatorname{Cos}[2 e + 3 f x] + 75 a^2 f x \operatorname{Cos}[4 e + 3 f x] + 15 a^2 f x \operatorname{Cos}[4 e + 5 f x] + 15 a^2 f x \operatorname{Cos}[6 e + 5 f x] - 180 a^2 \operatorname{Sin}[f x] + 80 a b \operatorname{Sin}[f x] - 20 b^2 \operatorname{Sin}[f x] + 120 a^2 \operatorname{Sin}[2 e + f x] - 120 a b \operatorname{Sin}[2 e + f x] - 60 b^2 \operatorname{Sin}[2 e + f x] - 120 a^2 \operatorname{Sin}[2 e + 3 f x] + 40 a b \operatorname{Sin}[2 e + 3 f x] + 20 b^2 \operatorname{Sin}[2 e + 3 f x] + 30 a^2 \operatorname{Sin}[4 e + 3 f x] - 60 a b \operatorname{Sin}[4 e + 3 f x] - 30 a^2 \operatorname{Sin}[4 e + 5 f x] + 20 a b \operatorname{Sin}[4 e + 5 f x] + 4 b^2 \operatorname{Sin}[4 e + 5 f x] \right)$$

**Problem 334: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b(2a + b) \operatorname{Tan}[e + f x]}{f} + \frac{b^2 \operatorname{Tan}[e + f x]^3}{3f}$$

Result (type 3, 106 leaves):

$$\left( 4 (b + a \operatorname{Cos}[e + f x]^2)^2 \operatorname{Sec}[e + f x]^3 \left( 3 a^2 f x \operatorname{Cos}[e + f x]^3 + b^2 \operatorname{Sec}[e] \operatorname{Sin}[f x] + 2 b (3 a + b) \operatorname{Cos}[e + f x]^2 \operatorname{Sec}[e] \operatorname{Sin}[f x] + b^2 \operatorname{Cos}[e + f x] \operatorname{Tan}[e] \right) \right) / \left( 3 f (a + 2 b + a \operatorname{Cos}[2(e + f x)])^2 \right)$$

**Problem 335: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e + f x]^2 (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-a^2 x - \frac{(a+b)^2 \operatorname{Cot}[e+fx]}{f} + \frac{b^2 \operatorname{Tan}[e+fx]}{f}$$

Result (type 3, 82 leaves):

$$-\left(4(b+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e+fx] \left(a^2 f x \operatorname{Cos}[e+fx] - \left((a+b)^2 \operatorname{Cot}[e+fx] \operatorname{Csc}[e] + b^2 \operatorname{Sec}[e]\right) \operatorname{Sin}[fx]\right) \right) / \left(f(a+2b+a \operatorname{Cos}[2(e+fx)])^2\right)$$

**Problem 336: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+fx]^4 (a+b \operatorname{Sec}[e+fx])^2 dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$a^2 x + \frac{(a^2 - b^2) \operatorname{Cot}[e+fx]}{f} - \frac{(a+b)^2 \operatorname{Cot}[e+fx]^3}{3f}$$

Result (type 3, 160 leaves):

$$\frac{1}{24f} \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^3 \left(9a^2 f x \operatorname{Cos}[fx] - 9a^2 f x \operatorname{Cos}[2e+fx] - 3a^2 f x \operatorname{Cos}[2e+3fx] + 3a^2 f x \operatorname{Cos}[4e+3fx] - 12a^2 \operatorname{Sin}[fx] + 12b^2 \operatorname{Sin}[fx] - 12a^2 \operatorname{Sin}[2e+fx] - 12ab \operatorname{Sin}[2e+fx] + 8a^2 \operatorname{Sin}[2e+3fx] + 4ab \operatorname{Sin}[2e+3fx] - 4b^2 \operatorname{Sin}[2e+3fx]\right)$$

**Problem 337: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+fx]^6 (a+b \operatorname{Sec}[e+fx])^2 dx$$

Optimal (type 3, 65 leaves, 4 steps):

$$-a^2 x - \frac{a^2 \operatorname{Cot}[e+fx]}{f} + \frac{(a^2 - b^2) \operatorname{Cot}[e+fx]^3}{3f} - \frac{(a+b)^2 \operatorname{Cot}[e+fx]^5}{5f}$$

Result (type 3, 256 leaves):

$$\frac{1}{480f} \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^5 \left(-150a^2 f x \operatorname{Cos}[fx] + 150a^2 f x \operatorname{Cos}[2e+fx] + 75a^2 f x \operatorname{Cos}[2e+3fx] - 75a^2 f x \operatorname{Cos}[4e+3fx] - 15a^2 f x \operatorname{Cos}[4e+5fx] + 15a^2 f x \operatorname{Cos}[6e+5fx] + 280a^2 \operatorname{Sin}[fx] + 120ab \operatorname{Sin}[fx] + 20b^2 \operatorname{Sin}[fx] + 180a^2 \operatorname{Sin}[2e+fx] - 60b^2 \operatorname{Sin}[2e+fx] - 140a^2 \operatorname{Sin}[2e+3fx] + 20b^2 \operatorname{Sin}[2e+3fx] - 90a^2 \operatorname{Sin}[4e+3fx] - 60ab \operatorname{Sin}[4e+3fx] + 46a^2 \operatorname{Sin}[4e+5fx] + 12ab \operatorname{Sin}[4e+5fx] - 4b^2 \operatorname{Sin}[4e+5fx]\right)$$

**Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[e + f x]^5}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 69 leaves, 4 steps):

$$\frac{(a + 2 b) \text{Log}[\text{Cos}[e + f x]]}{b^2 f} - \frac{(a + b)^2 \text{Log}[b + a \text{Cos}[e + f x]^2]}{2 a b^2 f} + \frac{\text{Sec}[e + f x]^2}{2 b f}$$

Result (type 3, 180 leaves):

$$\frac{1}{8 a b^2 f (a + b \text{Sec}[e + f x]^2)} (a + 2 b + a \text{Cos}[2 (e + f x)]) (2 a b + 2 a (a + 2 b) \text{Log}[\text{Cos}[e + f x]] - a^2 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] - 2 a b \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] - b^2 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + \text{Cos}[2 (e + f x)] (2 a (a + 2 b) \text{Log}[\text{Cos}[e + f x]] - (a + b)^2 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]])) \text{Sec}[e + f x]^4$$

**Problem 343: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^5}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{(2 a + 3 b) \text{Csc}[e + f x]^2}{2 (a + b)^2 f} - \frac{\text{Csc}[e + f x]^4}{4 (a + b) f} + \frac{b^3 \text{Log}[b + a \text{Cos}[e + f x]^2]}{2 a (a + b)^3 f} + \frac{(a^2 + 3 a b + 3 b^2) \text{Log}[\text{Sin}[e + f x]]}{(a + b)^3 f}$$

Result (type 3, 464 leaves):

$$\frac{1}{32 a (a + b)^3 f (-1 + \text{Cot}[e]^2) (a + b \text{Sec}[e + f x]^2)} \text{Cos}[2 e] (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Csc}[e]^2 \text{Csc}[e + f x]^4 \text{Sec}[e + f x]^2 (4 a^3 + 12 a^2 b + 8 a b^2 + 6 i a^3 f x + 18 i a^2 b f x + 18 i a b^2 f x + 2 i a^3 f x \text{Cos}[4 (e + f x)] + 6 i a^2 b f x \text{Cos}[4 (e + f x)] + 6 i a b^2 f x \text{Cos}[4 (e + f x)] + 3 b^3 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + b^3 \text{Cos}[4 (e + f x)] \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + 3 a^3 \text{Log}[\text{Sin}[e + f x]^2] + 9 a^2 b \text{Log}[\text{Sin}[e + f x]^2] + 9 a b^2 \text{Log}[\text{Sin}[e + f x]^2] + a^3 \text{Cos}[4 (e + f x)] \text{Log}[\text{Sin}[e + f x]^2] + 3 a^2 b \text{Cos}[4 (e + f x)] \text{Log}[\text{Sin}[e + f x]^2] + 3 a b^2 \text{Cos}[4 (e + f x)] \text{Log}[\text{Sin}[e + f x]^2] + 4 \text{Cos}[2 (e + f x)] (-b^3 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + a (a^2 (-2 - 2 i f x) + 3 b^2 (-1 - 2 i f x) + a b (-5 - 6 i f x) - (a^2 + 3 a b + 3 b^2) \text{Log}[\text{Sin}[e + f x]^2])) - 16 i a (a^2 + 3 a b + 3 b^2) \text{ArcTan}[\text{Tan}[e + f x]] \text{Sin}[e + f x]^4)$$

**Problem 344: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**



$$\int \frac{\tan[e + f x]^6}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 83 leaves, 7 steps):

$$-\frac{x}{a} + \frac{(a+b)^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{a b^{5/2} f} - \frac{(a+2b) \tan[e+fx]}{b^2 f} + \frac{\tan[e+fx]^3}{3 b f}$$

Result (type 3, 229 leaves):

$$\left( (a+2b+a \cos[2(e+fx)]) \sec[e+fx]^2 \left( -\frac{3x}{a} - \left( 3(a+b)^{5/2} \operatorname{ArcTan}\left[ \frac{\sec[fx] (\cos[2e] - i \sin[2e])}{(- (a+2b) \sin[fx] + a \sin[2e+fx])} \right] \right) \right) \right) / \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \left( \cos[2e] - i \sin[2e] \right) / \left( a b^2 f \sqrt{b (\cos[e] - i \sin[e])^4} \right) - \frac{(3a+7b) \sec[e] \sec[e+fx] \sin[fx]}{b^2 f} + \frac{\sec[e] \sec[e+fx]^3 \sin[fx]}{b f} + \frac{\sec[e+fx]^2 \tan[e]}{b f} \right) / \left( 6 (a+b \sec[e+fx]^2) \right)$$

**Problem 345: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^4}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$\frac{x}{a} - \frac{(a+b)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{a b^{3/2} f} + \frac{\tan[e+fx]}{b f}$$

Result (type 3, 206 leaves):

$$\left( (a+2b+a \cos[2(e+fx)]) \sec[e+fx]^2 \left( (a+b)^2 \operatorname{ArcTan}\left[ \frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])}{\left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) (\cos[2e] - i \sin[2e]) + \sqrt{a+b} \sqrt{b (i \cos[e] + \sin[e])^4} (b f x + a \sec[e] \sec[e+fx] \sin[fx])} \right] \right) \right) / \left( 2 a b \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right)$$

**Problem 346: Result unnecessarily involves complex numbers and more than**

twice size of optimal antiderivative.

$$\int \frac{\tan[e + f x]^2}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$-\frac{x}{a} + \frac{\sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{a \sqrt{b} f}$$

Result (type 3, 184 leaves):

$$-\left(\left(\left(a + 2b + a \cos[2(e + fx)]\right) \sec[e + fx]^2 \left(\sqrt{a+b} f x \sqrt{b (\cos[e] - i \sin[e])^4} + (a+b) \operatorname{ArcTan}\left[(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])\right]\right) / \left(2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}\right) (\cos[2e] - i \sin[2e])\right)\right) / \left(2 a \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4}\right)\right)$$

**Problem 347: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{x}{a} + \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \cot[e+fx]}{\sqrt{b}}\right]}{a \sqrt{a+b} f}$$

Result (type 3, 182 leaves):

$$\left(\left(a + 2b + a \cos[2(e + fx)]\right) \sec[e + fx]^2 \left(\sqrt{a+b} f x \sqrt{b (\cos[e] - i \sin[e])^4} + b \operatorname{ArcTan}\left[(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])\right]\right) / \left(2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}\right) (\cos[2e] - i \sin[2e])\right)\right) / \left(2 a \sqrt{a+b} f (a+b \sec[e+fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4}\right)$$

**Problem 348: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^2}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 62 leaves, 6 steps):

$$-\frac{x}{a} + \frac{b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a(a+b)^{3/2} f} - \frac{\operatorname{Cot}[e+fx]}{(a+b) f}$$

Result (type 3, 204 leaves):

$$-\left(\left((a+2b+a\cos[2(e+fx)])\sec[e+fx]^2\right.\right. \\ \left.\left.(b^2 \operatorname{ArcTan}\left[(\sec[fx](\cos[2e]-i\sin[2e])(- (a+2b)\sin[fx]+a\sin[2e+fx])\right]\right] / \right. \\ \left.\left.(2\sqrt{a+b}\sqrt{b(\cos[e]-i\sin[e])^4}\right)(\cos[2e]-i\sin[2e]) + \right. \\ \left.\left.\sqrt{a+b}\sqrt{b(\cos[e]-i\sin[e])^4}((a+b)fx-a\csc[e]\csc[e+fx]\sin[fx])\right)\right) / \\ \left.(2a(a+b)^{3/2}f(a+b\sec[e+fx]^2)\sqrt{b(\cos[e]-i\sin[e])^4}\right)$$

**Problem 349: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+fx]^4}{a+b\sec[e+fx]^2} dx$$

Optimal (type 3, 86 leaves, 7 steps):

$$\frac{x}{a} - \frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a(a+b)^{5/2} f} + \frac{(a+2b)\operatorname{Cot}[e+fx]}{(a+b)^2 f} - \frac{\operatorname{Cot}[e+fx]^3}{3(a+b) f}$$

Result (type 3, 587 leaves):

$$\begin{aligned}
 & x \frac{(a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^2}{2a(a + b \sec[e + fx]^2)} - \\
 & \frac{(a + 2b + a \cos[2e + 2fx]) \cot[e] \csc[e + fx]^2 \sec[e + fx]^2}{6(a + b) f (a + b \sec[e + fx]^2)} + \\
 & \left( (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^2 \left( \left( b^3 \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \right. \\
 & \left. \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \cos[2e] \right) \right] \right) / \right. \\
 & \left. \left( 2a \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i b^3 \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \right. \\
 & \left. \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) \right] \right) / \right. \\
 & \left. \left. \left. \left. \left( 2a \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) \right) \right) / \left( (a + b)^2 (a + b \sec[e + fx]^2) \right) + \\
 & \frac{(a + 2b + a \cos[2e + 2fx]) \csc[e] \csc[e + fx]^3 \sec[e + fx]^2 \sin[fx]}{6(a + b) f (a + b \sec[e + fx]^2)} + \\
 & \frac{(a + 2b + a \cos[2e + 2fx]) \csc[e] \csc[e + fx] \sec[e + fx]^2 (-4a \sin[fx] - 7b \sin[fx])}{6(a + b)^2 f (a + b \sec[e + fx]^2)}
 \end{aligned}$$

**Problem 350: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + fx]^6}{a + b \sec[e + fx]^2} dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{x}{a} + \frac{b^{7/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{a(a + b)^{7/2} f} - \\
 & \frac{(a^2 + 3ab + 3b^2) \cot[e + fx]}{(a + b)^3 f} + \frac{(a + 2b) \cot[e + fx]^3}{3(a + b)^2 f} - \frac{\cot[e + fx]^5}{5(a + b) f}
 \end{aligned}$$

Result (type 3, 671 leaves):

$$\frac{1}{960 a (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)} (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2$$

$$\left( - \left( \left( 480 b^4 \operatorname{ArcTan} \left[ \left( \operatorname{Sec}[f x] \left( \operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e] \right) \right] \left( - (a+2 b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e+f x] \right) \right] \right) \right) / \right.$$

$$\left. \left( 2 \sqrt{a+b} \sqrt{b \left( \operatorname{Cos}[e] - i \operatorname{Sin}[e] \right)^4} \right) \left( \operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e] \right) \right) / \left( \sqrt{a+b} \sqrt{b \left( \operatorname{Cos}[e] - i \operatorname{Sin}[e] \right)^4} \right) + \operatorname{Csc}[e] \operatorname{Csc}[e+f x]^5$$

$$\left( -150 (a+b)^3 f x \operatorname{Cos}[f x] + 150 (a+b)^3 f x \operatorname{Cos}[2 e+f x] + 75 a^3 f x \operatorname{Cos}[2 e+3 f x] + \right.$$

$$225 a^2 b f x \operatorname{Cos}[2 e+3 f x] + 225 a b^2 f x \operatorname{Cos}[2 e+3 f x] +$$

$$75 b^3 f x \operatorname{Cos}[2 e+3 f x] - 75 a^3 f x \operatorname{Cos}[4 e+3 f x] - 225 a^2 b f x \operatorname{Cos}[4 e+3 f x] -$$

$$225 a b^2 f x \operatorname{Cos}[4 e+3 f x] - 75 b^3 f x \operatorname{Cos}[4 e+3 f x] - 15 a^3 f x \operatorname{Cos}[4 e+5 f x] -$$

$$45 a^2 b f x \operatorname{Cos}[4 e+5 f x] - 45 a b^2 f x \operatorname{Cos}[4 e+5 f x] - 15 b^3 f x \operatorname{Cos}[4 e+5 f x] +$$

$$15 a^3 f x \operatorname{Cos}[6 e+5 f x] + 45 a^2 b f x \operatorname{Cos}[6 e+5 f x] + 45 a b^2 f x \operatorname{Cos}[6 e+5 f x] +$$

$$15 b^3 f x \operatorname{Cos}[6 e+5 f x] + 280 a^3 \operatorname{Sin}[f x] + 780 a^2 b \operatorname{Sin}[f x] + 680 a b^2 \operatorname{Sin}[f x] +$$

$$180 a^3 \operatorname{Sin}[2 e+f x] + 540 a^2 b \operatorname{Sin}[2 e+f x] + 480 a b^2 \operatorname{Sin}[2 e+f x] -$$

$$140 a^3 \operatorname{Sin}[2 e+3 f x] - 420 a^2 b \operatorname{Sin}[2 e+3 f x] - 400 a b^2 \operatorname{Sin}[2 e+3 f x] -$$

$$90 a^3 \operatorname{Sin}[4 e+3 f x] - 240 a^2 b \operatorname{Sin}[4 e+3 f x] - 180 a b^2 \operatorname{Sin}[4 e+3 f x] +$$

$$46 a^3 \operatorname{Sin}[4 e+5 f x] + 132 a^2 b \operatorname{Sin}[4 e+5 f x] + 116 a b^2 \operatorname{Sin}[4 e+5 f x] \left. \right)$$

### Problem 355: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cot}[e+f x]^3}{(a+b \operatorname{Sec}[e+f x]^2)^2} dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$-\frac{b^3}{2 a^2 (a+b)^2 f (b+a \operatorname{Cos}[e+f x]^2)} - \frac{\operatorname{Csc}[e+f x]^2}{2 (a+b)^2 f}$$

$$\frac{b^2 (3 a+b) \operatorname{Log}[b+a \operatorname{Cos}[e+f x]^2]}{2 a^2 (a+b)^3 f} - \frac{(a+3 b) \operatorname{Log}[\operatorname{Sin}[e+f x]]}{(a+b)^3 f}$$

Result (type 3, 306 leaves):

$$-\frac{1}{8 a^2 (a+b)^3 f (a+2 b+a \operatorname{Cos}[2(e+f x)])}$$

$$\operatorname{Csc}[e+f x]^2 \left( 4 a^4 + 12 a^3 b + 8 a^2 b^2 + 4 a b^3 + 4 b^4 + 3 a^2 b^2 \operatorname{Log}[a+2 b+a \operatorname{Cos}[2(e+f x)]] \right) +$$

$$13 a b^3 \operatorname{Log}[a+2 b+a \operatorname{Cos}[2(e+f x)]] + 4 b^4 \operatorname{Log}[a+2 b+a \operatorname{Cos}[2(e+f x)]] + 2 a^4$$

$$\operatorname{Log}[\operatorname{Sin}[e+f x]] + 14 a^3 b \operatorname{Log}[\operatorname{Sin}[e+f x]] + 24 a^2 b^2 \operatorname{Log}[\operatorname{Sin}[e+f x]] - a \operatorname{Cos}[4(e+f x)]$$

$$\left( b^2 (3 a+b) \operatorname{Log}[a+2 b+a \operatorname{Cos}[2(e+f x)]] + 2 a^2 (a+3 b) \operatorname{Log}[\operatorname{Sin}[e+f x]] \right) +$$

$$4 \operatorname{Cos}[2(e+f x)] \left( a^4 + a^3 b - a b^3 - b^4 - b^3 (3 a+b) \operatorname{Log}[a+2 b+a \operatorname{Cos}[2(e+f x)]] \right) -$$

$$2 a^2 b (a+3 b) \operatorname{Log}[\operatorname{Sin}[e+f x]] \left. \right)$$

**Problem 356: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$\frac{b^4}{2 a^2 (a + b)^3 f (b + a \text{Cos}[e + f x]^2)} + \frac{(a + 2 b) \text{Csc}[e + f x]^2}{(a + b)^3 f} - \frac{\text{Csc}[e + f x]^4}{4 (a + b)^2 f} + \frac{b^3 (4 a + b) \text{Log}[b + a \text{Cos}[e + f x]^2]}{2 a^2 (a + b)^4 f} + \frac{(a^2 + 4 a b + 6 b^2) \text{Log}[\text{Sin}[e + f x]]}{(a + b)^4 f}$$

Result (type 3, 292 leaves):

$$\frac{1}{16 a^2 (a + b)^4 f (a + b \text{Sec}[e + f x]^2)^2} (a + 2 b + a \text{Cos}[2 (e + f x)]) (4 b^4 (a + b) + 4 i a^2 (a^2 + 4 a b + 6 b^2) f x (a + 2 b + a \text{Cos}[2 (e + f x)])) - 4 i a^2 (a^2 + 4 a b + 6 b^2) \text{ArcTan}[\text{Tan}[e + f x]] (a + 2 b + a \text{Cos}[2 (e + f x)]) + 4 a^2 (a + b) (a + 2 b) (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Csc}[e + f x]^2 - a^2 (a + b)^2 (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Csc}[e + f x]^4 + 2 b^3 (4 a + b) (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + 2 a^2 (a^2 + 4 a b + 6 b^2) (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Log}[\text{Sin}[e + f x]^2]) \text{Sec}[e + f x]^4$$

**Problem 357: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$-\frac{x}{a^2} - \frac{(3 a - 2 b) (a + b)^{3/2} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{2 a^2 b^{5/2} f} + \frac{(3 a + b) \text{Tan}[e + f x]}{2 a b^2 f} - \frac{(a + b) \text{Tan}[e + f x]^3}{2 a b f (a + b + b \text{Tan}[e + f x]^2)}$$

Result (type 3, 593 leaves):

$$\begin{aligned}
 & - \frac{x (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4}{4a^2 (a + b \sec[e + fx]^2)^2} + \\
 & \left( (3a - 2b) (a + b)^2 (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left( \left( \operatorname{ArcTan} \left[ \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \cos[2e] \right) \right] \right) \right) / \\
 & \left( 8a^2 b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i \operatorname{ArcTan} \left[ \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) \right] \right) \right) / \\
 & \left( 8a^2 b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \left. \right) / (a + b \sec[e + fx]^2)^2 + \\
 & \frac{(a + 2b + a \cos[2e + 2fx])^2 \sec[e] \sec[e + fx]^5 \sin[fx]}{4b^2 f (a + b \sec[e + fx]^2)^2} + \\
 & \left( (a + 2b + a \cos[2e + 2fx]) \right. \\
 & \quad \left. \sec[e + fx]^4 \right. \\
 & \quad \left. (-a^3 \sin[2e] - 4a^2 b \sin[2e] - 5a b^2 \sin[2e] - 2b^3 \sin[2e] + \right. \\
 & \quad \left. a^3 \sin[2fx] + 2a^2 b \sin[2fx] + a b^2 \sin[2fx]) \right) / \\
 & \left( 8a^2 b^2 f (a + b \sec[e + fx]^2)^2 (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right)
 \end{aligned}$$

**Problem 358: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + fx]^4}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$\frac{x}{a^2} + \frac{(a - 2b) \sqrt{a+b} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}} \right]}{2a^2 b^{3/2} f} - \frac{(a+b) \tan[e+fx]}{2abf (a+b \tan[e+fx]^2)}$$

Result (type 3, 249 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^4 \right. \\ \left. \left( 2x(a + 2b + a \cos[2(e + fx)]) + (-a^2 + ab + 2b^2) \operatorname{ArcTan} \left[ \frac{(\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx]))}{(2\sqrt{a+b} \sqrt{b(\cos[e] - i \sin[e])^4})} \right] (a + 2b + a \cos[2(e + fx)]) \right. \right. \\ \left. \left. (\cos[2e] - i \sin[2e]) \right) \right) / \left( b\sqrt{a+b} f \sqrt{b(\cos[e] - i \sin[e])^4} \right) + \\ \left. \frac{(a+b) ((a+2b) \sin[2e] - a \sin[2fx])}{bf(\cos[e] - \sin[e])(\cos[e] + \sin[e])} \right) / (8a^2(a+b \sec[e+fx]^2)^2)$$

**Problem 359: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + fx]^2}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{x}{a^2} + \frac{(a + 2b) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{2a^2 \sqrt{b} \sqrt{a+b} f} + \frac{\tan[e + fx]}{2af(a + b + b \tan[e + fx]^2)}$$

Result (type 3, 388 leaves):



$$\begin{aligned}
 & - \left( \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \right. \right. \\
 & \quad \left. \left( 16x + \left( (-a^3 + 6a^2b + 24ab^2 + 16b^3) \operatorname{ArcTan} \left[ \frac{\sec[fx] (\cos[2e] - i \sin[2e])}{(- (a + 2b) \sin[fx] + a \sin[2e + fx])} \right] \right) \right) \right. \\
 & \quad \left. \left. \left( 2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) \right) \right) / \\
 & \quad \left( \cos[2e] - i \sin[2e] \right) \left( b (a+b)^{3/2} f \sqrt{b (\cos[e] - i \sin[e])^4} + \right. \\
 & \quad \left. \left( (a^2 + 8ab + 8b^2) ((a + 2b) \sin[2e] - a \sin[2fx]) \right) / \right. \\
 & \quad \left. \left( b (a+b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right) \right) \left. \right) / \\
 & \quad \left( 64 a^2 (a + b \sec[e + fx]^2)^2 \right) + \left( (a + 2b + a \cos[2e + 2fx])^2 \right. \\
 & \quad \left. \sec[e + fx]^4 \right. \\
 & \quad \left. \left( \frac{(a + 2b) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} - \frac{a \sqrt{b} \sin[2(e + fx)]}{(a+b) (a + 2b + a \cos[2(e + fx)])} \right) \right) / \left( 64 \right. \\
 & \quad \left. b^{3/2} \right. \\
 & \quad \left. f \right. \\
 & \quad \left. (a + b \sec[e + fx]^2)^2 \right)
 \end{aligned}$$

**Problem 360: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{2a^2 (a+b)^{3/2} f} - \frac{b \tan[e + fx]}{2a (a+b) f (a + b + b \tan[e + fx]^2)}$$

Result (type 3, 240 leaves):

$$\left( (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^4 \right. \\ \left. \left( 2x(a + 2b + a \cos[2(e + fx)]) + (b(3a + 2b) \operatorname{ArcTan}[(\sec[fx] (\cos[2e] - i \sin[2e]) \right. \right. \\ \left. \left. (- (a + 2b) \sin[fx] + a \sin[2e + fx])]) / (2\sqrt{a+b} \sqrt{b(\cos[e] - i \sin[e])^4}) \right) \right. \\ \left. (a + 2b + a \cos[2(e + fx)]) (\cos[2e] - i \sin[2e]) \right) / \\ \left( (a + b)^{3/2} f \sqrt{b(\cos[e] - i \sin[e])^4} + \right. \\ \left. \frac{b((a + 2b) \sin[2e] - a \sin[2fx])}{(a + b) f (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) / (8a^2 (a + b \sec[e + fx]^2)^2)$$

**Problem 361: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + fx]^2}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$-\frac{x}{a^2} + \frac{b^{3/2} (5a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{5/2} f} - \\ \frac{(2a-b) \cot[e+fx]}{2a(a+b)^2 f} - \frac{b \cot[e+fx]}{2a(a+b) f (a+b+b \tan[e+fx]^2)}$$

Result (type 3, 564 leaves):

$$\begin{aligned}
 & - \frac{x (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4}{4a^2 (a + b \sec[e + fx]^2)^2} + \\
 & \left( (5a + 2b) (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left( - \left( \left( b^2 \operatorname{ArcTan}[\sec[fx]] \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
 & \quad \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) \right) / \\
 & \quad \left( 8a^2 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) + \left( i b^2 \operatorname{ArcTan} \left[ \right. \right. \\
 & \quad \left. \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
 & \quad \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) \right) / \\
 & \quad \left( 8a^2 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \left. \right) / \\
 & \left( (a+b)^2 (a + b \sec[e + fx]^2)^2 \right) + \left( (a + 2b + a \cos[2e + 2fx])^2 \right. \\
 & \quad \left. \operatorname{Csc}[e] \right. \\
 & \quad \left. \operatorname{Csc}[e + fx] \right. \\
 & \quad \left. \sec[e + fx]^4 \right. \\
 & \quad \left. \sin[fx] \right) / \left( 4 \right. \\
 & \quad \left. (a+b)^2 \right. \\
 & \quad \left. f \right. \\
 & \quad \left. (a + b \sec[e + fx]^2)^2 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^4 \right. \\
 & \quad \left. (-a b^2 \sin[2e] - 2b^3 \sin[2e] + a b^2 \sin[2fx]) \right) / \\
 & \left( 8a^2 (a+b)^2 f (a + b \sec[e + fx]^2)^2 (\cos[e] - \sin[e]) \right. \\
 & \quad \left. (\cos[e] + \sin[e]) \right)
 \end{aligned}$$

**Problem 362: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + fx]^4}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 160 leaves, 8 steps):

$$\frac{x}{a^2} - \frac{b^{5/2} (7a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{7/2} f} + \frac{(2a^2 + 6ab - b^2) \operatorname{Cot}[e+fx]}{2a (a+b)^3 f} - \frac{(2a - 3b) \operatorname{Cot}[e+fx]^3}{6a (a+b)^2 f} - \frac{b \operatorname{Cot}[e+fx]^3}{2a (a+b) f (a+b + b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 1896 leaves):

$$\left( (7a + 2b) (a + 2b + a \operatorname{Cos}[2e + 2fx])^2 \operatorname{Sec}[e + fx]^4 \left( \left( b^3 \operatorname{ArcTan}\left[ \operatorname{Sec}[fx] \left( \frac{\operatorname{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} \right) \right. \right. \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right) \operatorname{Cos}[2e] \right) \right] / \\ \left( 8a^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]} \right) - \left( i b^3 \operatorname{ArcTan}\left[ \operatorname{Sec}[fx] \left( \frac{\operatorname{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} \right) \right. \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right) \operatorname{Sin}[2e] \right) \right] / \\ \left( 8a^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]} \right) \right) \Bigg) / \\ \left( (a+b)^3 (a+b \operatorname{Sec}[e+fx]^2)^2 \right) + \frac{1}{384 a^2 (a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)^2} \\ (a + 2b + a \operatorname{Cos}[2e + 2fx]) \\ \operatorname{Csc}[ \\ e] \operatorname{Csc}[e + fx]^3 \operatorname{Sec}[ \\ 2e] \operatorname{Sec}[e + fx]^4 \\ (-6 a^4 f x \operatorname{Cos}[fx] - 54 a^3 b f x \operatorname{Cos}[fx] - 126 a^2 b^2 f x \operatorname{Cos}[fx] - \\ 114 a b^3 f x \operatorname{Cos}[fx] - 36 b^4 f x \operatorname{Cos}[fx] + 3 a^4 f x \operatorname{Cos}[3fx] - \\ 3 a^3 b f x \operatorname{Cos}[3fx] - 27 a^2 b^2 f x \operatorname{Cos}[3fx] - 33 a b^3 f x \operatorname{Cos}[3fx] - \\ 12 b^4 f x \operatorname{Cos}[3fx] + 6 a^4 f x \operatorname{Cos}[2e - fx] + 54 a^3 b f x \operatorname{Cos}[2e - fx] + \\ 126 a^2 b^2 f x \operatorname{Cos}[2e - fx] + 114 a b^3 f x \operatorname{Cos}[2e - fx] + 36 b^4 f x \operatorname{Cos}[2e - fx] + \\ 6 a^4 f x \operatorname{Cos}[2e + fx] + 54 a^3 b f x \operatorname{Cos}[2e + fx] + 126 a^2 b^2 f x \operatorname{Cos}[2e + fx] + \\ 114 a b^3 f x \operatorname{Cos}[2e + fx] + 36 b^4 f x \operatorname{Cos}[2e + fx] - 6 a^4 f x \operatorname{Cos}[4e + fx] - \\ 54 a^3 b f x \operatorname{Cos}[4e + fx] - 126 a^2 b^2 f x \operatorname{Cos}[4e + fx] - 114 a b^3 f x \operatorname{Cos}[4e + fx] - \\ 36 b^4 f x \operatorname{Cos}[4e + fx] - 3 a^4 f x \operatorname{Cos}[2e + 3fx] + 3 a^3 b f x \operatorname{Cos}[2e + 3fx] + \\ 27 a^2 b^2 f x \operatorname{Cos}[2e + 3fx] + 33 a b^3 f x \operatorname{Cos}[2e + 3fx] + 12 b^4 f x \operatorname{Cos}[2e + 3fx] + \\ 3 a^4 f x \operatorname{Cos}[4e + 3fx] - 3 a^3 b f x \operatorname{Cos}[4e + 3fx] - 27 a^2 b^2 f x \operatorname{Cos}[4e + 3fx] - \\ 33 a b^3 f x \operatorname{Cos}[4e + 3fx] - 12 b^4 f x \operatorname{Cos}[4e + 3fx] - 3 a^4 f x \operatorname{Cos}[6e + 3fx] + \\ 3 a^3 b f x \operatorname{Cos}[6e + 3fx] + 27 a^2 b^2 f x \operatorname{Cos}[6e + 3fx] + 33 a b^3 f x \operatorname{Cos}[6e + 3fx] + \\ 12 b^4 f x \operatorname{Cos}[6e + 3fx] - 3 a^4 f x \operatorname{Cos}[2e + 5fx] - 9 a^3 b f x \operatorname{Cos}[2e + 5fx] - \\ 9 a^2 b^2 f x \operatorname{Cos}[2e + 5fx] - 3 a b^3 f x \operatorname{Cos}[2e + 5fx] + 3 a^4 f x \operatorname{Cos}[4e + 5fx] + \\ 9 a^3 b f x \operatorname{Cos}[4e + 5fx] + 9 a^2 b^2 f x \operatorname{Cos}[4e + 5fx] + 3 a b^3 f x \operatorname{Cos}[4e + 5fx] - \\ 3 a^4 f x \operatorname{Cos}[6e + 5fx] - 9 a^3 b f x \operatorname{Cos}[6e + 5fx] - 9 a^2 b^2 f x \operatorname{Cos}[6e + 5fx] - \\ 3 a b^3 f x \operatorname{Cos}[6e + 5fx] + 3 a^4 f x \operatorname{Cos}[8e + 5fx] + 9 a^3 b f x \operatorname{Cos}[8e + 5fx] +$$

$$\begin{aligned}
 & 9 a^2 b^2 f x \operatorname{Cos}[8 e+5 f x]+3 a b^3 f x \operatorname{Cos}[8 e+5 f x]-12 a^4 \operatorname{Sin}[f x]-60 a^3 b \operatorname{Sin}[f x]- \\
 & 96 a^2 b^2 \operatorname{Sin}[f x]+18 b^4 \operatorname{Sin}[f x]+4 a^4 \operatorname{Sin}[3 f x]+36 a^3 b \operatorname{Sin}[3 f x]+ \\
 & 80 a^2 b^2 \operatorname{Sin}[3 f x]-6 a b^3 \operatorname{Sin}[3 f x]+6 b^4 \operatorname{Sin}[3 f x]+4 a^4 \operatorname{Sin}[2 e-f x]+ \\
 & 76 a^3 b \operatorname{Sin}[2 e-f x]+144 a^2 b^2 \operatorname{Sin}[2 e-f x]+18 b^4 \operatorname{Sin}[2 e-f x]-4 a^4 \operatorname{Sin}[2 e+f x]- \\
 & 76 a^3 b \operatorname{Sin}[2 e+f x]-144 a^2 b^2 \operatorname{Sin}[2 e+f x]+6 a b^3 \operatorname{Sin}[2 e+f x]+ \\
 & 18 b^4 \operatorname{Sin}[2 e+f x]-12 a^4 \operatorname{Sin}[4 e+f x]-60 a^3 b \operatorname{Sin}[4 e+f x]-96 a^2 b^2 \operatorname{Sin}[4 e+f x]- \\
 & 6 a b^3 \operatorname{Sin}[4 e+f x]-18 b^4 \operatorname{Sin}[4 e+f x]-12 a^4 \operatorname{Sin}[2 e+3 f x]-24 a^3 b \operatorname{Sin}[2 e+3 f x]+ \\
 & 6 a b^3 \operatorname{Sin}[2 e+3 f x]-6 b^4 \operatorname{Sin}[2 e+3 f x]+4 a^4 \operatorname{Sin}[4 e+3 f x]+36 a^3 b \operatorname{Sin}[4 e+3 f x]+ \\
 & 80 a^2 b^2 \operatorname{Sin}[4 e+3 f x]-3 a b^3 \operatorname{Sin}[4 e+3 f x]-6 b^4 \operatorname{Sin}[4 e+3 f x]- \\
 & 12 a^4 \operatorname{Sin}[6 e+3 f x]-24 a^3 b \operatorname{Sin}[6 e+3 f x]+3 a b^3 \operatorname{Sin}[6 e+3 f x]+ \\
 & 6 b^4 \operatorname{Sin}[6 e+3 f x]+8 a^4 \operatorname{Sin}[2 e+5 f x]+20 a^3 b \operatorname{Sin}[2 e+5 f x]+3 a b^3 \operatorname{Sin}[2 e+5 f x]- \\
 & 3 a b^3 \operatorname{Sin}[4 e+5 f x]+8 a^4 \operatorname{Sin}[6 e+5 f x]+20 a^3 b \operatorname{Sin}[6 e+5 f x]
 \end{aligned}$$

**Problem 363: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x]^2)^2} dx$$

Optimal (type 3, 207 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{x}{a^2} + \frac{b^{7/2} (9 a+2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{2 a^2 (a+b)^{9/2} f} - \frac{(2 a^3+8 a^2 b+12 a b^2-b^3) \operatorname{Cot}[e+f x]}{2 a (a+b)^4 f} + \\
 & \frac{(2 a^2+6 a b-3 b^2) \operatorname{Cot}[e+f x]^3}{6 a (a+b)^3 f} - \frac{(2 a-5 b) \operatorname{Cot}[e+f x]^5}{10 a (a+b)^2 f} - \frac{b \operatorname{Cot}[e+f x]^5}{2 a (a+b) f (a+b+b \operatorname{Tan}[e+f x]^2)}
 \end{aligned}$$

Result (type 3, 3028 leaves):

$$\begin{aligned}
 & \left( (9 a+2 b) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^2 \operatorname{Sec}[e+f x]^4 \right. \\
 & \left. - \left( \left( b^4 \operatorname{ArcTan}\left[\operatorname{Sec}[f x]\right] \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} \right) \right. \right. \right. \\
 & \left. \left. \left. (-a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \operatorname{Cos}[2 e] \right) / \right. \right. \\
 & \left. \left. \left( 8 a^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) \right) + \left( i b^4 \operatorname{ArcTan}\left[ \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sec}[f x] \left( \frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} \right) \right. \right. \right. \\
 & \left. \left. \left. (-a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \operatorname{Sin}[2 e] \right) / \right. \right. \\
 & \left. \left. \left( 8 a^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) \right) \right) /
 \end{aligned}$$

$$\left( (a+b)^4 (a+b \operatorname{Sec}[e+fx]^2)^2 \right) + \frac{1}{7680 a^2 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^2}$$

$$(a +$$

$$2 b + a \operatorname{Cos}[2 e + 2 f x])$$

$$\operatorname{Csc}[e] \operatorname{Csc}[e+fx]^5 \operatorname{Sec}[2 e]$$

$$\operatorname{Sec}[e+fx]^4$$

$$(75 a^5 f x \operatorname{Cos}[f x] + 900 a^4 b f x \operatorname{Cos}[f x] +$$

$$2850 a^3 b^2 f x \operatorname{Cos}[f x] + 3900 a^2 b^3 f x \operatorname{Cos}[f x] +$$

$$2475 a b^4 f x \operatorname{Cos}[f x] + 600 b^5 f x \operatorname{Cos}[f x] - 15 a^5 f x \operatorname{Cos}[3 f x] +$$

$$240 a^4 b f x \operatorname{Cos}[3 f x] + 1110 a^3 b^2 f x \operatorname{Cos}[3 f x] +$$

$$1740 a^2 b^3 f x \operatorname{Cos}[3 f x] + 1185 a b^4 f x \operatorname{Cos}[3 f x] +$$

$$300 b^5 f x \operatorname{Cos}[3 f x] - 75 a^5 f x \operatorname{Cos}[2 e - f x] -$$

$$900 a^4 b f x \operatorname{Cos}[2 e - f x] - 2850 a^3 b^2 f x \operatorname{Cos}[2 e - f x] -$$

$$3900 a^2 b^3 f x \operatorname{Cos}[2 e - f x] - 2475 a b^4 f x \operatorname{Cos}[2 e - f x] -$$

$$600 b^5 f x \operatorname{Cos}[2 e - f x] - 75 a^5 f x \operatorname{Cos}[2 e + f x] - 900 a^4 b f x \operatorname{Cos}[2 e + f x] -$$

$$2850 a^3 b^2 f x \operatorname{Cos}[2 e + f x] - 3900 a^2 b^3 f x \operatorname{Cos}[2 e + f x] -$$

$$2475 a b^4 f x \operatorname{Cos}[2 e + f x] - 600 b^5 f x \operatorname{Cos}[2 e + f x] +$$

$$75 a^5 f x \operatorname{Cos}[4 e + f x] + 900 a^4 b f x \operatorname{Cos}[4 e + f x] +$$

$$2850 a^3 b^2 f x \operatorname{Cos}[4 e + f x] + 3900 a^2 b^3 f x \operatorname{Cos}[4 e + f x] +$$

$$2475 a b^4 f x \operatorname{Cos}[4 e + f x] + 600 b^5 f x \operatorname{Cos}[4 e + f x] + 15 a^5 f x \operatorname{Cos}[2 e + 3 f x] -$$

$$240 a^4 b f x \operatorname{Cos}[2 e + 3 f x] - 1110 a^3 b^2 f x \operatorname{Cos}[2 e + 3 f x] -$$

$$1740 a^2 b^3 f x \operatorname{Cos}[2 e + 3 f x] - 1185 a b^4 f x \operatorname{Cos}[2 e + 3 f x] -$$

$$300 b^5 f x \operatorname{Cos}[2 e + 3 f x] - 15 a^5 f x \operatorname{Cos}[4 e + 3 f x] + 240 a^4 b f x \operatorname{Cos}[4 e + 3 f x] +$$

$$1110 a^3 b^2 f x \operatorname{Cos}[4 e + 3 f x] + 1740 a^2 b^3 f x \operatorname{Cos}[4 e + 3 f x] +$$

$$1185 a b^4 f x \operatorname{Cos}[4 e + 3 f x] + 300 b^5 f x \operatorname{Cos}[4 e + 3 f x] + 15 a^5 f x \operatorname{Cos}[6 e + 3 f x] -$$

$$240 a^4 b f x \operatorname{Cos}[6 e + 3 f x] - 1110 a^3 b^2 f x \operatorname{Cos}[6 e + 3 f x] -$$

$$1740 a^2 b^3 f x \operatorname{Cos}[6 e + 3 f x] - 1185 a b^4 f x \operatorname{Cos}[6 e + 3 f x] - 300 b^5 f x \operatorname{Cos}[6 e + 3 f x] +$$

$$45 a^5 f x \operatorname{Cos}[2 e + 5 f x] + 120 a^4 b f x \operatorname{Cos}[2 e + 5 f x] + 30 a^3 b^2 f x \operatorname{Cos}[2 e + 5 f x] -$$

$$180 a^2 b^3 f x \operatorname{Cos}[2 e + 5 f x] - 195 a b^4 f x \operatorname{Cos}[2 e + 5 f x] - 60 b^5 f x \operatorname{Cos}[2 e + 5 f x] -$$

$$45 a^5 f x \operatorname{Cos}[4 e + 5 f x] - 120 a^4 b f x \operatorname{Cos}[4 e + 5 f x] - 30 a^3 b^2 f x \operatorname{Cos}[4 e + 5 f x] +$$

$$180 a^2 b^3 f x \operatorname{Cos}[4 e + 5 f x] + 195 a b^4 f x \operatorname{Cos}[4 e + 5 f x] + 60 b^5 f x \operatorname{Cos}[4 e + 5 f x] +$$

$$45 a^5 f x \operatorname{Cos}[6 e + 5 f x] + 120 a^4 b f x \operatorname{Cos}[6 e + 5 f x] + 30 a^3 b^2 f x \operatorname{Cos}[6 e + 5 f x] -$$

$$180 a^2 b^3 f x \operatorname{Cos}[6 e + 5 f x] - 195 a b^4 f x \operatorname{Cos}[6 e + 5 f x] - 60 b^5 f x \operatorname{Cos}[6 e + 5 f x] -$$

$$45 a^5 f x \operatorname{Cos}[8 e + 5 f x] - 120 a^4 b f x \operatorname{Cos}[8 e + 5 f x] - 30 a^3 b^2 f x \operatorname{Cos}[8 e + 5 f x] +$$

$$180 a^2 b^3 f x \operatorname{Cos}[8 e + 5 f x] + 195 a b^4 f x \operatorname{Cos}[8 e + 5 f x] + 60 b^5 f x \operatorname{Cos}[8 e + 5 f x] -$$

$$15 a^5 f x \operatorname{Cos}[4 e + 7 f x] - 60 a^4 b f x \operatorname{Cos}[4 e + 7 f x] - 90 a^3 b^2 f x \operatorname{Cos}[4 e + 7 f x] -$$

$$60 a^2 b^3 f x \operatorname{Cos}[4 e + 7 f x] - 15 a b^4 f x \operatorname{Cos}[4 e + 7 f x] + 15 a^5 f x \operatorname{Cos}[6 e + 7 f x] +$$

$$60 a^4 b f x \operatorname{Cos}[6 e + 7 f x] + 90 a^3 b^2 f x \operatorname{Cos}[6 e + 7 f x] + 60 a^2 b^3 f x \operatorname{Cos}[6 e + 7 f x] +$$

$$15 a b^4 f x \operatorname{Cos}[6 e + 7 f x] - 15 a^5 f x \operatorname{Cos}[8 e + 7 f x] - 60 a^4 b f x \operatorname{Cos}[8 e + 7 f x] -$$

$$90 a^3 b^2 f x \operatorname{Cos}[8 e + 7 f x] - 60 a^2 b^3 f x \operatorname{Cos}[8 e + 7 f x] - 15 a b^4 f x \operatorname{Cos}[8 e + 7 f x] +$$

$$15 a^5 f x \operatorname{Cos}[10 e + 7 f x] + 60 a^4 b f x \operatorname{Cos}[10 e + 7 f x] + 90 a^3 b^2 f x \operatorname{Cos}[10 e + 7 f x] +$$

$$60 a^2 b^3 f x \operatorname{Cos}[10 e + 7 f x] + 15 a b^4 f x \operatorname{Cos}[10 e + 7 f x] - 10 a^5 \operatorname{Sin}[f x] +$$

$$860 a^4 b \operatorname{Sin}[f x] + 3120 a^3 b^2 \operatorname{Sin}[f x] + 3600 a^2 b^3 \operatorname{Sin}[f x] - 300 b^5 \operatorname{Sin}[f x] +$$

$$46 a^5 \operatorname{Sin}[3 f x] - 508 a^4 b \operatorname{Sin}[3 f x] - 2324 a^3 b^2 \operatorname{Sin}[3 f x] - 3120 a^2 b^3 \operatorname{Sin}[3 f x] +$$

$$75 a b^4 \operatorname{Sin}[3 f x] - 150 b^5 \operatorname{Sin}[3 f x] - 240 a^5 \operatorname{Sin}[2 e - f x] - 1840 a^4 b \operatorname{Sin}[2 e - f x] -$$

$$4840 a^3 b^2 \operatorname{Sin}[2 e - f x] - 5040 a^2 b^3 \operatorname{Sin}[2 e - f x] - 300 b^5 \operatorname{Sin}[2 e - f x] +$$

$$240 a^5 \operatorname{Sin}[2 e + f x] + 1840 a^4 b \operatorname{Sin}[2 e + f x] + 4840 a^3 b^2 \operatorname{Sin}[2 e + f x] +$$

$$5040 a^2 b^3 \operatorname{Sin}[2 e + f x] - 75 a b^4 \operatorname{Sin}[2 e + f x] - 300 b^5 \operatorname{Sin}[2 e + f x] - 10 a^5 \operatorname{Sin}[4 e + f x] +$$

$$860 a^4 b \operatorname{Sin}[4 e + f x] + 3120 a^3 b^2 \operatorname{Sin}[4 e + f x] + 3600 a^2 b^3 \operatorname{Sin}[4 e + f x] +$$

$$75 a b^4 \operatorname{Sin}[4 e + f x] + 300 b^5 \operatorname{Sin}[4 e + f x] - 240 a^4 b \operatorname{Sin}[2 e + 3 f x] -$$

$$\begin{aligned}
 & 900 a^3 b^2 \sin[2 e + 3 f x] - 1200 a^2 b^3 \sin[2 e + 3 f x] - 75 a b^4 \sin[2 e + 3 f x] + \\
 & 150 b^5 \sin[2 e + 3 f x] + 46 a^5 \sin[4 e + 3 f x] - 508 a^4 b \sin[4 e + 3 f x] - \\
 & 2324 a^3 b^2 \sin[4 e + 3 f x] - 3120 a^2 b^3 \sin[4 e + 3 f x] + 60 a b^4 \sin[4 e + 3 f x] + \\
 & 150 b^5 \sin[4 e + 3 f x] - 240 a^4 b \sin[6 e + 3 f x] - 900 a^3 b^2 \sin[6 e + 3 f x] - \\
 & 1200 a^2 b^3 \sin[6 e + 3 f x] - 60 a b^4 \sin[6 e + 3 f x] - 150 b^5 \sin[6 e + 3 f x] - \\
 & 48 a^5 \sin[2 e + 5 f x] - 32 a^4 b \sin[2 e + 5 f x] + 340 a^3 b^2 \sin[2 e + 5 f x] + \\
 & 864 a^2 b^3 \sin[2 e + 5 f x] - 60 a b^4 \sin[2 e + 5 f x] + 30 b^5 \sin[2 e + 5 f x] - \\
 & 90 a^5 \sin[4 e + 5 f x] - 300 a^4 b \sin[4 e + 5 f x] - 300 a^3 b^2 \sin[4 e + 5 f x] + \\
 & 60 a b^4 \sin[4 e + 5 f x] - 30 b^5 \sin[4 e + 5 f x] - 48 a^5 \sin[6 e + 5 f x] - \\
 & 32 a^4 b \sin[6 e + 5 f x] + 340 a^3 b^2 \sin[6 e + 5 f x] + 864 a^2 b^3 \sin[6 e + 5 f x] - \\
 & 15 a b^4 \sin[6 e + 5 f x] - 30 b^5 \sin[6 e + 5 f x] - 90 a^5 \sin[8 e + 5 f x] - \\
 & 300 a^4 b \sin[8 e + 5 f x] - 300 a^3 b^2 \sin[8 e + 5 f x] + 15 a b^4 \sin[8 e + 5 f x] + \\
 & 30 b^5 \sin[8 e + 5 f x] + 46 a^5 \sin[4 e + 7 f x] + 172 a^4 b \sin[4 e + 7 f x] + \\
 & 216 a^3 b^2 \sin[4 e + 7 f x] + 15 a b^4 \sin[4 e + 7 f x] - 15 a b^4 \sin[6 e + 7 f x] + \\
 & 46 a^5 \sin[8 e + 7 f x] + 172 a^4 b \sin[8 e + 7 f x] + 216 a^3 b^2 \sin[8 e + 7 f x]
 \end{aligned}$$

**Problem 367: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[e + f x]}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{b^3}{4 a^3 (a + b) f (b + a \cos[e + f x]^2)^2} + \frac{b^2 (3 a + 2 b)}{2 a^3 (a + b)^2 f (b + a \cos[e + f x]^2)} + \\
 & \frac{b (3 a^2 + 3 a b + b^2) \operatorname{Log}[b + a \cos[e + f x]^2]}{2 a^3 (a + b)^3 f} + \frac{\operatorname{Log}[\sin[e + f x]]}{(a + b)^3 f}
 \end{aligned}$$

Result (type 3, 253 leaves):

$$\begin{aligned}
 & \frac{1}{32 a^3 (a + b)^3 f (a + b \sec[e + f x]^2)^3} \\
 & (a + 2 b + a \cos[2(e + f x)]) (-4 b^3 (a + b)^2 + 4 b^2 (a + b) (3 a + 2 b) (a + 2 b + a \cos[2(e + f x)])) + \\
 & 4 i b (3 a^2 + 3 a b + b^2) f x (a + 2 b + a \cos[2(e + f x)])^2 - \\
 & 2 i b (3 a^2 + 3 a b + b^2) \operatorname{ArcTan}[\tan[2(e + f x)]] (a + 2 b + a \cos[2(e + f x)])^2 + \\
 & b (3 a^2 + 3 a b + b^2) (a + 2 b + a \cos[2(e + f x)])^2 \operatorname{Log}[(a + 2 b + a \cos[2(e + f x)])^2] + \\
 & 4 a^3 (a + 2 b + a \cos[2(e + f x)])^2 \operatorname{Log}[\sin[e + f x]] \sec[e + f x]^6
 \end{aligned}$$

**Problem 368: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^3}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$\frac{b^4}{4 a^3 (a+b)^2 f (b+a \operatorname{Cos}[e+f x]^2)^2} - \frac{b^3 (2 a+b)}{a^3 (a+b)^3 f (b+a \operatorname{Cos}[e+f x]^2)} -$$

$$\frac{\operatorname{Csc}[e+f x]^2}{2 (a+b)^3 f} - \frac{b^2 (6 a^2+4 a b+b^2) \operatorname{Log}[b+a \operatorname{Cos}[e+f x]^2]}{2 a^3 (a+b)^4 f} - \frac{(a+4 b) \operatorname{Log}[\operatorname{Sin}[e+f x]]}{(a+b)^4 f}$$

Result (type 3, 1045 leaves):

$$\frac{b^4 (a+2 b+a \operatorname{Cos}[2 e+2 f x]) \operatorname{Sec}[e+f x]^6}{8 a^3 (a+b)^2 f (a+b \operatorname{Sec}[e+f x]^2)^3} - \frac{b^3 (2 a+b) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^2 \operatorname{Sec}[e+f x]^6}{4 a^3 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} -$$

$$\left( (-6 a^2 b^2 - 4 a b^3 - b^4) \operatorname{ArcTan}[\operatorname{Tan}[2 e+2 f x]] (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \right) /$$

$$\left( 16 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3 \right) - \frac{(a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Csc}[e+f x]^2 \operatorname{Sec}[e+f x]^6}{16 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} +$$

$$\left( (-6 a^2 b^2 - 4 a b^3 - b^4) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Log}[(a+2 b+a \operatorname{Cos}[2 e+2 f x])^2] \right.$$

$$\left. \operatorname{Sec}[e+f x]^6 \right) / \left( 32 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3 \right) +$$

$$\left( (-a-4 b) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Log}[\operatorname{Sin}[e+f x]] \operatorname{Sec}[e+f x]^6 \right) /$$

$$\left( 8 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3 \right) +$$

$$\frac{1}{(a+b \operatorname{Sec}[e+f x]^2)^3} x (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6$$

$$\left( \frac{a \operatorname{Cot}[e]}{8 (a+b)^4} + \frac{b \operatorname{Cot}[e]}{2 (a+b)^4} - \frac{3 i b^2 \operatorname{Cos}[e]^2}{4 a (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} - \frac{i b^3 \operatorname{Cos}[e]^2}{2 a^2 (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} - \right.$$

$$\frac{i b^4 \operatorname{Cos}[e]^2}{8 a^3 (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} - \frac{3 b^2 \operatorname{Cos}[e] \operatorname{Sin}[e]}{2 a (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} -$$

$$\frac{b^3 \operatorname{Cos}[e] \operatorname{Sin}[e]}{a^2 (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} - \frac{b^4 \operatorname{Cos}[e] \operatorname{Sin}[e]}{4 a^3 (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} +$$

$$\frac{3 i b^2 \operatorname{Sin}[e]^2}{4 a (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} + \frac{i b^3 \operatorname{Sin}[e]^2}{2 a^2 (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} +$$

$$\frac{i b^4 \operatorname{Sin}[e]^2}{8 a^3 (a+b)^4 (\operatorname{Cos}[e]^2 - \operatorname{Sin}[e]^2)} - \frac{i (a+a \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e])}{8 (a+b)^4 (-1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e])} -$$

$$\frac{i (b+b \operatorname{Cos}[2 e] + i b \operatorname{Sin}[2 e])}{2 (a+b)^4 (-1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e])} - \frac{3 i (-b^2 + b^2 \operatorname{Cos}[4 e] + i b^2 \operatorname{Sin}[4 e])}{4 a (a+b)^4 (1 + \operatorname{Cos}[4 e] + i \operatorname{Sin}[4 e])} -$$

$$\left. \frac{i (-b^3 + b^3 \operatorname{Cos}[4 e] + i b^3 \operatorname{Sin}[4 e])}{2 a^2 (a+b)^4 (1 + \operatorname{Cos}[4 e] + i \operatorname{Sin}[4 e])} - \frac{i (-b^4 + b^4 \operatorname{Cos}[4 e] + i b^4 \operatorname{Sin}[4 e])}{8 a^3 (a+b)^4 (1 + \operatorname{Cos}[4 e] + i \operatorname{Sin}[4 e])} \right)$$

**Problem 369: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+f x]^5}{(a+b \operatorname{Sec}[e+f x]^2)^3} dx$$



Optimal (type 3, 192 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{b^5}{4 a^3 (a+b)^3 f (b+a \operatorname{Cos}[e+f x]^2)^2} + \\
 & \frac{b^4 (5 a+2 b)}{2 a^3 (a+b)^4 f (b+a \operatorname{Cos}[e+f x]^2)} + \frac{(2 a+5 b) \operatorname{Csc}[e+f x]^2}{2 (a+b)^4 f} - \frac{\operatorname{Csc}[e+f x]^4}{4 (a+b)^3 f} + \\
 & \frac{b^3 (10 a^2+5 a b+b^2) \operatorname{Log}[b+a \operatorname{Cos}[e+f x]^2]}{2 a^3 (a+b)^5 f} + \frac{(a^2+5 a b+10 b^2) \operatorname{Log}[\operatorname{Sin}[e+f x]]}{(a+b)^5 f}
 \end{aligned}$$

Result (type 3, 1286 leaves):

$$\begin{aligned}
 & - \frac{b^5 (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^6}{8a^3 (a + b)^3 f (a + b \sec[e + fx]^2)^3} + \\
 & \frac{b^4 (5a + 2b) (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6}{8a^3 (a + b)^4 f (a + b \sec[e + fx]^2)^3} - \\
 & \left( \frac{i (a^2 + 5ab + 10b^2) \operatorname{ArcTan}[\tan[e + fx]] (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{(8(a + b)^5 f (a + b \sec[e + fx]^2)^3)} - \right. \\
 & \left. \frac{i (10a^2 b^3 + 5ab^4 + b^5) \operatorname{ArcTan}[\tan[2e + 2fx]] (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{(16a^3 (a + b)^5 f (a + b \sec[e + fx]^2)^3)} + \right. \\
 & \left. \frac{(2a + 5b) (a + 2b + a \cos[2e + 2fx])^3 \operatorname{Csc}[e + fx]^2 \sec[e + fx]^6}{16(a + b)^4 f (a + b \sec[e + fx]^2)^3} - \right. \\
 & \left. \frac{(a + 2b + a \cos[2e + 2fx])^3 \operatorname{Csc}[e + fx]^4 \sec[e + fx]^6}{32(a + b)^3 f (a + b \sec[e + fx]^2)^3} + \right. \\
 & \left. \frac{((10a^2 b^3 + 5ab^4 + b^5) (a + 2b + a \cos[2e + 2fx])^3 \operatorname{Log}[(a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6])}{(32a^3 (a + b)^5 f (a + b \sec[e + fx]^2)^3)} + \right. \\
 & \left. \frac{((a^2 + 5ab + 10b^2) (a + 2b + a \cos[2e + 2fx])^3 \operatorname{Log}[\sin[e + fx]^2 \sec[e + fx]^6])}{(16(a + b)^5 f (a + b \sec[e + fx]^2)^3)} + \right. \\
 & \left. \frac{1}{(a + b \sec[e + fx]^2)^3} x (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \frac{i a^2}{8(a + b)^5} + \frac{5 i a b}{8(a + b)^5} + \right. \right. \\
 & \frac{5 i b^2}{4(a + b)^5} - \frac{a^2 \cot[e]}{8(a + b)^5} - \frac{5 a b \cot[e]}{8(a + b)^5} - \frac{5 b^2 \cot[e]}{4(a + b)^5} + \frac{5 i b^3 \cos[e]^2}{4a(a + b)^5 (\cos[e]^2 - \sin[e]^2)} + \\
 & \frac{5 i b^4 \cos[e]^2}{8a^2(a + b)^5 (\cos[e]^2 - \sin[e]^2)} + \frac{i b^5 \cos[e]^2}{8a^3(a + b)^5 (\cos[e]^2 - \sin[e]^2)} + \\
 & \frac{5 b^3 \cos[e] \sin[e]}{2a(a + b)^5 (\cos[e]^2 - \sin[e]^2)} + \frac{5 b^4 \cos[e] \sin[e]}{4a^2(a + b)^5 (\cos[e]^2 - \sin[e]^2)} + \\
 & \frac{b^5 \cos[e] \sin[e]}{4a^3(a + b)^5 (\cos[e]^2 - \sin[e]^2)} - \frac{5 i b^3 \sin[e]^2}{4a(a + b)^5 (\cos[e]^2 - \sin[e]^2)} - \\
 & \frac{5 i b^4 \sin[e]^2}{8a^2(a + b)^5 (\cos[e]^2 - \sin[e]^2)} - \frac{i b^5 \sin[e]^2}{8a^3(a + b)^5 (\cos[e]^2 - \sin[e]^2)} + \\
 & \left. \left. \frac{(i (a^2 + 5ab + a^2 \cos[2e] + 5ab \cos[2e] + i a^2 \sin[2e] + 5 i a b \sin[2e]))}{(8(a + b)^5 (-1 + \cos[2e] + i \sin[2e]))} + \right. \right. \\
 & \frac{5 i (b^2 + b^2 \cos[2e] + i b^2 \sin[2e])}{4(a + b)^5 (-1 + \cos[2e] + i \sin[2e])} + \frac{5 i (-b^3 + b^3 \cos[4e] + i b^3 \sin[4e])}{4a(a + b)^5 (1 + \cos[4e] + i \sin[4e])} + \\
 & \left. \left. \frac{5 i (-b^4 + b^4 \cos[4e] + i b^4 \sin[4e])}{8a^2(a + b)^5 (1 + \cos[4e] + i \sin[4e])} + \frac{i (-b^5 + b^5 \cos[4e] + i b^5 \sin[4e])}{8a^3(a + b)^5 (1 + \cos[4e] + i \sin[4e])} \right) \right)
 \end{aligned}$$

Problem 370: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 147 leaves, 7 steps):

$$-\frac{x}{a^3} + \frac{\sqrt{a+b} (3a^2 - 4ab + 8b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8a^3 b^{5/2} f} - \frac{(a+b) \text{Tan}[e+fx]^3}{4abf(a+b+b \text{Tan}[e+fx]^2)^2} - \frac{(3a-4b)(a+b) \text{Tan}[e+fx]}{8a^2 b^2 f(a+b+b \text{Tan}[e+fx]^2)}$$

Result (type 3, 760 leaves):

$$\left( (-3a^3 + a^2b - 4ab^2 - 8b^3) (a + 2b + a \text{Cos}[2e + 2fx])^3 \text{Sec}[e + fx]^6 \left( \left( \text{ArcTan}\left[ \frac{\text{Sec}[fx] \left( \frac{\text{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \text{Cos}[4e] - i b \text{Sin}[4e]} - \frac{i \text{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \text{Cos}[4e] - i b \text{Sin}[4e]}} \right)}{(-a \text{Sin}[fx] - 2b \text{Sin}[fx] + a \text{Sin}[2e + fx]) \text{Cos}[2e]} \right]} \right) / \right. \right. \\ \left. \left. \left( 64a^3 b^2 \sqrt{a+b} f \sqrt{b \text{Cos}[4e] - i b \text{Sin}[4e]} \right) - \left( i \text{ArcTan}\left[ \frac{\text{Sec}[fx] \left( \frac{\text{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \text{Cos}[4e] - i b \text{Sin}[4e]} - \frac{i \text{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \text{Cos}[4e] - i b \text{Sin}[4e]}} \right)}{(-a \text{Sin}[fx] - 2b \text{Sin}[fx] + a \text{Sin}[2e + fx]) \text{Sin}[2e]} \right]} \right) / \right. \right. \\ \left. \left. \left( 64a^3 b^2 \sqrt{a+b} f \sqrt{b \text{Cos}[4e] - i b \text{Sin}[4e]} \right) \right) \right) / \\ (a + b \text{Sec}[e + fx]^2)^3 + \frac{1}{128a^3 b^2 f (a + b \text{Sec}[e + fx]^2)^3} \\ (a + 2b + a \text{Cos}[2e + 2fx]) \\ \text{Sec}[ \\ 2e] \text{Sec}[e + fx]^6 \\ (-24a^2 b^2 f x \text{Cos}[2e] - 64ab^3 f x \text{Cos}[2e] - 64b^4 f x \text{Cos}[2e] - \\ 16a^2 b^2 f x \text{Cos}[2fx] - 32ab^3 f x \text{Cos}[2fx] - 16a^2 b^2 f x \text{Cos}[4e + 2fx] - \\ 32ab^3 f x \text{Cos}[4e + 2fx] - 4a^2 b^2 f x \text{Cos}[2e + 4fx] - \\ 4a^2 b^2 f x \text{Cos}[6e + 4fx] + 9a^4 \text{Sin}[2e] + 15a^3 b \text{Sin}[2e] - \\ 18a^2 b^2 \text{Sin}[2e] - 72ab^3 \text{Sin}[2e] - 48b^4 \text{Sin}[2e] - 9a^4 \text{Sin}[2fx] - \\ 13a^3 b \text{Sin}[2fx] + 28a^2 b^2 \text{Sin}[2fx] + 32ab^3 \text{Sin}[2fx] + 3a^4 \text{Sin}[4e + 2fx] - \\ a^3 b \text{Sin}[4e + 2fx] - 20a^2 b^2 \text{Sin}[4e + 2fx] - 16ab^3 \text{Sin}[4e + 2fx] - \\ 3a^4 \text{Sin}[2e + 4fx] + 3a^3 b \text{Sin}[2e + 4fx] + 6a^2 b^2 \text{Sin}[2e + 4fx])$$

**Problem 371: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 137 leaves, 7 steps):

$$\frac{x}{a^3} + \frac{(a^2 - 4 a b - 8 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 b^{3/2} \sqrt{a + b} f} - \frac{(a + b) \text{Tan}[e + f x]}{4 a b f (a + b + b \text{Tan}[e + f x]^2)^2} + \frac{(a - 4 b) \text{Tan}[e + f x]}{8 a^2 b f (a + b + b \text{Tan}[e + f x]^2)}$$

Result (type 3, 1744 leaves):

$$\left( (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \left( \frac{(3 a^2 + 8 a b + 8 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{(a + b)^{5/2}} - \frac{(a \sqrt{b} (3 a^2 + 16 a b + 16 b^2 + 3 a (a + 2 b) \text{Cos}[2 (e + f x)]) \text{Sin}[2 (e + f x)])}{((a + b)^2 (a + 2 b + a \text{Cos}[2 (e + f x)])^2)} \right) \right) / (1024 b^{5/2} f (a + b \text{Sec}[e + f x]^2)^3) - \left( (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \left( - \frac{3 a (a + 2 b) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{(a + b)^{5/2}} + \frac{(\sqrt{b} (3 a^3 + 14 a^2 b + 24 a b^2 + 16 b^3 + a (3 a^2 + 4 a b + 4 b^2) \text{Cos}[2 (e + f x)]) \text{Sin}[2 (e + f x)])}{((a + b)^2 (a + 2 b + a \text{Cos}[2 (e + f x)])^2)} \right) \right) / (2048 b^{5/2} f (a + b \text{Sec}[e + f x]^2)^3) + \frac{1}{32 (a + b \text{Sec}[e + f x]^2)^3} (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \left( \frac{1}{(a + b)^2} (3 a^5 - 10 a^4 b + 80 a^3 b^2 + 480 a^2 b^3 + 640 a b^4 + 256 b^5) \left( \left( \text{ArcTan}[\text{Sec}[f x]] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) - a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x] \right) \text{Cos}[2 e] \right) \right) / (64 a^3 b^2 \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}) - (i \text{ArcTan}[\text{Sec}[f x]]$$

$$\begin{aligned}
 & \left( \frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e]-ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e]-ib\sin[4e]}} \right) \\
 & \left( -a\sin[fx] - 2b\sin[fx] + a\sin[2e+fx] \right) \sin[2e] \Bigg/ \left( 64a^3b^2\sqrt{a+b}f \right. \\
 & \left. \sqrt{b\cos[4e]-ib\sin[4e]} \right) + \frac{1}{128a^3b^2(a+b)^2f(a+2b+a\cos[2e+2fx])^2} \\
 & \text{Sec}[2e] \left( 768a^4b^2fx\cos[2e] + 3584a^3b^3fx\cos[2e] + 6912a^2b^4fx\cos[2e] + \right. \\
 & 6144ab^5fx\cos[2e] + 2048b^6fx\cos[2e] + 512a^4b^2fx\cos[2fx] + \\
 & 2048a^3b^3fx\cos[2fx] + 2560a^2b^4fx\cos[2fx] + 1024ab^5fx\cos[2fx] + \\
 & 512a^4b^2fx\cos[4e+2fx] + 2048a^3b^3fx\cos[4e+2fx] + 2560a^2b^4fx\cos[4e+2fx] + \\
 & 1024ab^5fx\cos[4e+2fx] + 128a^4b^2fx\cos[2e+4fx] + 256a^3b^3fx\cos[2e+4fx] + \\
 & 128a^2b^4fx\cos[2e+4fx] + 128a^4b^2fx\cos[6e+4fx] + 256a^3b^3fx\cos[6e+4fx] + \\
 & 128a^2b^4fx\cos[6e+4fx] - 9a^6\sin[2e] + 12a^5b\sin[2e] + 684a^4b^2\sin[2e] + \\
 & 2880a^3b^3\sin[2e] + 5280a^2b^4\sin[2e] + 4608ab^5\sin[2e] + 1536b^6\sin[2e] + \\
 & 9a^6\sin[2fx] - 14a^5b\sin[2fx] - 608a^4b^2\sin[2fx] - 2112a^3b^3\sin[2fx] - \\
 & 2560a^2b^4\sin[2fx] - 1024ab^5\sin[2fx] - 3a^6\sin[4e+2fx] + 10a^5b\sin[4e+2fx] + \\
 & 304a^4b^2\sin[4e+2fx] + 1056a^3b^3\sin[4e+2fx] + 1280a^2b^4\sin[4e+2fx] + \\
 & 512ab^5\sin[4e+2fx] + 3a^6\sin[2e+4fx] - 12a^5b\sin[2e+4fx] - \\
 & \left. 204a^4b^2\sin[2e+4fx] - 384a^3b^3\sin[2e+4fx] - 192a^2b^4\sin[2e+4fx] \right) - \\
 & \left( (a+2b+a\cos[2e+2fx])^3 \text{Sec}[e+fx]^6 \left( - \left( \left( 6a^2 \text{ArcTan}[(\text{Sec}[fx](\cos[2e]-i\sin[2e]) \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left. - (a+2b)\sin[fx] + a\sin[2e+fx] \right) \right) \right) \right) \right) \Bigg/ \left( 2\sqrt{a+b}\sqrt{b(\cos[e]-i\sin[e])^4} \right) \Bigg] \\
 & \left( \cos[2e]-i\sin[2e] \right) \Bigg/ \left( \sqrt{a+b}\sqrt{b(\cos[e]-i\sin[e])^4} \right) \Bigg) + \\
 & (a\text{Sec}[2e] \left( (-9a^4-16a^3b+48a^2b^2+128ab^3+64b^4)\sin[2fx] + a(-3a^3+2a^2b+24a \right. \\
 & \left. b^2+16b^3)\sin[2(e+2fx)] + (3a^4-64a^2b^2-128ab^3-64b^4)\sin[4e+2fx] \right) + \\
 & \left. (9a^5+18a^4b-64a^3b^2-256a^2b^3-320ab^4-128b^5)\tan[2e] \right) \Bigg/ \\
 & \left. \left( a^2(a+2b+a\cos[2(e+fx)])^2 \right) \right) \Bigg/ \left( 2048b^2(a+b)^2f(a+b\text{Sec}[e+fx]^2)^3 \right)
 \end{aligned}$$

**Problem 372: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e+fx]^2}{(a+b\sec[e+fx]^2)^3} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{x}{a^3} + \frac{(3a^2+12ab+8b^2)\text{ArcTan}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b}}\right]}{8a^3\sqrt{b}(a+b)^{3/2}f} + \\
 & \frac{\tan[e+fx]}{4af(a+b+b\tan[e+fx]^2)^2} + \frac{(3a+4b)\tan[e+fx]}{8a^2(a+b)f(a+b+b\tan[e+fx]^2)}
 \end{aligned}$$

Result (type 3, 1745 leaves):

$$\begin{aligned}
 & \left( (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \right. \right. \\
 & \quad \left. \left. (a\sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \cos[2(e+fx)]) \sin[2(e+fx)]) \right) / \right. \\
 & \quad \left. \left. \left( (a+b)^2 (a+2b+a \cos[2(e+fx)])^2 \right) \right) \right) / \left( 1024 b^{5/2} f (a+b \sec[e+fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( - \frac{3a(a+2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \right. \right. \\
 & \quad \left. \left. (\sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \cos[2(e+fx)]) \sin[2(e+fx)]) \right) / \right. \\
 & \quad \left. \left. \left( (a+b)^2 (a+2b+a \cos[2(e+fx)])^2 \right) \right) \right) / \\
 & \left( 2048 b^{5/2} f (a+b \sec[e+fx]^2)^3 \right) + \frac{1}{32 (a+b \sec[e+fx]^2)^3} \\
 & (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \\
 & \left( - \frac{1}{(a+b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right. \\
 & \quad \left. \left( \left( \operatorname{ArcTan}[\sec[fx]] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{ib \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) (-a \sin[fx] - 2b \sin[fx] + \right. \right. \\
 & \quad \left. \left. a \sin[2e + fx]) \right) \cos[2e] \right) / \left( 64 a^3 b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]} \right) - \\
 & \quad \left( ib \operatorname{ArcTan}[\sec[fx]] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \right. \right. \\
 & \quad \left. \left. \frac{ib \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) (-a \sin[fx] - 2b \sin[fx] + \right. \\
 & \quad \left. a \sin[2e + fx]) \right) \sin[2e] \right) / \left( 64 a^3 b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]} \right) \right) - \\
 & \frac{1}{128 a^3 b^2 (a+b)^2 f (a+2b+a \cos[2e+2fx])^2} \sec[2e] (768 a^4 b^2 f x \cos[2e] + \\
 & 3584 a^3 b^3 f x \cos[2e] + 6912 a^2 b^4 f x \cos[2e] + 6144 a b^5 f x \cos[2e] + \\
 & 2048 b^6 f x \cos[2e] + 512 a^4 b^2 f x \cos[2fx] + 2048 a^3 b^3 f x \cos[2fx] + \\
 & 2560 a^2 b^4 f x \cos[2fx] + 1024 a b^5 f x \cos[2fx] + 512 a^4 b^2 f x \cos[4e+2fx] + \\
 & 2048 a^3 b^3 f x \cos[4e+2fx] + 2560 a^2 b^4 f x \cos[4e+2fx] + 1024 a b^5 f x \cos[4e+2fx] + \\
 & 128 a^4 b^2 f x \cos[2e+4fx] + 256 a^3 b^3 f x \cos[2e+4fx] + 128 a^2 b^4 f x \cos[2e+4fx] + \\
 & 128 a^4 b^2 f x \cos[6e+4fx] + 256 a^3 b^3 f x \cos[6e+4fx] + 128 a^2 b^4 f x \cos[6e+4fx] -
 \end{aligned}$$

$$\begin{aligned}
 & 9 a^6 \sin [2 e] + 12 a^5 b \sin [2 e] + 684 a^4 b^2 \sin [2 e] + 2880 a^3 b^3 \sin [2 e] + \\
 & 5280 a^2 b^4 \sin [2 e] + 4608 a b^5 \sin [2 e] + 1536 b^6 \sin [2 e] + 9 a^6 \sin [2 f x] - \\
 & 14 a^5 b \sin [2 f x] - 608 a^4 b^2 \sin [2 f x] - 2112 a^3 b^3 \sin [2 f x] - 2560 a^2 b^4 \sin [2 f x] - \\
 & 1024 a b^5 \sin [2 f x] - 3 a^6 \sin [4 e + 2 f x] + 10 a^5 b \sin [4 e + 2 f x] + \\
 & 304 a^4 b^2 \sin [4 e + 2 f x] + 1056 a^3 b^3 \sin [4 e + 2 f x] + 1280 a^2 b^4 \sin [4 e + 2 f x] + \\
 & 512 a b^5 \sin [4 e + 2 f x] + 3 a^6 \sin [2 e + 4 f x] - 12 a^5 b \sin [2 e + 4 f x] - \\
 & 204 a^4 b^2 \sin [2 e + 4 f x] - 384 a^3 b^3 \sin [2 e + 4 f x] - 192 a^2 b^4 \sin [2 e + 4 f x] \Big) - \\
 & \left( (a + 2 b + a \cos [2 e + 2 f x])^3 \sec [e + f x]^6 \right. \\
 & \left. - \left( \left( 6 a^2 \operatorname{ArcTan} \left[ (\sec [f x] (\cos [2 e] - i \sin [2 e])) (- (a + 2 b) \sin [f x] + a \sin [2 e + f x]) \right] \right) / \right. \right. \\
 & \left. \left( 2 \sqrt{a + b} \sqrt{b (\cos [e] - i \sin [e])^4} \right) \right) \\
 & \left. (\cos [2 e] - i \sin [2 e]) \right) / \left( \sqrt{a + b} \sqrt{b (\cos [e] - i \sin [e])^4} \right) \Big) + \\
 & (a \sec [2 e] \left( (-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \sin [2 f x] + a (-3 a^3 + 2 a^2 b + 24 a \right. \\
 & \left. b^2 + 16 b^3) \sin [2 (e + 2 f x)] + (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \sin [4 e + 2 f x] \right) + \\
 & (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \tan [2 e] \Big) / \\
 & \left( a^2 (a + 2 b + a \cos [2 (e + f x)])^2 \right) \Big) / \left( 2048 b^2 (a + b)^2 f (a + b \sec [e + f x])^3 \right)
 \end{aligned}$$

**Problem 373: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec [e + f x])^3} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\begin{aligned}
 & \frac{x}{a^3} - \frac{\sqrt{b} (15 a^2 + 20 a b + 8 b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan [e + f x]}{\sqrt{a + b}} \right]}{8 a^3 (a + b)^{5/2} f} - \\
 & \frac{b \tan [e + f x]}{4 a (a + b) f (a + b + b \tan [e + f x])^2} - \frac{b (7 a + 4 b) \tan [e + f x]}{8 a^2 (a + b)^2 f (a + b + b \tan [e + f x])^2}
 \end{aligned}$$

Result (type 3, 627 leaves):

$$\begin{aligned} & \frac{x (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{8a^3 (a + b \sec[e + fx]^2)^3} + \\ & \left( (15a^2 + 20ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( \left( b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b}} - \frac{i \sin[2e]}{2\sqrt{a+b}} \right] \right. \right. \right. \\ & \left. \left. \left. \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\ & \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) \right) / \\ & \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \left( i b \operatorname{ArcTan} \left[ \frac{\cos[2e]}{2\sqrt{a+b}} - \frac{i \sin[2e]}{2\sqrt{a+b}} \right] \right. \\ & \left. \left. \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \\ & \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \\ & \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \Big) / \left( (a+b)^2 (a+b \sec[e + fx]^2)^3 \right) + \\ & \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6 (9a^2 b \sin[2e] + 28ab^2 \sin[2e] + \right. \\ & \left. 16b^3 \sin[2e] - 9a^2 b \sin[2fx] - 6ab^2 \sin[2fx]) \right) / \\ & \left( 64a^3 (a+b)^2 f (a+b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) \right. \\ & \left. (\cos[e] + \sin[e]) \right) + \\ & \left( (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^6 (-a^2 b \sin[2e] - 2b^3 \sin[2e] + a^2 b \sin[2fx]) \right) / \\ & \left( 16a^3 (a+b) f (a+b \sec[e + fx]^2)^3 \right. \\ & \left. (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right) \end{aligned}$$

**Problem 374: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + fx]^2}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$\begin{aligned} & -\frac{x}{a^3} + \frac{b^{3/2} (35a^2 + 28ab + 8b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{8a^3 (a+b)^{7/2} f} - \frac{(8a^2 - 11ab - 4b^2) \cot[e + fx]}{8a^2 (a+b)^3 f} \\ & \frac{b \cot[e + fx]}{4a (a+b) f (a+b + b \tan[e + fx]^2)^2} - \frac{b (9a + 4b) \cot[e + fx]}{8a^2 (a+b)^2 f (a+b + b \tan[e + fx]^2)} \end{aligned}$$

Result (type 3, 2089 leaves):

$$\left( (35a^2 + 28ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \right)$$



$$\begin{aligned}
 & \text{Sec}[e + f x]^6 \left( - \left( \left( b^2 \text{ArcTan}[\text{Sec}[f x] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right. \right. \right. \\
 & \quad \left. \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Cos}[2 e] \right) \right) / \\
 & \quad \left( 64 a^3 \sqrt{a+b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) + \left( i b^2 \text{ArcTan}[\right. \\
 & \quad \left. \text{Sec}[f x] \left( \frac{\text{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right. \\
 & \quad \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Sin}[2 e] \right) / \\
 & \quad \left. \left( 64 a^3 \sqrt{a+b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) \right) / \\
 & \left( (a+b)^3 (a+b \text{Sec}[e + f x]^2)^3 \right) + \frac{1}{512 a^3 (a+b)^3 f (a+b \text{Sec}[e + f x]^2)^3} \\
 & (a + \\
 & \quad 2 b + a \text{Cos}[2 e + 2 f x]) \\
 & \text{Csc}[e] \text{Csc}[e + f x] \text{Sec}[2 e] \\
 & \text{Sec}[e + f x]^6 \\
 & (8 a^5 f x \text{Cos}[f x] + 56 a^4 b f x \text{Cos}[f x] + \\
 & \quad 184 a^3 b^2 f x \text{Cos}[f x] + 296 a^2 b^3 f x \text{Cos}[f x] + \\
 & \quad 224 a b^4 f x \text{Cos}[f x] + 64 b^5 f x \text{Cos}[f x] - 12 a^5 f x \text{Cos}[3 f x] - \\
 & \quad 68 a^4 b f x \text{Cos}[3 f x] - 132 a^3 b^2 f x \text{Cos}[3 f x] - \\
 & \quad 108 a^2 b^3 f x \text{Cos}[3 f x] - 32 a b^4 f x \text{Cos}[3 f x] - \\
 & \quad 8 a^5 f x \text{Cos}[2 e - f x] - 56 a^4 b f x \text{Cos}[2 e - f x] - \\
 & \quad 184 a^3 b^2 f x \text{Cos}[2 e - f x] - 296 a^2 b^3 f x \text{Cos}[2 e - f x] - \\
 & \quad 224 a b^4 f x \text{Cos}[2 e - f x] - 64 b^5 f x \text{Cos}[2 e - f x] - 8 a^5 f x \text{Cos}[2 e + f x] - \\
 & \quad 56 a^4 b f x \text{Cos}[2 e + f x] - 184 a^3 b^2 f x \text{Cos}[2 e + f x] - \\
 & \quad 296 a^2 b^3 f x \text{Cos}[2 e + f x] - 224 a b^4 f x \text{Cos}[2 e + f x] - \\
 & \quad 64 b^5 f x \text{Cos}[2 e + f x] + 8 a^5 f x \text{Cos}[4 e + f x] + 56 a^4 b f x \text{Cos}[4 e + f x] + \\
 & \quad 184 a^3 b^2 f x \text{Cos}[4 e + f x] + 296 a^2 b^3 f x \text{Cos}[4 e + f x] + \\
 & \quad 224 a b^4 f x \text{Cos}[4 e + f x] + 64 b^5 f x \text{Cos}[4 e + f x] + 12 a^5 f x \text{Cos}[2 e + 3 f x] + \\
 & \quad 68 a^4 b f x \text{Cos}[2 e + 3 f x] + 132 a^3 b^2 f x \text{Cos}[2 e + 3 f x] + \\
 & \quad 108 a^2 b^3 f x \text{Cos}[2 e + 3 f x] + 32 a b^4 f x \text{Cos}[2 e + 3 f x] - \\
 & \quad 12 a^5 f x \text{Cos}[4 e + 3 f x] - 68 a^4 b f x \text{Cos}[4 e + 3 f x] - 132 a^3 b^2 f x \text{Cos}[4 e + 3 f x] - \\
 & \quad 108 a^2 b^3 f x \text{Cos}[4 e + 3 f x] - 32 a b^4 f x \text{Cos}[4 e + 3 f x] + \\
 & \quad 12 a^5 f x \text{Cos}[6 e + 3 f x] + 68 a^4 b f x \text{Cos}[6 e + 3 f x] + 132 a^3 b^2 f x \text{Cos}[6 e + 3 f x] + \\
 & \quad 108 a^2 b^3 f x \text{Cos}[6 e + 3 f x] + 32 a b^4 f x \text{Cos}[6 e + 3 f x] - 4 a^5 f x \text{Cos}[2 e + 5 f x] - \\
 & \quad 12 a^4 b f x \text{Cos}[2 e + 5 f x] - 12 a^3 b^2 f x \text{Cos}[2 e + 5 f x] - 4 a^2 b^3 f x \text{Cos}[2 e + 5 f x] + \\
 & \quad 4 a^5 f x \text{Cos}[4 e + 5 f x] + 12 a^4 b f x \text{Cos}[4 e + 5 f x] + 12 a^3 b^2 f x \text{Cos}[4 e + 5 f x] + \\
 & \quad 4 a^2 b^3 f x \text{Cos}[4 e + 5 f x] - 4 a^5 f x \text{Cos}[6 e + 5 f x] - 12 a^4 b f x \text{Cos}[6 e + 5 f x] - \\
 & \quad 12 a^3 b^2 f x \text{Cos}[6 e + 5 f x] - 4 a^2 b^3 f x \text{Cos}[6 e + 5 f x] + 4 a^5 f x \text{Cos}[8 e + 5 f x] + \\
 & \quad 12 a^4 b f x \text{Cos}[8 e + 5 f x] + 12 a^3 b^2 f x \text{Cos}[8 e + 5 f x] + 4 a^2 b^3 f x \text{Cos}[8 e + 5 f x] - \\
 & \quad 32 a^5 \text{Sin}[f x] - 64 a^4 b \text{Sin}[f x] - 30 a^2 b^3 \text{Sin}[f x] - 120 a b^4 \text{Sin}[f x] - \\
 & \quad 48 b^5 \text{Sin}[f x] + 32 a^5 \text{Sin}[3 f x] + 64 a^4 b \text{Sin}[3 f x] + 26 a^3 b^2 \text{Sin}[3 f x] +
 \end{aligned}$$

$$\begin{aligned}
 & 86 a^2 b^3 \sin[3 f x] + 32 a b^4 \sin[3 f x] - 48 a^5 \sin[2 e - f x] - 128 a^4 b \sin[2 e - f x] - \\
 & 128 a^3 b^2 \sin[2 e - f x] - 30 a^2 b^3 \sin[2 e - f x] - 120 a b^4 \sin[2 e - f x] - \\
 & 48 b^5 \sin[2 e - f x] + 48 a^5 \sin[2 e + f x] + 128 a^4 b \sin[2 e + f x] + \\
 & 102 a^3 b^2 \sin[2 e + f x] - 86 a^2 b^3 \sin[2 e + f x] - 136 a b^4 \sin[2 e + f x] - \\
 & 48 b^5 \sin[2 e + f x] - 32 a^5 \sin[4 e + f x] - 64 a^4 b \sin[4 e + f x] + 26 a^3 b^2 \sin[4 e + f x] + \\
 & 86 a^2 b^3 \sin[4 e + f x] + 136 a b^4 \sin[4 e + f x] + 48 b^5 \sin[4 e + f x] - 8 a^5 \sin[2 e + 3 f x] - \\
 & 26 a^3 b^2 \sin[2 e + 3 f x] - 86 a^2 b^3 \sin[2 e + 3 f x] - 32 a b^4 \sin[2 e + 3 f x] + \\
 & 32 a^5 \sin[4 e + 3 f x] + 64 a^4 b \sin[4 e + 3 f x] - 13 a^3 b^2 \sin[4 e + 3 f x] - \\
 & 36 a^2 b^3 \sin[4 e + 3 f x] - 16 a b^4 \sin[4 e + 3 f x] - 8 a^5 \sin[6 e + 3 f x] + \\
 & 13 a^3 b^2 \sin[6 e + 3 f x] + 36 a^2 b^3 \sin[6 e + 3 f x] + 16 a b^4 \sin[6 e + 3 f x] + \\
 & 8 a^5 \sin[2 e + 5 f x] + 13 a^3 b^2 \sin[2 e + 5 f x] + 6 a^2 b^3 \sin[2 e + 5 f x] - \\
 & 13 a^3 b^2 \sin[4 e + 5 f x] - 6 a^2 b^3 \sin[4 e + 5 f x] + 8 a^5 \sin[6 e + 5 f x]
 \end{aligned}$$

**Problem 375: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^4}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 230 leaves, 9 steps):

$$\begin{aligned}
 & \frac{x}{a^3} - \frac{b^{5/2} (63 a^2 + 36 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{9/2} f} + \\
 & \frac{(8 a^3 + 32 a^2 b - 15 a b^2 - 4 b^3) \cot[e + f x]}{8 a^2 (a + b)^4 f} - \frac{(8 a^2 - 39 a b - 12 b^2) \cot[e + f x]^3}{24 a^2 (a + b)^3 f} - \\
 & \frac{b \cot[e + f x]^3}{4 a (a + b) f (a + b + b \tan[e + f x]^2)^2} - \frac{b (11 a + 4 b) \cot[e + f x]^3}{8 a^2 (a + b)^2 f (a + b + b \tan[e + f x]^2)}
 \end{aligned}$$

Result (type 3, 3340 leaves):

$$\begin{aligned}
 & \left( (63 a^2 + 36 a b + 8 b^2) (a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \left( \left( b^3 \operatorname{ArcTan}\left[ \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sec[f x] \left( \frac{\cos[2 e]}{2 \sqrt{a + b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a + b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right. \right. \right. \right. \\
 & \left. \left. \left. \left. (-a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x]) \right] \cos[2 e] \right) \right) \right) / \\
 & \left( 64 a^3 \sqrt{a + b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) - \left( i b^3 \operatorname{ArcTan}\left[ \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sec[f x] \left( \frac{\cos[2 e]}{2 \sqrt{a + b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a + b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right. \right. \right. \right. \\
 & \left. \left. \left. \left. (-a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x]) \right] \sin[2 e] \right) \right) \right) / \\
 & \left( 64 a^3 \sqrt{a + b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) \left. \right) \left. \right) \left. \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( (a+b)^4 (a+b \operatorname{Sec}[e+fx]^2)^3 \right) + \frac{1}{6144 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} \\
 & (a+2b+a \operatorname{Cos}[2e+2fx]) \\
 & \operatorname{Csc}[ \\
 & \quad e] \operatorname{Csc}[e+fx]^3 \operatorname{Sec}[ \\
 & \quad 2e] \operatorname{Sec}[e+fx]^6 \\
 & (-36 a^6 f x \operatorname{Cos}[fx] - 336 a^5 b f x \operatorname{Cos}[fx] - 1560 a^4 b^2 f x \operatorname{Cos}[fx] - \\
 & \quad 3600 a^3 b^3 f x \operatorname{Cos}[fx] - 4260 a^2 b^4 f x \operatorname{Cos}[fx] - 2496 a b^5 f x \operatorname{Cos}[fx] - \\
 & \quad 576 b^6 f x \operatorname{Cos}[fx] + 36 a^6 f x \operatorname{Cos}[3fx] + 240 a^5 b f x \operatorname{Cos}[3fx] + \\
 & \quad 408 a^4 b^2 f x \operatorname{Cos}[3fx] - 48 a^3 b^3 f x \operatorname{Cos}[3fx] - 732 a^2 b^4 f x \operatorname{Cos}[3fx] - \\
 & \quad 672 a b^5 f x \operatorname{Cos}[3fx] - 192 b^6 f x \operatorname{Cos}[3fx] + 36 a^6 f x \operatorname{Cos}[2e-fx] + \\
 & \quad 336 a^5 b f x \operatorname{Cos}[2e-fx] + 1560 a^4 b^2 f x \operatorname{Cos}[2e-fx] + 3600 a^3 b^3 f x \operatorname{Cos}[2e-fx] + \\
 & \quad 4260 a^2 b^4 f x \operatorname{Cos}[2e-fx] + 2496 a b^5 f x \operatorname{Cos}[2e-fx] + \\
 & \quad 576 b^6 f x \operatorname{Cos}[2e-fx] + 36 a^6 f x \operatorname{Cos}[2e+fx] + 336 a^5 b f x \operatorname{Cos}[2e+fx] + \\
 & \quad 1560 a^4 b^2 f x \operatorname{Cos}[2e+fx] + 3600 a^3 b^3 f x \operatorname{Cos}[2e+fx] + \\
 & \quad 4260 a^2 b^4 f x \operatorname{Cos}[2e+fx] + 2496 a b^5 f x \operatorname{Cos}[2e+fx] + 576 b^6 f x \operatorname{Cos}[2e+fx] - \\
 & \quad 36 a^6 f x \operatorname{Cos}[4e+fx] - 336 a^5 b f x \operatorname{Cos}[4e+fx] - 1560 a^4 b^2 f x \operatorname{Cos}[4e+fx] - \\
 & \quad 3600 a^3 b^3 f x \operatorname{Cos}[4e+fx] - 4260 a^2 b^4 f x \operatorname{Cos}[4e+fx] - 2496 a b^5 f x \operatorname{Cos}[4e+fx] - \\
 & \quad 576 b^6 f x \operatorname{Cos}[4e+fx] - 36 a^6 f x \operatorname{Cos}[2e+3fx] - 240 a^5 b f x \operatorname{Cos}[2e+3fx] - \\
 & \quad 408 a^4 b^2 f x \operatorname{Cos}[2e+3fx] + 48 a^3 b^3 f x \operatorname{Cos}[2e+3fx] + 732 a^2 b^4 f x \operatorname{Cos}[2e+3fx] + \\
 & \quad 672 a b^5 f x \operatorname{Cos}[2e+3fx] + 192 b^6 f x \operatorname{Cos}[2e+3fx] + 36 a^6 f x \operatorname{Cos}[4e+3fx] + \\
 & \quad 240 a^5 b f x \operatorname{Cos}[4e+3fx] + 408 a^4 b^2 f x \operatorname{Cos}[4e+3fx] - 48 a^3 b^3 f x \operatorname{Cos}[4e+3fx] - \\
 & \quad 732 a^2 b^4 f x \operatorname{Cos}[4e+3fx] - 672 a b^5 f x \operatorname{Cos}[4e+3fx] - 192 b^6 f x \operatorname{Cos}[4e+3fx] - \\
 & \quad 36 a^6 f x \operatorname{Cos}[6e+3fx] - 240 a^5 b f x \operatorname{Cos}[6e+3fx] - 408 a^4 b^2 f x \operatorname{Cos}[6e+3fx] + \\
 & \quad 48 a^3 b^3 f x \operatorname{Cos}[6e+3fx] + 732 a^2 b^4 f x \operatorname{Cos}[6e+3fx] + 672 a b^5 f x \operatorname{Cos}[6e+3fx] + \\
 & \quad 192 b^6 f x \operatorname{Cos}[6e+3fx] - 12 a^6 f x \operatorname{Cos}[2e+5fx] - 144 a^5 b f x \operatorname{Cos}[2e+5fx] - \\
 & \quad 456 a^4 b^2 f x \operatorname{Cos}[2e+5fx] - 624 a^3 b^3 f x \operatorname{Cos}[2e+5fx] - 396 a^2 b^4 f x \operatorname{Cos}[2e+5fx] - \\
 & \quad 96 a b^5 f x \operatorname{Cos}[2e+5fx] + 12 a^6 f x \operatorname{Cos}[4e+5fx] + 144 a^5 b f x \operatorname{Cos}[4e+5fx] + \\
 & \quad 456 a^4 b^2 f x \operatorname{Cos}[4e+5fx] + 624 a^3 b^3 f x \operatorname{Cos}[4e+5fx] + 396 a^2 b^4 f x \operatorname{Cos}[4e+5fx] + \\
 & \quad 96 a b^5 f x \operatorname{Cos}[4e+5fx] - 12 a^6 f x \operatorname{Cos}[6e+5fx] - 144 a^5 b f x \operatorname{Cos}[6e+5fx] - \\
 & \quad 456 a^4 b^2 f x \operatorname{Cos}[6e+5fx] - 624 a^3 b^3 f x \operatorname{Cos}[6e+5fx] - 396 a^2 b^4 f x \operatorname{Cos}[6e+5fx] - \\
 & \quad 96 a b^5 f x \operatorname{Cos}[6e+5fx] + 12 a^6 f x \operatorname{Cos}[8e+5fx] + 144 a^5 b f x \operatorname{Cos}[8e+5fx] + \\
 & \quad 456 a^4 b^2 f x \operatorname{Cos}[8e+5fx] + 624 a^3 b^3 f x \operatorname{Cos}[8e+5fx] + 396 a^2 b^4 f x \operatorname{Cos}[8e+5fx] + \\
 & \quad 96 a b^5 f x \operatorname{Cos}[8e+5fx] - 12 a^6 f x \operatorname{Cos}[4e+7fx] - 48 a^5 b f x \operatorname{Cos}[4e+7fx] - \\
 & \quad 72 a^4 b^2 f x \operatorname{Cos}[4e+7fx] - 48 a^3 b^3 f x \operatorname{Cos}[4e+7fx] - 12 a^2 b^4 f x \operatorname{Cos}[4e+7fx] + \\
 & \quad 12 a^6 f x \operatorname{Cos}[6e+7fx] + 48 a^5 b f x \operatorname{Cos}[6e+7fx] + 72 a^4 b^2 f x \operatorname{Cos}[6e+7fx] + \\
 & \quad 48 a^3 b^3 f x \operatorname{Cos}[6e+7fx] + 12 a^2 b^4 f x \operatorname{Cos}[6e+7fx] - 12 a^6 f x \operatorname{Cos}[8e+7fx] - \\
 & \quad 48 a^5 b f x \operatorname{Cos}[8e+7fx] - 72 a^4 b^2 f x \operatorname{Cos}[8e+7fx] - 48 a^3 b^3 f x \operatorname{Cos}[8e+7fx] - \\
 & \quad 12 a^2 b^4 f x \operatorname{Cos}[8e+7fx] + 12 a^6 f x \operatorname{Cos}[10e+7fx] + 48 a^5 b f x \operatorname{Cos}[10e+7fx] + \\
 & \quad 72 a^4 b^2 f x \operatorname{Cos}[10e+7fx] + 48 a^3 b^3 f x \operatorname{Cos}[10e+7fx] + 12 a^2 b^4 f x \operatorname{Cos}[10e+7fx] - \\
 & \quad 128 a^6 \operatorname{Sin}[fx] - 440 a^5 b \operatorname{Sin}[fx] - 1152 a^4 b^2 \operatorname{Sin}[fx] - 1920 a^3 b^3 \operatorname{Sin}[fx] + \\
 & \quad 228 a^2 b^4 \operatorname{Sin}[fx] + 1320 a b^5 \operatorname{Sin}[fx] + 432 b^6 \operatorname{Sin}[fx] + 48 a^6 \operatorname{Sin}[3fx] + \\
 & \quad 104 a^5 b \operatorname{Sin}[3fx] + 640 a^4 b^2 \operatorname{Sin}[3fx] + 1511 a^3 b^3 \operatorname{Sin}[3fx] - 528 a^2 b^4 \operatorname{Sin}[3fx] + \\
 & \quad 264 a b^5 \operatorname{Sin}[3fx] + 144 b^6 \operatorname{Sin}[3fx] - 32 a^6 \operatorname{Sin}[2e-fx] + 384 a^5 b \operatorname{Sin}[2e-fx] + \\
 & \quad 2048 a^4 b^2 \operatorname{Sin}[2e-fx] + 3072 a^3 b^3 \operatorname{Sin}[2e-fx] + 228 a^2 b^4 \operatorname{Sin}[2e-fx] + \\
 & \quad 1320 a b^5 \operatorname{Sin}[2e-fx] + 432 b^6 \operatorname{Sin}[2e-fx] + 32 a^6 \operatorname{Sin}[2e+fx] - 384 a^5 b \operatorname{Sin}[2e+fx] - \\
 & \quad 2048 a^4 b^2 \operatorname{Sin}[2e+fx] - 2919 a^3 b^3 \operatorname{Sin}[2e+fx] + 642 a^2 b^4 \operatorname{Sin}[2e+fx] + \\
 & \quad 1416 a b^5 \operatorname{Sin}[2e+fx] + 432 b^6 \operatorname{Sin}[2e+fx] - 128 a^6 \operatorname{Sin}[4e+fx] - \\
 & \quad 440 a^5 b \operatorname{Sin}[4e+fx] - 1152 a^4 b^2 \operatorname{Sin}[4e+fx] - 2073 a^3 b^3 \operatorname{Sin}[4e+fx] - \\
 & \quad 642 a^2 b^4 \operatorname{Sin}[4e+fx] - 1416 a b^5 \operatorname{Sin}[4e+fx] - 432 b^6 \operatorname{Sin}[4e+fx] - \\
 & \quad 144 a^6 \operatorname{Sin}[2e+3fx] - 672 a^5 b \operatorname{Sin}[2e+3fx] - 960 a^4 b^2 \operatorname{Sin}[2e+3fx] +
 \end{aligned}$$

$$\begin{aligned}
& 153 a^3 b^3 \sin[2 e + 3 f x] + 528 a^2 b^4 \sin[2 e + 3 f x] - 264 a b^5 \sin[2 e + 3 f x] - \\
& 144 b^6 \sin[2 e + 3 f x] + 48 a^6 \sin[4 e + 3 f x] + 104 a^5 b \sin[4 e + 3 f x] + \\
& 640 a^4 b^2 \sin[4 e + 3 f x] + 1664 a^3 b^3 \sin[4 e + 3 f x] - 66 a^2 b^4 \sin[4 e + 3 f x] - \\
& 408 a b^5 \sin[4 e + 3 f x] - 144 b^6 \sin[4 e + 3 f x] - 144 a^6 \sin[6 e + 3 f x] - \\
& 672 a^5 b \sin[6 e + 3 f x] - 960 a^4 b^2 \sin[6 e + 3 f x] + 66 a^2 b^4 \sin[6 e + 3 f x] + \\
& 408 a b^5 \sin[6 e + 3 f x] + 144 b^6 \sin[6 e + 3 f x] + 80 a^6 \sin[2 e + 5 f x] + \\
& 480 a^5 b \sin[2 e + 5 f x] + 832 a^4 b^2 \sin[2 e + 5 f x] + 294 a^2 b^4 \sin[2 e + 5 f x] + \\
& 96 a b^5 \sin[2 e + 5 f x] - 48 a^6 \sin[4 e + 5 f x] - 120 a^5 b \sin[4 e + 5 f x] - \\
& 294 a^2 b^4 \sin[4 e + 5 f x] - 96 a b^5 \sin[4 e + 5 f x] + 80 a^6 \sin[6 e + 5 f x] + \\
& 480 a^5 b \sin[6 e + 5 f x] + 832 a^4 b^2 \sin[6 e + 5 f x] - 51 a^3 b^3 \sin[6 e + 5 f x] - \\
& 132 a^2 b^4 \sin[6 e + 5 f x] - 48 a b^5 \sin[6 e + 5 f x] - 48 a^6 \sin[8 e + 5 f x] - \\
& 120 a^5 b \sin[8 e + 5 f x] + 51 a^3 b^3 \sin[8 e + 5 f x] + 132 a^2 b^4 \sin[8 e + 5 f x] + \\
& 48 a b^5 \sin[8 e + 5 f x] + 32 a^6 \sin[4 e + 7 f x] + 104 a^5 b \sin[4 e + 7 f x] + \\
& 51 a^3 b^3 \sin[4 e + 7 f x] + 18 a^2 b^4 \sin[4 e + 7 f x] - 51 a^3 b^3 \sin[6 e + 7 f x] - \\
& 18 a^2 b^4 \sin[6 e + 7 f x] + 32 a^6 \sin[8 e + 7 f x] + 104 a^5 b \sin[8 e + 7 f x]
\end{aligned}$$

**Problem 376: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^6}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 285 leaves, 10 steps):

$$\begin{aligned}
& -\frac{x}{a^3} + \frac{b^{7/2} (99 a^2 + 44 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{11/2} f} - \\
& \frac{(8 a^4 + 40 a^3 b + 80 a^2 b^2 - 19 a b^3 - 4 b^4) \cot[e + f x]}{8 a^2 (a + b)^5 f} + \\
& \frac{(8 a^3 + 32 a^2 b - 51 a b^2 - 12 b^3) \cot[e + f x]^3}{24 a^2 (a + b)^4 f} - \frac{(8 a^2 - 75 a b - 20 b^2) \cot[e + f x]^5}{40 a^2 (a + b)^3 f} - \\
& \frac{b \cot[e + f x]^5}{4 a (a + b) f (a + b + b \tan[e + f x]^2)^2} - \frac{b (13 a + 4 b) \cot[e + f x]^5}{8 a^2 (a + b)^2 f (a + b + b \tan[e + f x]^2)}
\end{aligned}$$

Result (type 3, 976 leaves):

$$\begin{aligned}
 & - \frac{x (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{8a^3 (a + b \sec[e + fx]^2)^3} + \\
 & \left( \frac{(11a \cos[e] + 26b \cos[e]) (a + 2b + a \cos[2e + 2fx])^3 \csc[e] \csc[e + fx]^2 \sec[e + fx]^6}{(120 (a + b)^4 f (a + b \sec[e + fx]^2)^3)} - \right. \\
 & \left. \frac{(a + 2b + a \cos[2e + 2fx])^3 \cot[e] \csc[e + fx]^4 \sec[e + fx]^6}{40 (a + b)^3 f (a + b \sec[e + fx]^2)^3} + \right. \\
 & \left( (99a^2 + 44ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left( - \left( b^4 \operatorname{ArcTan}[\sec[fx]] \right. \right. \right. \\
 & \left. \left. \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
 & \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \cos[2e] \right) \right) / \\
 & \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) + \left( i b^4 \operatorname{ArcTan}[\right. \\
 & \left. \sec[fx] \left( \frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \\
 & \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) / \\
 & \left. \left( 64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) / \left( (a + b)^5 (a + b \sec[e + fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^3 \csc[e] \csc[e + fx]^5 \sec[e + fx]^6 \sin[fx] \right) / \\
 & \left( 40 (a + b)^3 f (a + b \sec[e + fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^3 \csc[e] \csc[e + fx]^3 \right. \\
 & \left. \sec[e + fx]^6 (-11a \sin[fx] - 26b \sin[fx]) \right) / \\
 & \left( 120 (a + b)^4 f (a + b \sec[e + fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^3 \csc[e] \csc[e + fx] \sec[e + fx]^6 \right. \\
 & \left. (23a^2 \sin[fx] + 106ab \sin[fx] + 173b^2 \sin[fx]) \right) / \\
 & \left( 120 (a + b)^5 f (a + b \sec[e + fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx]) \sec[2e] \sec[e + fx]^6 \right. \\
 & \left. (a b^5 \sin[2e] + 2b^6 \sin[2e] - a b^5 \sin[2fx]) \right) / \\
 & \left( 16a^3 (a + b)^4 f (a + b \sec[e + fx]^2)^3 \right) + \\
 & \left( (a + 2b + a \cos[2e + 2fx])^2 \sec[2e] \sec[e + fx]^6 \right. \\
 & \left. (-21a^2 b^4 \sin[2e] - 52a b^5 \sin[2e] - 16b^6 \sin[2e] + 21a^2 b^4 \sin[2fx] + 6a b^5 \sin[2fx]) \right) / \\
 & \left( 64a^3 (a + b)^5 f (a + b \sec[e + fx]^2)^3 \right)
 \end{aligned}$$

### Problem 377: Unable to integrate problem.

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^5 dx$$

Optimal (type 3, 111 leaves, 7 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} - \frac{(a+2b)(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3b^2 f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{5/2}}{5b^2 f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^5 dx$$

### Problem 378: Unable to integrate problem.

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^3 dx$$

Optimal (type 3, 80 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} - \frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3bf}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^3 dx$$

### Problem 379: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x] dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f}$$

Result (type 3, 307 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx] \left( \frac{2}{1 + e^{2i(e+fx)}} + \right. \right. \\ \left. \left. \left( i \sqrt{a} \left( 2fx + i \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] + \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] \right) \right) \right) / \\ \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \sqrt{a + b \operatorname{Sec}[e+fx]^2} \Big/ \\ \left( \sqrt{2} f \sqrt{a + 2b + a \cos[2e + 2fx]} \right)$$

**Problem 380: Unable to integrate problem.**

$$\int \cot[e+fx] \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 70 leaves, 7 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{f} - \frac{\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{f}$$

Result (type 8, 25 leaves):

$$\int \cot[e+fx] \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

**Problem 381: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^3 \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 109 leaves, 8 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{f} + \\ \frac{(2a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{2\sqrt{a+b}f} - \frac{\cot[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves):

$$\frac{1}{\sqrt{2} f \sqrt{a+2 b+a \operatorname{Cos}[2 e+2 f x]}}$$

$$e^{i(e+f x)} \sqrt{4 b+a e^{-2 i(e+f x)}\left(1+e^{2 i(e+f x)}\right)^2} \operatorname{Cos}[e+f x] \left( \frac{1+e^{2 i(e+f x)}}{\left(-1+e^{2 i(e+f x)}\right)^2} - \right.$$

$$\left. \frac{1}{\sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}} \left( -2 i \sqrt{a} \sqrt{a+b} f x+(2 a+b) \operatorname{Log}\left[1-e^{2 i(e+f x)}\right] + \right.$$

$$\left. \sqrt{a} \sqrt{a+b} \operatorname{Log}\left[a+2 b+a e^{2 i(e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right] + \right.$$

$$\left. \sqrt{a} \sqrt{a+b} \operatorname{Log}\left[a+a e^{2 i(e+f x)}+2 b e^{2 i(e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right] - \right.$$

$$\left. 2 a \operatorname{Log}\left[a+b+a e^{2 i(e+f x)}+b e^{2 i(e+f x)}+\sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right] - \right.$$

$$\left. b \operatorname{Log}\left[a+b+a e^{2 i(e+f x)}+b e^{2 i(e+f x)}+\sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right] \right) \sqrt{a+b} \operatorname{Sec}[e+f x]^2$$

**Problem 382: Unable to integrate problem.**

$$\int \operatorname{Cot}[e+f x]^5 \sqrt{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 161 leaves, 9 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} - \frac{\left(8 a^2+12 a b+3 b^2\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{3 / 2} f} +$$

$$\frac{(4 a+3 b) \operatorname{Cot}[e+f x]^2 \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{8(a+b) f} - \frac{\operatorname{Cot}[e+f x]^4 \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{4 f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Cot}[e+f x]^5 \sqrt{a+b \operatorname{Sec}[e+f x]^2} dx$$

**Problem 383: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+f x]^2} \operatorname{Tan}[e+f x]^6 dx$$

Optimal (type 3, 219 leaves, 10 steps):



$$\begin{aligned}
 & - \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{(a^3 + 5 a^2 b + 15 a b^2 - 5 b^3) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16 b^{5/2} f} \\
 & + \frac{(a-b)(a+5b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16 b^2 f} \\
 & + \frac{(a-5b) \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24 b f} + \frac{\operatorname{Tan}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{6 f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^6 dx$$

**Problem 384: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^4 dx$$

Optimal (type 3, 165 leaves, 9 steps):

$$\begin{aligned}
 & \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(a^2 + 6 a b - 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8 b^{3/2} f} \\
 & + \frac{(a-3b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8 b f} + \frac{\operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4 f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^4 dx$$

**Problem 385: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^2 dx$$

Optimal (type 3, 118 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \\
 & \frac{(a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2 \sqrt{b} f} + \frac{\operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2 f}
 \end{aligned}$$

Result (type 3, 526 leaves):

$$\frac{1}{\sqrt{2} f \sqrt{a+2 b+a \operatorname{Cos}[2 e+2 f x]}} e^{i(e+f x)} \sqrt{4 b+a e^{-2 i(e+f x)}\left(1+e^{2 i(e+f x)}\right)^2} \operatorname{Cos}[e+f x]$$

$$\left(-\frac{i\left(-1+e^{2 i(e+f x)}\right)}{\left(1+e^{2 i(e+f x)}\right)^2}+\frac{1}{\sqrt{b} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}}\left(-2 \sqrt{a} \sqrt{b} f x+\right.\right.$$

$$i \sqrt{a} \sqrt{b} \operatorname{Log}\left[a+2 b+a e^{2 i(e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right]-$$

$$i \sqrt{a} \sqrt{b} \operatorname{Log}\left[a+a e^{2 i(e+f x)}+2 b e^{2 i(e+f x)}+\sqrt{a} \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right]-$$

$$a \operatorname{Log}\left[\left(2\left(\sqrt{b}\left(-1+e^{2 i(e+f x)}\right)-i \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right) f\right) /$$

$$\left.\left(\left(a-b\right)\left(1+e^{2 i(e+f x)}\right)\right)\right]+$$

$$b \operatorname{Log}\left[\left(2\left(\sqrt{b}\left(-1+e^{2 i(e+f x)}\right)-i \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}\right) f\right) /$$

$$\left.\left(\left(a-b\right)\left(1+e^{2 i(e+f x)}\right)\right)\right]\right] \sqrt{a+b \operatorname{Sec}[e+f x]^2}$$

**Problem 386: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+f x]^2} d x$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{f}+\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{f}$$

Result (type 8, 18 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+f x]^2} d x$$

**Problem 387: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+f x]^2 \sqrt{a+b \operatorname{Sec}[e+f x]^2} d x$$

Optimal (type 3, 69 leaves, 6 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{f}-\frac{\operatorname{Cot}[e+f x] \sqrt{a+b \operatorname{Tan}[e+f x]^2}}{f}$$

Result (type 3, 306 leaves):

$$\left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx] \left( -\frac{2i}{-1 + e^{2i(e+fx)}} + \right. \right. \\ \left. \left. \left( \sqrt{a} \left( -2fx + i \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] - \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] \right) \right) \right) / \\ \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \left( \sqrt{a + b \operatorname{Sec}[e+fx]^2} \right) / \\ \left( \sqrt{2} f \sqrt{a + 2b + a \cos[2e + 2fx]} \right)$$

**Problem 388: Unable to integrate problem.**

$$\int \cot[e+fx]^4 \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 114 leaves, 7 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \\ \frac{(3a + 2b) \cot[e+fx] \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{3(a+b)f} - \frac{\cot[e+fx]^3 \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{3f}$$

Result (type 8, 27 leaves):

$$\int \cot[e+fx]^4 \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

**Problem 389: Unable to integrate problem.**

$$\int \cot[e+fx]^6 \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 167 leaves, 8 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(15a^2 + 25ab + 8b^2) \cot[e+fx] \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{15(a+b)^2 f} \\ \frac{(b - 5(a+b)) \cot[e+fx]^3 \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{15(a+b)f} - \frac{\cot[e+fx]^5 \sqrt{a + b \operatorname{Tan}[e+fx]^2}}{5f}$$

Result (type 8, 27 leaves):

$$\int \cot[e+fx]^6 \sqrt{a + b \operatorname{Sec}[e+fx]^2} dx$$

### Problem 390: Unable to integrate problem.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^5 dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$-\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{a \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3 f} - \frac{(a+2 b)(a+b \operatorname{Sec}[e+f x]^2)^{5/2}}{5 b^2 f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{7/2}}{7 b^2 f}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^5 dx$$

### Problem 391: Unable to integrate problem.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^3 dx$$

Optimal (type 3, 104 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} - \frac{a \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} - \frac{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3 f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{5/2}}{5 b f}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^3 dx$$

### Problem 392: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x] dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{a \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3 f}$$

Result (type 3, 343 leaves):

$$\left( \sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \left( \frac{8(b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2)}{(1+e^{2i(e+fx)})^3} + \right. \right. \\ \left. \left. \left( 3i a^{3/2} \left( 2fx + i \operatorname{Log}[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] + \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] \right) \right) \right) / \\ \left( \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \left( a+b \operatorname{Sec}[e+fx]^2 \right)^{3/2} / \\ \left( 3f(a+2b+a \cos[2e+2fx])^{3/2} \right)$$

**Problem 393: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 91 leaves, 8 steps):

$$\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{f} - \frac{(a+b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{f} + \frac{b \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f}$$

Result (type 3, 506 leaves):

$$\frac{1}{f(a+2b+a \cos[2e+2fx])^{3/2}} \sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \\ \left( \frac{2b}{1+e^{2i(e+fx)}} + \frac{1}{\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}} \left( -2i a^{3/2} fx + 2(a+b)^{3/2} \operatorname{Log}[1-e^{2i(e+fx)}] + \right. \right. \\ \left. \left. a^{3/2} \operatorname{Log}[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] + \right. \right. \\ \left. \left. a^{3/2} \operatorname{Log}[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] - \right. \right. \\ \left. \left. 2a \sqrt{a+b} \operatorname{Log}[a+b+a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] - \right. \right. \\ \left. \left. 2b \sqrt{a+b} \operatorname{Log}[a+b+a e^{2i(e+fx)} + b e^{2i(e+fx)} + \right. \right. \\ \left. \left. \left. \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right) \right) \left( a+b \operatorname{Sec}[e+fx]^2 \right)^{3/2}$$

**Problem 394: Result unnecessarily involves complex numbers and more than**

twice size of optimal antiderivative.

$$\int \text{Cot}[e + f x]^3 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 114 leaves, 8 steps):

$$-\frac{a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{(2a-b) \sqrt{a+b} \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{2f} - \frac{(a+b) \text{Cot}[e+f x]^2 \sqrt{a+b \text{Sec}[e+f x]^2}}{2f}$$

Result (type 3, 622 leaves):

$$\frac{1}{f (a + 2b + a \text{Cos}[2e + 2fx])^{3/2}} \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2}$$

$$\text{Cos}[e + fx]^3 \left( \frac{(a+b) (1 + e^{2i(e+fx)})}{(-1 + e^{2i(e+fx)})^2} - \frac{1}{\sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} \right)$$

$$\left( -2i a^{3/2} \sqrt{a+b} f x + (2a^2 + ab - b^2) \text{Log}[1 - e^{2i(e+fx)}] + a^{3/2} \sqrt{a+b} \text{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] + a^{3/2} \sqrt{a+b} \text{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] - 2a^2 \text{Log}[a + b + a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] - ab \text{Log}[a + b + a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] + b^2 \text{Log}[a + b + a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] \right) (a+b \text{Sec}[e + fx]^2)^{3/2}$$

Problem 395: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Cot}[e + f x]^5 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 159 leaves, 9 steps):

$$\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{f} - \frac{(8a^2 + 4ab - b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8\sqrt{a+b}f} +$$

$$\frac{(4a-b) \operatorname{Cot}[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{8f} - \frac{(a+b) \operatorname{Cot}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{4f}$$

Result (type 3, 684 leaves):

$$\frac{1}{2\sqrt{2}f(a+2b+a\cos[2e+2fx])^{3/2}} e^{i(e+fx)} \sqrt{4b+ae^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} \cos[e+fx]^3$$

$$\left( -\left( (1+e^{2i(e+fx)}) (b(1+6e^{2i(e+fx)}+e^{4i(e+fx)})+a(6-4e^{2i(e+fx)}+6e^{4i(e+fx)})) \right) / \right.$$

$$\left. (-1+e^{2i(e+fx)})^4 \right) + \frac{1}{\sqrt{a+b} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}}$$

$$\left( -8ia^{3/2}\sqrt{a+b}fx + (8a^2+4ab-b^2) \operatorname{Log}[1-e^{2i(e+fx)}] + \right.$$

$$4a^{3/2}\sqrt{a+b} \operatorname{Log}[a+2b+ae^{2i(e+fx)}+\sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] +$$

$$4a^{3/2}\sqrt{a+b} \operatorname{Log}[a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] -$$

$$8a^2 \operatorname{Log}[a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] -$$

$$4ab \operatorname{Log}[a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] +$$

$$\left. b^2 \operatorname{Log}[a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] \right) (a+b$$

$$\operatorname{Sec}[e+fx]^2)^{3/2}$$

**Problem 396: Unable to integrate problem.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Tan}[e+fx]^6 dx$$

Optimal (type 3, 290 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{(3 a^4 + 20 a^3 b + 90 a^2 b^2 - 60 a b^3 - 5 b^4) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{128 b^{5/2} f} \\
 & + \frac{(3 a^3 + 17 a^2 b - 55 a b^2 - 5 b^3) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{128 b^2 f} + \\
 & + \frac{(3 a^2 - 50 a b - 5 b^2) \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{192 b f} + \\
 & + \frac{(9 a + b) \operatorname{Tan}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{48 f} + \frac{b \operatorname{Tan}[e+fx]^7 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8 f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^6 dx$$

### Problem 397: Unable to integrate problem.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^4 dx$$

Optimal (type 3, 214 leaves, 10 steps):

$$\begin{aligned}
 & \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(a - b) (a^2 + 10 a b + b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16 b^{3/2} f} + \\
 & + \frac{(a^2 - 8 a b - b^2) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16 b f} + \\
 & + \frac{(7 a + b) \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24 f} + \frac{b \operatorname{Tan}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{6 f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^4 dx$$

### Problem 398: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Tan}[e + f x]^2 dx$$

Optimal (type 3, 166 leaves, 9 steps):



$$\begin{aligned}
 & - \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{(3a^2 - 6ab - b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8\sqrt{b}f} + \\
 & \frac{(5a+b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} + \frac{b \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4f}
 \end{aligned}$$

Result (type 3, 702 leaves):

$$\begin{aligned}
 & \frac{1}{2\sqrt{2}f(a+2b+a\cos[2(e+fx)])^{3/2}} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \\
 & \left( -\frac{1}{(1+e^{2i(e+fx)})^4} i(-1+e^{2i(e+fx)}) (5a(1+e^{2i(e+fx)})^2 - b(1-6e^{2i(e+fx)}+e^{4i(e+fx)})) + \right. \\
 & \frac{1}{\sqrt{b}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \left( -8a^{3/2}\sqrt{b}fx + \right. \\
 & 4i a^{3/2}\sqrt{b} \operatorname{Log}[a+2b+a e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] - \\
 & 4i a^{3/2}\sqrt{b} \operatorname{Log}[a+a e^{2i(e+fx)}+2b e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}] - \\
 & 3a^2 \operatorname{Log}\left[4\left(\sqrt{b}(-1+e^{2i(e+fx)})-i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)f\right] / \\
 & ((3a^2-6ab-b^2)(1+e^{2i(e+fx)})) + \\
 & 6ab \operatorname{Log}\left[4\left(\sqrt{b}(-1+e^{2i(e+fx)})-i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)f\right] / \\
 & ((3a^2-6ab-b^2)(1+e^{2i(e+fx)})) + \\
 & b^2 \operatorname{Log}\left[4\left(\sqrt{b}(-1+e^{2i(e+fx)})-i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)f\right] / \\
 & ((3a^2-6ab-b^2)(1+e^{2i(e+fx)})) \left. \right] \left. \right) (a+b \operatorname{Sec}[e+fx]^2)^{3/2}
 \end{aligned}$$

**Problem 399: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 118 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} (3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves):

$$\frac{1}{f (a + 2b + a \operatorname{Cos}[2e + 2fx])^{3/2}} \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \operatorname{Cos}[e+fx]^3 \left( -\frac{i b (-1 + e^{2i(e+fx)})}{(1 + e^{2i(e+fx)})^2} + \frac{1}{\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} \right) \left( 2 a^{3/2} f x - i a^{3/2} \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] + i a^{3/2} \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] - 3 a \sqrt{b} \operatorname{Log}\left[ \left( -2 \sqrt{b} (-1 + e^{2i(e+fx)}) f + 2 i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) / (b (3a+b) (1 + e^{2i(e+fx)})) \right] - b^{3/2} \operatorname{Log}\left[ \left( -2 \sqrt{b} (-1 + e^{2i(e+fx)}) f + 2 i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) / (b (3a+b) (1 + e^{2i(e+fx)})) \right] \right) \left( a + b \operatorname{Sec}[e+fx]^2 \right)^{3/2}$$

**Problem 400: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+fx]^2 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 111 leaves, 8 steps):

$$-\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(a+b) \operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f}$$

Result (type 3, 410 leaves):

$$\begin{aligned}
 & \left( \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\
 & \left( -\frac{2i(a+b)}{-1 + e^{2i(e+fx)}} + \left( i a^{3/2} \operatorname{Log} \left[ a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right] - \right. \right. \\
 & \quad \left. \left. i a^{3/2} \operatorname{Log} \left[ a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right] - \right. \right. \\
 & \quad \left. \left. 2 \left( a^{3/2} f x + b^{3/2} \operatorname{Log} \left[ \left( \left( \sqrt{b} (-1 + e^{2i(e+fx)}) - i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) f \right) / \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (b^2 (1 + e^{2i(e+fx)})) \right) \right] \right) \right) \right) / \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \\
 & \left. (a + b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / \left( f (a + 2b + a \cos[2e + 2fx])^{3/2} \right)
 \end{aligned}$$

**Problem 401: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^4 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 112 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan} \left[ \frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{f} + \frac{(3a-b) \cot[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{3f} - \frac{(a+b) \cot[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{3f}$$

Result (type 3, 354 leaves):

$$\left( \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right. \\ \left. \cos[e+fx]^3 \left( \frac{8i (b e^{2i(e+fx)} + a (1 - e^{2i(e+fx)} + e^{4i(e+fx)}))}{(-1 + e^{2i(e+fx)})^3} \right) + \right. \\ \left. \left( 3 a^{3/2} \left( 2fx - i \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] + \right. \right. \right. \\ \left. \left. i \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] \right) \right) \Big/ \\ \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \Big) (a + b \operatorname{Sec}[e+fx]^2)^{3/2} \Big/ \\ (3f (a + 2b + a \cos[2e + 2fx])^{3/2})$$

**Problem 402: Unable to integrate problem.**

$$\int \cot[e+fx]^6 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(15a^2 + 10ab - 2b^2) \cot[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{15(a+b)f} + \\ \frac{(5a-b) \cot[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{15f} - \frac{(a+b) \cot[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{5f}$$

Result (type 8, 27 leaves):

$$\int \cot[e+fx]^6 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

**Problem 403: Unable to integrate problem.**

$$\int \frac{\operatorname{Tan}[e+fx]^5}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} - \frac{(a+2b) \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{b^2 f} + \frac{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{3b^2 f}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e + f x]^5}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Problem 404: Unable to integrate problem.

$$\int \frac{\tan[e + f x]^3}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} + \frac{\sqrt{a+b \sec[e+f x]^2}}{b f}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e + f x]^3}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Problem 405: Unable to integrate problem.

$$\int \frac{\tan[e + f x]}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a}}\right]}{\sqrt{a} f}$$

Result (type 8, 25 leaves):

$$\int \frac{\tan[e + f x]}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Problem 406: Unable to integrate problem.

$$\int \frac{\cot[e + f x]}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 70 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b} f}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cot}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Problem 407: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} + \frac{(2a+3b) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{2(a+b)^{3/2} f} - \frac{\text{Cot}[e+fx]^2 \sqrt{a+b \text{Sec}[e+fx]^2}}{2(a+b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Problem 408: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} - \frac{(8a^2 + 20ab + 15b^2) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{5/2} f} + \frac{(4a+7b) \text{Cot}[e+fx]^2 \sqrt{a+b \text{Sec}[e+fx]^2}}{8(a+b)^2 f} - \frac{\text{Cot}[e+fx]^4 \sqrt{a+b \text{Sec}[e+fx]^2}}{4(a+b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Problem 409: Unable to integrate problem.

$$\int \frac{\text{Tan}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 173 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[ex]}{\sqrt{a+b \tan^2[ex]}}\right]}{\sqrt{a} f} + \frac{(3a^2 + 10ab + 15b^2) \text{ArcTanh}\left[\frac{\sqrt{b} \tan[ex]}{\sqrt{a+b \tan^2[ex]}}\right]}{8b^{5/2} f} \\
 & \frac{(3a + 7b) \tan[ex] \sqrt{a+b \tan^2[ex]}}{8b^2 f} + \frac{\tan^3[ex] \sqrt{a+b \tan^2[ex]}}{4bf}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan^6[ex]}{\sqrt{a+b \sec^2[ex]}} dx$$

Problem 410: Unable to integrate problem.

$$\int \frac{\tan^4[ex]}{\sqrt{a+b \sec^2[ex]}} dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$\begin{aligned}
 & \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[ex]}{\sqrt{a+b \tan^2[ex]}}\right]}{\sqrt{a} f} - \\
 & \frac{(a + 3b) \text{ArcTanh}\left[\frac{\sqrt{b} \tan[ex]}{\sqrt{a+b \tan^2[ex]}}\right]}{2b^{3/2} f} + \frac{\tan[ex] \sqrt{a+b \tan^2[ex]}}{2bf}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan^4[ex]}{\sqrt{a+b \sec^2[ex]}} dx$$

Problem 411: Unable to integrate problem.

$$\int \frac{\tan^2[ex]}{\sqrt{a+b \sec^2[ex]}} dx$$

Optimal (type 3, 80 leaves, 7 steps):

$$- \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[ex]}{\sqrt{a+b \tan^2[ex]}}\right]}{\sqrt{a} f} + \frac{\text{ArcTanh}\left[\frac{\sqrt{b} \tan[ex]}{\sqrt{a+b \tan^2[ex]}}\right]}{\sqrt{b} f}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan^2[ex]}{\sqrt{a+b \sec^2[ex]}} dx$$

### Problem 412: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{\sqrt{a} f}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

### Problem 413: Unable to integrate problem.

$$\int \frac{\operatorname{Cot}[e + f x]^2}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{\sqrt{a} f} - \frac{\operatorname{Cot}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{(a + b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Cot}[e + f x]^2}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

### Problem 414: Unable to integrate problem.

$$\int \frac{\operatorname{Cot}[e + f x]^4}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{\sqrt{a} f} + \frac{(3a + 5b) \operatorname{Cot}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{3(a + b)^2 f} - \frac{\operatorname{Cot}[e + f x]^3 \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{3(a + b) f}$$

Result (type 8, 27 leaves):



$$\int \frac{\text{Cot}[e + f x]^4}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Problem 415: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 172 leaves, 8 steps):

$$\begin{aligned} & - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{\sqrt{a} f} - \frac{(15 a^2 + 40 a b + 33 b^2) \text{Cot}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{15 (a + b)^3 f} + \\ & \frac{(5 a + 9 b) \text{Cot}[e + f x]^3 \sqrt{a + b \text{Tan}[e + f x]^2}}{15 (a + b)^2 f} - \frac{\text{Cot}[e + f x]^5 \sqrt{a + b \text{Tan}[e + f x]^2}}{5 (a + b) f} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Problem 416: Unable to integrate problem.

$$\int \frac{\text{Tan}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$\begin{aligned} & - \frac{\text{ArcTanh}\left[\frac{\sqrt{a + b \text{Sec}[e + f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} + \frac{(a + b)^2}{a b^2 f \sqrt{a + b \text{Sec}[e + f x]^2}} + \frac{\sqrt{a + b \text{Sec}[e + f x]^2}}{b^2 f} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Tan}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Problem 417: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{3/2} f} - \frac{a+b}{a b f \sqrt{a+b \text{Sec}[e+fx]^2}}$$

Result (type 6, 1695 leaves):

$$\begin{aligned} & \left( 3 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Sin}[e+fx]^3 \text{Tan}[e+fx]^4 \right) / \\ & \left( 4 \sqrt{2} f (a+b \text{Sec}[e+fx]^2)^{3/2} (a+b-a \text{Sin}[e+fx]^2)^{3/2} \right. \\ & \left( 6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + \right. \\ & \left. \left( 3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + \right. \right. \\ & \left. \left. (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \right) \text{Sin}[e+fx]^2 \right) \\ & \left( \left( 9 a (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Sin}[e+fx]^5 \right) / \right. \\ & \left( 4 \sqrt{2} (a+b-a \text{Sin}[e+fx]^2)^{5/2} \left( 6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \right. \right. \right. \\ & \left. \left. \frac{a \text{Sin}[e+fx]^2}{a+b} \right] + \left( 3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] + \right. \right. \\ & \left. \left. (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] \right) \text{Sin}[e+fx]^2 \right) \right) + \\ & \left( 9 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Sin}[e+fx]^3 \right) / \\ & \left( 4 \sqrt{2} (a+b-a \text{Sin}[e+fx]^2)^{3/2} \left( 6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \right. \right. \right. \\ & \left. \left. \frac{a \text{Sin}[e+fx]^2}{a+b} \right] + \left( 3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] + \right. \right. \\ & \left. \left. (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] \right) \text{Sin}[e+fx]^2 \right) \right) + \\ & \left( 3 (a+b) \text{Sin}[e+fx]^3 \left( \frac{1}{a+b} 2 a f \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] \right. \right. \\ & \left. \left. \text{Cos}[e+fx] \text{Sin}[e+fx] + \frac{2}{3} f \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] \right. \right. \\ & \left. \left. \text{Cos}[e+fx] \text{Sin}[e+fx] \right) \text{Tan}[e+fx] \right) / \left( 4 \sqrt{2} f (a+b-a \text{Sin}[e+fx]^2)^{3/2} \right. \\ & \left( 6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] + \right. \\ & \left( 3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] + \right. \\ & \left. \left. (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b} \right] \right) \text{Sin}[e+fx]^2 \right) \right) - \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^3 \right. \\
 & \quad \left( 2 f \left( 3 a \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + (a+b) \right. \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \cos[e+fx] \sin[e+fx] + \right. \\
 & \quad 6 (a+b) \left( \frac{1}{a+b} 2 a f \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \right. \\
 & \quad \quad \left. \sin[e+fx] + \frac{2}{3} f \operatorname{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \\
 & \quad \quad \left. \cos[e+fx] \sin[e+fx] \right) + \sin[e+fx]^2 \left( 3 a \left( \frac{1}{4 (a+b)} 15 a f \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, \right. \right. \right. \\
 & \quad \quad \quad \left. \left. 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] + \frac{3}{4} f \operatorname{AppellF1}\left[ \right. \right. \\
 & \quad \quad \quad \left. \left. 4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \\
 & \quad \quad \left( \frac{1}{4 (a+b)} 9 a f \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right. \\
 & \quad \quad \left. \cos[e+fx] \sin[e+fx] + \frac{9}{4} f \operatorname{AppellF1}\left[4, \frac{5}{2}, \frac{3}{2}, 5, \sin[e+fx]^2, \right. \right. \\
 & \quad \quad \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \left. \right) \left. \right) \tan[e+fx] \Big/ \\
 & \left( 4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{3/2} \left( 6 (a+b) \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \sin[e+fx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{a \sin[e+fx]^2}{a+b}\right] + \left( 3 a \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \\
 & \quad \quad \left. \left. (a+b) \operatorname{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right)^2 \Big) + \\
 & \left( 3 (a+b) \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx] \tan[e+fx]^2 \right) \Big/ \\
 & \left( 4 \sqrt{2} (a+b - a \sin[e+fx]^2)^{3/2} \right. \\
 & \quad \left( 6 (a+b) \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left( 3 a \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \quad \left. \left. (a+b) \operatorname{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \Big) \Big) \Big) \Big)
 \end{aligned}$$

Problem 418: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 57 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} + \frac{1}{a f \sqrt{a+b \text{Sec}[e+f x]^2}}$$

Result (type 3, 425 leaves):

$$-\frac{(a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \text{Sec}[e + f x]^2}{8 b f \sqrt{a + 2 b + a \text{Cos}[2 (e + f x)]} (a + b \text{Sec}[e + f x]^2)^{3/2}} +$$

$$\left( e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} \right.$$

$$(a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \left( \frac{\sqrt{a} (a + 4 b) (1 + e^{2 i (e+f x)})}{b (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)} + \right.$$

$$\left. \left( 4 i f x - 2 \text{Log}[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] - \right.$$

$$\left. \left. 2 \text{Log}[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] \right) \right) /$$

$$\left( \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \text{Sec}[e + f x]^3 / \left( 8 \sqrt{2} a^{3/2} f (a + b \text{Sec}[e + f x]^2)^{3/2} \right)$$

**Problem 419: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 100 leaves, 8 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{(a + b)^{3/2} f} - \frac{b}{a (a + b) f \sqrt{a + b \text{Sec}[e + f x]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cot}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

### Problem 420: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 153 leaves, 9 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{3/2} f} + \frac{(2a+5b)\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{2(a+b)^{5/2} f} - \frac{(a-2b)b}{2a(a+b)^2 f \sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{\text{Cot}[e+fx]^2}{2(a+b) f \sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

### Problem 421: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 213 leaves, 10 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{3/2} f} - \frac{(8a^2 + 28ab + 35b^2)\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{7/2} f} + \frac{b(4a^2 + 11ab - 8b^2)}{8a(a+b)^3 f \sqrt{a+b\text{Sec}[e+fx]^2}} + \frac{(4a+9b)\text{Cot}[e+fx]^2}{8(a+b)^2 f \sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{\text{Cot}[e+fx]^4}{4(a+b) f \sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

### Problem 422: Unable to integrate problem.

$$\int \frac{\text{Tan}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 172 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b+b\tan[e+fx]^2}}\right]}{a^{3/2}f}-\frac{(3a+5b)\text{ArcTanh}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b+b\tan[e+fx]^2}}\right]}{2b^{5/2}f}-\frac{(a+b)\tan[e+fx]^3}{abf\sqrt{a+b+b\tan[e+fx]^2}}+\frac{(3a+2b)\tan[e+fx]\sqrt{a+b+b\tan[e+fx]^2}}{2ab^2f}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^6}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

Problem 423: Unable to integrate problem.

$$\int \frac{\tan[e+fx]^4}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b+b\tan[e+fx]^2}}\right]}{a^{3/2}f}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b+b\tan[e+fx]^2}}\right]}{b^{3/2}f}-\frac{(a+b)\tan[e+fx]}{abf\sqrt{a+b+b\tan[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^4}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

Problem 424: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\tan[e+fx]^2}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b+b\tan[e+fx]^2}}\right]}{a^{3/2}f}+\frac{\tan[e+fx]}{af\sqrt{a+b+b\tan[e+fx]^2}}$$

Result (type 3, 764 leaves):

$$\begin{aligned}
 & - \left( \left( e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} (a + 2b + a \cos[2e + 2fx])^{3/2} \right. \right. \\
 & \quad \left( -3i a^{3/2} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} - 4i \sqrt{a} b \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \right. \\
 & \quad 3i a^{3/2} e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \\
 & \quad 4i \sqrt{a} b e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + 4a^2 f x + 4ab f x + \\
 & \quad 8a^2 e^{2i(e+fx)} f x + 24ab e^{2i(e+fx)} f x + 16b^2 e^{2i(e+fx)} f x + 4a^2 e^{4i(e+fx)} f x + \\
 & \quad \left. \left. 4ab e^{4i(e+fx)} f x - 2i(a+b) (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2) \right) \right. \\
 & \quad \left. \left. \log \left[ e^{-2i} e \left( a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right. \\
 & \quad \left. 2i(a+b) (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2) \right. \\
 & \quad \left. \left. \log \left[ e^{-2i} e \left( a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \right) \right] \\
 & \quad \sec[e+fx]^3 \Big/ \left( 8\sqrt{2} a^{3/2} (a+b) (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \right. \\
 & \quad \left. f (a+b \sec[e+fx]^2)^{3/2} \right) + \\
 & \quad \frac{(a + 2b + a \cos[2e + 2fx])^{3/2} \sec[e+fx]^2 \tan[e+fx]}{8(a+b) f \sqrt{a + 2b + a \cos[2(e+fx)]} (a+b \sec[e+fx]^2)^{3/2}}
 \end{aligned}$$

### Problem 425: Unable to integrate problem.

$$\int \frac{1}{(a + b \sec[e + fx])^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\text{ArcTan} \left[ \frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}} \right]}{a^{3/2} f} - \frac{b \tan[e+fx]}{a(a+b) f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{(a + b \sec[e + fx])^{3/2}} dx$$

### Problem 426: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^2}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b + b \text{Tan}[e + f x]^2}}\right]}{a^{3/2} f} - \frac{b \text{Cot}[e + f x]}{a (a + b) f \sqrt{a + b + b \text{Tan}[e + f x]^2}} - \frac{(a - b) \text{Cot}[e + f x] \sqrt{a + b + b \text{Tan}[e + f x]^2}}{a (a + b)^2 f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^2}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

### Problem 427: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b + b \text{Tan}[e + f x]^2}}\right]}{a^{3/2} f} - \frac{b \text{Cot}[e + f x]^3}{a (a + b) f \sqrt{a + b + b \text{Tan}[e + f x]^2}} + \frac{(3a - b)(a + 3b) \text{Cot}[e + f x] \sqrt{a + b + b \text{Tan}[e + f x]^2}}{3a (a + b)^3 f} - \frac{(a - 3b) \text{Cot}[e + f x]^3 \sqrt{a + b + b \text{Tan}[e + f x]^2}}{3a (a + b)^2 f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

### Problem 428: Unable to integrate problem.

$$\int \frac{\text{Cot}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 241 leaves, 9 steps):



$$\begin{aligned}
 & - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b+b \tan[e+fx]^2}}\right]}{a^{3/2} f} - \frac{b \cot[e+fx]^5}{a(a+b) f \sqrt{a+b+b \tan[e+fx]^2}} - \\
 & \frac{(15 a^3 + 55 a^2 b + 73 a b^2 - 15 b^3) \cot[e+fx] \sqrt{a+b+b \tan[e+fx]^2}}{15 a (a+b)^4 f} + \\
 & \frac{(5 a^2 + 14 a b - 15 b^2) \cot[e+fx]^3 \sqrt{a+b+b \tan[e+fx]^2}}{15 a (a+b)^3 f} - \\
 & \frac{(a-5 b) \cot[e+fx]^5 \sqrt{a+b+b \tan[e+fx]^2}}{5 a (a+b)^2 f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cot[e+fx]^6}{(a+b \sec[e+fx]^2)^{3/2}} dx$$

**Problem 429: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e+fx]^5}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 97 leaves, 6 steps):

$$- \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} + \frac{(a+b)^2}{3 a b^2 f (a+b \sec[e+fx]^2)^{3/2}} + \frac{\frac{1}{a^2} - \frac{1}{b^2}}{f \sqrt{a+b \sec[e+fx]^2}}$$

Result (type 6, 1699 leaves):

$$\begin{aligned}
 & \left( (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^5 \tan[e+fx]^6 \right) / \\
 & \left( 3 \sqrt{2} f (a+b \sec[e+fx]^2)^{5/2} (a+b - a \sin[e+fx]^2)^{5/2} \right. \\
 & \left( 8 (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \\
 & \left. \left( 5 a \text{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \\
 & \left. \left. (a+b) \text{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \\
 & \left( \left( 5 a (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^7 \right) / \right. \\
 & \left. \left( 3 \sqrt{2} (a+b - a \sin[e+fx]^2)^{7/2} \left( 8 (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] + \left( 5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right. \\
 & \left. (a + b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Sin}[e + f x]^2 \right) + \\
 & \left( 5 (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Sin}[e + f x]^5 \right) / \\
 & \left( 3 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \left( 8 (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] + \left( 5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right. \right. \\
 & \left. \left. (a + b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Sin}[e + f x]^2 \right) \right) + \\
 & \left( (a + b) \operatorname{Sin}[e + f x]^5 \left( \frac{1}{4 (a + b)} 15 a f \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right. \right. \\
 & \left. \left. \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] + \frac{3}{4} f \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right. \right. \\
 & \left. \left. \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \right) \operatorname{Tan}[e + f x] \right) / \left( 3 \sqrt{2} f (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right. \\
 & \left. \left( 8 (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right. \right. \\
 & \left. \left( 5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \right. \right. \\
 & \left. \left. (a + b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Sin}[e + f x]^2 \right) \right) - \\
 & \left( (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Sin}[e + f x]^5 \right. \\
 & \left. \left( 2 f \left( 5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + (a + b) \right. \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] + \right. \\
 & \left. 8 (a + b) \left( \frac{1}{4 (a + b)} 15 a f \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right. \right. \\
 & \left. \left. \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] + \frac{3}{4} f \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \right) + \operatorname{Sin}[e + f x]^2 \\
 & \left. \left( 5 a \left( \frac{1}{5 (a + b)} 28 a f \operatorname{AppellF1}\left[5, \frac{1}{2}, \frac{9}{2}, 6, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x] \right. \right. \right. \\
 & \left. \left. \operatorname{Sin}[e + f x] + \frac{4}{5} f \operatorname{AppellF1}\left[5, \frac{3}{2}, \frac{7}{2}, 6, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[ \right. \right. \\
 & \left. \left. e + f x] \operatorname{Sin}[e + f x] \right) + (a + b) \left( \frac{1}{a + b} 4 a f \operatorname{AppellF1}\left[5, \frac{3}{2}, \frac{7}{2}, 6, \operatorname{Sin}[e + f x]^2, \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] + \frac{12}{5} f \operatorname{AppellF1}\left[5, \frac{5}{2}, \frac{5}{2}, 6, \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \right) \right) \operatorname{Tan}[e + f x] \Bigg/ \\
 & \left( 3 \sqrt{2} f (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \left( 8 (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. \left. \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] + \left( 5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] + \right. \right. \right. \\
 & \left. \left. \left. (a + b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] \right) \operatorname{Sin}[e + f x]^2 \right)^2 \right) + \\
 & \left( (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] \operatorname{Sin}[e + f x]^3 \operatorname{Tan}[e + f x]^2 \right) \Bigg/ \\
 & \left( 3 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right. \\
 & \left. \left( 8 (a + b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] + \right. \right. \\
 & \left. \left( 5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] + \right. \right. \\
 & \left. \left. \left. (a + b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b} \right] \right) \operatorname{Sin}[e + f x]^2 \right) \right) \Bigg) \Bigg) \Bigg)
 \end{aligned}$$

**Problem 430: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Sec}[e + f x]^2}}{\sqrt{a}}\right]}{a^{5/2} f} - \frac{a + b}{3 a b f (a + b \operatorname{Sec}[e + f x]^2)^{3/2}} - \frac{1}{a^2 f \sqrt{a + b \operatorname{Sec}[e + f x]^2}}$$

Result (type 3, 613 leaves):

$$\begin{aligned}
 & - \left( \left( (a + 3b + a \cos[2(e + fx)]) (a + 2b + a \cos[2e + 2fx])^{5/2} \sec[e + fx]^4 \right) / \right. \\
 & \quad \left. (48b^2 f (a + 2b + a \cos[2(e + fx)])^{3/2} (a + b \sec[e + fx]^2)^{5/2}) \right) + \\
 & \left( (a + b + (a - 2b) \cos[2(e + fx)]) (a + 2b + a \cos[2e + 2fx])^{5/2} \sec[e + fx]^4 \right) / \\
 & \quad (96b^2 f (a + 2b + a \cos[2(e + fx)])^{3/2} (a + b \sec[e + fx]^2)^{5/2}) - \\
 & \frac{1}{96\sqrt{2} a^{5/2} f (a + b \sec[e + fx]^2)^{5/2}} \\
 & e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} (a + 2b + a \cos[2e + 2fx])^{5/2} \\
 & \left( - \left( \left( \sqrt{a} (1 + e^{2i(e+fx)}) (-96b^3 e^{2i(e+fx)} + a^3 (1 + e^{2i(e+fx)})^2 - 32ab^2 (1 + e^{2i(e+fx)})^2 - \right. \right. \right. \\
 & \quad \left. \left. \left. 6a^2b (1 + e^{2i(e+fx)} + e^{4i(e+fx)}) \right) \right) / \left( b^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^2 \right) \right) + \\
 & \left( 24i f x - 12 \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] - \right. \\
 & \quad \left. 12 \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] \right) / \\
 & \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \sec[e + fx]^5
 \end{aligned}$$

**Problem 431: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[e + fx]}{(a + b \sec[e + fx]^2)^{5/2}} dx$$

Optimal (type 3, 83 leaves, 6 steps):

$$- \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} + \frac{1}{3 a f (a + b \sec[e + fx]^2)^{3/2}} + \frac{1}{a^2 f \sqrt{a + b \sec[e + fx]^2}}$$

Result (type 3, 613 leaves):

$$\begin{aligned}
 & - \left( \left( (a + 3b + a \cos[2(e + fx)]) (a + 2b + a \cos[2e + 2fx])^{5/2} \sec[e + fx]^4 \right) / \right. \\
 & \quad \left. (48b^2 f (a + 2b + a \cos[2(e + fx)])^{3/2} (a + b \sec[e + fx]^2)^{5/2}) \right) + \\
 & \left( (a + b + (a - 2b) \cos[2(e + fx)]) (a + 2b + a \cos[2e + 2fx])^{5/2} \sec[e + fx]^4 \right) / \\
 & \quad (32b^2 f (a + 2b + a \cos[2(e + fx)])^{3/2} (a + b \sec[e + fx]^2)^{5/2}) + \\
 & \frac{1}{96\sqrt{2} a^{5/2} f (a + b \sec[e + fx]^2)^{5/2}} \\
 & e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2 (a + 2b + a \cos[2e + 2fx])^{5/2}} \\
 & \left( - \left( \left( \sqrt{a} (1 + e^{2i(e+fx)}) (-96b^3 e^{2i(e+fx)} + a^3 (1 + e^{2i(e+fx)})^2 - 32ab^2 (1 + e^{2i(e+fx)})^2 - \right. \right. \right. \\
 & \quad \left. \left. \left. 6a^2b (1 + e^{2i(e+fx)} + e^{4i(e+fx)}) \right) \right) / \left( b^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^2 \right) \right) + \\
 & \left( 24i f x - 12 \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] - \right. \\
 & \quad \left. 12 \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] \right) / \\
 & \left( \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \sec[e + fx]^5
 \end{aligned}$$

**Problem 432: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e + fx]}{(a + b \sec[e + fx]^2)^{5/2}} dx$$

Optimal (type 3, 137 leaves, 9 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+fx]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2} f} - \frac{b}{3a(a+b)f(a+b \sec[e+fx]^2)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2 f \sqrt{a+b \sec[e+fx]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Cot}[e + fx]}{(a + b \sec[e + fx]^2)^{5/2}} dx$$

**Problem 433: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e + fx]^3}{(a + b \sec[e + fx]^2)^{5/2}} dx$$

Optimal (type 3, 200 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} + \\
 & \frac{(2a+7b) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{2(a+b)^{7/2} f} - \frac{(3a-2b)b}{6a(a+b)^2 f (a+b \text{Sec}[e+fx]^2)^{3/2}} - \\
 & \frac{\text{Cot}[e+fx]^2}{2(a+b) f (a+b \text{Sec}[e+fx]^2)^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3 f \sqrt{a+b \text{Sec}[e+fx]^2}}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e+fx]^3}{(a+b \text{Sec}[e+fx]^2)^{5/2}} dx$$

**Problem 434: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e+fx]^5}{(a+b \text{Sec}[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 268 leaves, 11 steps):

$$\begin{aligned}
 & \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} - \frac{(8a^2+36ab+63b^2) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{9/2} f} + \\
 & \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3 f (a+b \text{Sec}[e+fx]^2)^{3/2}} + \frac{(4a+11b) \text{Cot}[e+fx]^2}{8(a+b)^2 f (a+b \text{Sec}[e+fx]^2)^{3/2}} - \\
 & \frac{\text{Cot}[e+fx]^4}{4(a+b) f (a+b \text{Sec}[e+fx]^2)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4 f \sqrt{a+b \text{Sec}[e+fx]^2}}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e+fx]^5}{(a+b \text{Sec}[e+fx]^2)^{5/2}} dx$$

**Problem 435: Unable to integrate problem.**

$$\int \frac{\text{Tan}[e+fx]^6}{(a+b \text{Sec}[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 157 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} + \frac{\text{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{b^{5/2} f} - \\
 & \frac{(a+b) \tan[e+fx]^3}{3 a b f (a+b \tan[e+fx]^2)^{3/2}} + \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tan[e+fx]}{f \sqrt{a+b \tan[e+fx]^2}}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^6}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

**Problem 436: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e+fx]^4}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 120 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{(a+b) \tan[e+fx]}{3 a b f (a+b \tan[e+fx]^2)^{3/2}} + \frac{(a-3b) \tan[e+fx]}{3 a^2 b f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 3, 1414 leaves):

$$\begin{aligned}
& \left( i e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{5/2} \right. \\
& \left( -25 a^{7/2} - 58 a^{5/2} b - 32 a^{3/2} b^2 - 15 a^{7/2} e^{2i(e+fx)} - 108 a^{5/2} b e^{2i(e+fx)} - 192 a^{3/2} b^2 e^{2i(e+fx)} - \right. \\
& 96 \sqrt{a} b^3 e^{2i(e+fx)} + 15 a^{7/2} e^{4i(e+fx)} + 108 a^{5/2} b e^{4i(e+fx)} + 192 a^{3/2} b^2 e^{4i(e+fx)} + \\
& 96 \sqrt{a} b^3 e^{4i(e+fx)} + 25 a^{7/2} e^{6i(e+fx)} + 58 a^{5/2} b e^{6i(e+fx)} + 32 a^{3/2} b^2 e^{6i(e+fx)} - 24 i a^2 \\
& \left. \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} f x - 48 i a b \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} f x - \right. \\
& 24 i b^2 \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} f x - 12 a^2 \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} \\
& \left. \log \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \right. \\
& 24 a b \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} \\
& \left. \log \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \right. \\
& 12 b^2 \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} \\
& \left. \log \left[ e^{-2i e} \left( a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right. \\
& 12 a^2 \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} \\
& \left. \log \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right. \\
& 24 a b \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} \\
& \left. \log \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right. \\
& 12 b^2 \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^{3/2} \\
& \left. \log \left[ e^{-2i e} \left( a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \right) \\
& \sec[e+fx]^5 \Big/ \left( 96 \sqrt{2} a^{5/2} (a+b)^2 \left( 4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2 \right)^2 \right. \\
& \left. f (a+b \sec[e+fx]^2)^{5/2} \right) + \\
& \left( (2a+3b+a \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \right. \\
& \left. \sec[e+fx]^4 \tan[e+fx] \right) / \\
& \left( 48 (a+b)^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \sec[e+fx]^2)^{5/2} \right) - \\
& \left( (b+(3a+2b) \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \right. \\
& \left. \sec[e+fx]^4 \tan[e+fx] \right) / \\
& \left( 32 (a+b)^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \sec[e+fx]^2)^{5/2} \right)
\end{aligned}$$



Problem 437: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\tan[e+fx]^2}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} + \frac{\tan[e+fx]}{3 a f (a+b \tan[e+fx]^2)^{3/2}} + \frac{(2 a+3 b) \tan[e+fx]}{3 a^2 (a+b) f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 3, 1414 leaves):

$$\begin{aligned} & - \left( \left( i e^{i(e+fx)} \sqrt{4 b+a e^{-2 i(e+fx)} (1+e^{2 i(e+fx)})^2 (a+2 b+a \cos[2 e+2 f x])^{5/2}} \right. \right. \\ & \quad \left. \left. \begin{aligned} & -25 a^{7/2}-58 a^{5/2} b-32 a^{3/2} b^2-15 a^{7/2} e^{2 i(e+fx)}-108 a^{5/2} b e^{2 i(e+fx)}- \\ & 192 a^{3/2} b^2 e^{2 i(e+fx)}-96 \sqrt{a} b^3 e^{2 i(e+fx)}+15 a^{7/2} e^{4 i(e+fx)}+108 a^{5/2} b e^{4 i(e+fx)}+ \\ & 192 a^{3/2} b^2 e^{4 i(e+fx)}+96 \sqrt{a} b^3 e^{4 i(e+fx)}+25 a^{7/2} e^{6 i(e+fx)}+58 a^{5/2} b e^{6 i(e+fx)}+ \\ & 32 a^{3/2} b^2 e^{6 i(e+fx)}-24 i a^2\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} f x- \\ & 48 i a b\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} f x- \\ & 24 i b^2\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} f x-12 a^2\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} \\ & \quad \left. \log \left[ e^{-2 i e}\left(a+2 b+a e^{2 i(e+fx)}+\sqrt{a} \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2}\right) \right]- \right. \\ & \quad \left. 24 a b\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} \right. \\ & \quad \left. \log \left[ e^{-2 i e}\left(a+2 b+a e^{2 i(e+fx)}+\sqrt{a} \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2}\right) \right]- \right. \\ & \quad \left. 12 b^2\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} \right. \\ & \quad \left. \log \left[ e^{-2 i e}\left(a+2 b+a e^{2 i(e+fx)}+\sqrt{a} \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2}\right) \right]+ \right. \\ & \quad \left. 12 a^2\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} \right. \\ & \quad \left. \log \left[ e^{-2 i e}\left(a+a e^{2 i(e+fx)}+2 b e^{2 i(e+fx)}+\sqrt{a} \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2}\right) \right]+ \right. \\ & \quad \left. 24 a b\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} \right. \\ & \quad \left. \log \left[ e^{-2 i e}\left(a+a e^{2 i(e+fx)}+2 b e^{2 i(e+fx)}+\sqrt{a} \sqrt{4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2}\right) \right]+ \right. \\ & \quad \left. 12 b^2\left(4 b e^{2 i(e+fx)}+a\left(1+e^{2 i(e+fx)}\right)^2\right)^{3/2} \right) \end{aligned}$$

$$\begin{aligned} & \left( \text{Log} \left[ e^{-2 i} e \left( a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] \right) \\ & \left. \text{Sec} [e + f x]^5 \right) / \left( 96 \sqrt{2} a^{5/2} (a + b)^2 (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)^2 \right. \\ & \left. f (a + b \text{Sec} [e + f x]^2)^{5/2} \right) + \\ & \left( (2 a + 3 b + a \text{Cos} [2 (e + f x)]) (a + 2 b + a \text{Cos} [2 e + 2 f x])^{5/2} \right. \\ & \text{Sec} [e + f x]^4 \\ & \text{Tan} [e + f x] \left. \right) / (48 \\ & (a + b)^2 \\ & f \\ & (a + 2 b + a \text{Cos} [2 (e + f x)])^{3/2} \\ & (a + b \text{Sec} [e + f x]^2)^{5/2} - \\ & \left( (b + (3 a + 2 b) \text{Cos} [2 (e + f x)]) (a + 2 b + a \text{Cos} [2 e + 2 f x])^{5/2} \right. \\ & \text{Sec} [e + f x]^4 \\ & \text{Tan} [e + f x] \left. \right) / \\ & \left( 96 (a + b)^2 f (a + 2 b + a \text{Cos} [2 (e + f x)])^{3/2} \right. \\ & \left. (a + b \text{Sec} [e + f x]^2)^{5/2} \right) \end{aligned}$$

**Problem 438:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \text{Sec} [e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$\frac{\text{ArcTan} \left[ \frac{\sqrt{a} \text{Tan} [e+f x]}{\sqrt{a+b \text{Tan} [e+f x]^2}} \right]}{a^{5/2} f} - \frac{b \text{Tan} [e + f x]}{3 a (a + b) f (a + b + b \text{Tan} [e + f x]^2)^{3/2}} - \frac{b (5 a + 3 b) \text{Tan} [e + f x]}{3 a^2 (a + b)^2 f \sqrt{a + b + b \text{Tan} [e + f x]^2}}$$

Result (type 6, 1927 leaves):

$$\begin{aligned} & \left( 3 (a + b) \text{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \text{Sin} [e + f x]^2, \frac{a \text{Sin} [e + f x]^2}{a + b} \right] \text{Cos} [e + f x]^4 \text{Sin} [e + f x] \right) / \\ & \left( 4 \sqrt{2} f (a + b \text{Sec} [e + f x]^2)^{5/2} (a + b - a \text{Sin} [e + f x]^2)^{5/2} \right) \end{aligned}$$

$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
 & \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \\
 & \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \\
 & \left( \left( 15 a (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right. \right. \\
 & \left. \left. \cos[e+fx]^5 \sin[e+fx]^2 \right) / \left( 4 \sqrt{2} (a+b - a \sin[e+fx]^2)^{7/2} \right. \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
 & \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \\
 & \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) + \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \right) / \\
 & \left( 4 \sqrt{2} (a+b - a \sin[e+fx]^2)^{5/2} \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \right. \right. \right. \\
 & \left. \left. \frac{a \sin[e+fx]^2}{a+b} \right] + \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) - \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^3 \right. \\
 & \left. \sin[e+fx]^2 \right) / \left( \sqrt{2} (a+b - a \sin[e+fx]^2)^{5/2} \right. \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
 & \left( 5 a \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \\
 & \left. \left. 4 (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) + \\
 & \left( 3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left( \frac{1}{3(a+b)} 5 a f \operatorname{AppellF1} \left[ \frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] - \frac{4}{3} f \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] \text{Sin}[e + f x] \right) \right) \right) / \\
 & \left( 4 \sqrt{2} f (a + b - a \text{Sin}[e + f x]^2)^{5/2} \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \text{Sin}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. \frac{a \text{Sin}[e + f x]^2}{a + b}\right] + \left( 5 a \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] - \right. \right. \\
 & \left. \left. 4 (a + b) \text{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \right) \text{Sin}[e + f x]^2 \right) \right) - \\
 & \left( 3 (a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x]^4 \right. \\
 & \left. \text{Sin}[e + f x] \left( 2 f \left( 5 a \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] - \right. \right. \right. \\
 & \left. \left. 4 (a + b) \text{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \right) \right. \right. \\
 & \left. \text{Cos}[e + f x] \text{Sin}[e + f x] + 3 (a + b) \left( \frac{1}{3 (a + b)} 5 a f \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \right. \right. \right. \\
 & \left. \left. \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] \text{Sin}[e + f x] - \frac{4}{3} f \right. \right. \\
 & \left. \left. \text{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] \text{Sin}[e + f x] \right) \right) + \\
 & \left. \text{Sin}[e + f x]^2 \left( 5 a \left( \frac{1}{5 (a + b)} 21 a f \text{AppellF1}\left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \text{Sin}[e + f x]^2, \right. \right. \right. \right. \\
 & \left. \left. \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] \text{Sin}[e + f x] - \frac{12}{5} f \text{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \right. \right. \right. \\
 & \left. \left. \frac{7}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] \text{Sin}[e + f x] \right) - 4 (a + b) \right. \\
 & \left. \left( \frac{1}{a + b} 3 a f \text{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \text{Sin}[e + f x]^2, \frac{a \text{Sin}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] \right. \right.
 \end{aligned}$$

$$\begin{aligned} & \left( \sin[e+fx] - \left( 6(a+b)^3 f \cot[e+fx] \csc[e+fx]^4 \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2 \right. \right. \\ & \left. \left. \left( \frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right] \sin[e+fx]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} + \frac{a^2 \sin[e+fx]^4}{3(a+b)^2 \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2} + \right. \right. \right. \\ & \left. \left. \left. \frac{a \sin[e+fx]^2}{(a+b) \left( -1 + \frac{a \sin[e+fx]^2}{a+b} \right)} \right) \right) \right) \left/ \left( a^3 \left( 1 - \frac{a \sin[e+fx]^2}{a+b} \right)^{3/2} \right) \right) \right) \right) \left/ \right. \\ & \left. \left( 4\sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \right. \right. \right. \right. \\ & \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \right. \right. \\ & \left. \left. \left( 5a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4(a+b) \right. \right. \right. \\ & \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right)^2 \right) \right) \right) \right) \end{aligned}$$

### Problem 439: Unable to integrate problem.

$$\int \frac{\cot[e+fx]^2}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\begin{aligned} & - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \cot[e+fx]}{3a(a+b) f (a+b+b \tan[e+fx]^2)^{3/2}} - \\ & \frac{b(7a+3b) \cot[e+fx]}{3a^2(a+b)^2 f \sqrt{a+b+b \tan[e+fx]^2}} - \frac{(a-3b)(3a+b) \cot[e+fx] \sqrt{a+b+b \tan[e+fx]^2}}{3a^2(a+b)^3 f} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cot[e+fx]^2}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

**Problem 440: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 236 leaves, 9 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e+fx]}{\sqrt{a+b \text{Tan}[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \text{Cot}[e+fx]^3}{3 a (a+b) f (a+b+b \text{Tan}[e+fx]^2)^{3/2}} - \frac{b (3 a+b) \text{Cot}[e+fx]^3}{a^2 (a+b)^2 f \sqrt{a+b+b \text{Tan}[e+fx]^2}} + \frac{(a-b) (3 a^2+14 a b+3 b^2) \text{Cot}[e+fx] \sqrt{a+b+b \text{Tan}[e+fx]^2}}{3 a^2 (a+b)^4 f} - \frac{(a^2-10 a b-3 b^2) \text{Cot}[e+fx]^3 \sqrt{a+b+b \text{Tan}[e+fx]^2}}{3 a^2 (a+b)^3 f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

**Problem 441: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 315 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e+fx]}{\sqrt{a+b \text{Tan}[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \text{Cot}[e+fx]^5}{3 a (a+b) f (a+b+b \text{Tan}[e+fx]^2)^{3/2}} - \frac{b (11 a+3 b) \text{Cot}[e+fx]^5}{3 a^2 (a+b)^2 f \sqrt{a+b+b \text{Tan}[e+fx]^2}} - \frac{1}{15 a^2 (a+b)^5 f} + \frac{(15 a^4+70 a^3 b+128 a^2 b^2-70 a b^3-15 b^4) \text{Cot}[e+fx] \sqrt{a+b+b \text{Tan}[e+fx]^2}}{15 a^2 (a+b)^4 f} + \frac{(5 a^3+19 a^2 b-65 a b^2-15 b^3) \text{Cot}[e+fx]^3 \sqrt{a+b+b \text{Tan}[e+fx]^2}}{5 a^2 (a+b)^3 f} - \frac{(a^2-20 a b-5 b^2) \text{Cot}[e+fx]^5 \sqrt{a+b+b \text{Tan}[e+fx]^2}}{5 a^2 (a+b)^3 f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

**Problem 442: Result more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[e + f x]^2)^p (d \text{Tan}[e + f x])^m dx$$

Optimal (type 6, 105 leaves, 4 steps):

$$\frac{1}{d f (1+m)} \text{AppellF1}\left[\frac{1+m}{2}, 1, -p, \frac{3+m}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a+b}\right] \\ (d \text{Tan}[e + f x])^{1+m} (a + b + b \text{Tan}[e + f x]^2)^p \left(1 + \frac{b \text{Tan}[e + f x]^2}{a+b}\right)^{-p}$$

Result (type 6, 2929 leaves):

$$\left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right. \\ \left. \text{Cos}[e + f x] (a + 2b + a \text{Cos}[2(e + f x)])^p (\text{Sec}[e + f x]^2)^p \right. \\ \left. (a + b \text{Sec}[e + f x]^2)^p \text{Sin}[e + f x] \text{Tan}[e + f x]^m (d \text{Tan}[e + f x])^m \right) / \\ \left( f (1+m) \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + \right. \right. \\ \left. \left. 2 \left( b p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] - \right. \right. \right. \\ \left. \left. (a+b) \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \\ \left( \left( (a+b) m (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right. \right. \\ \left. \left. \text{Cos}[e + f x] (a + 2b + a \text{Cos}[2(e + f x)])^p (\text{Sec}[e + f x]^2)^{1+p} \text{Sin}[e + f x] \text{Tan}[e + f x]^{-1+m} \right) / \right. \\ \left( (1+m) \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + \right. \right. \\ \left. \left. 2 \left( b p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] - (a+b) \right. \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) + \\ \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right. \\ \left. (a + 2b + a \text{Cos}[2(e + f x)])^p (\text{Sec}[e + f x]^2)^{-1+p} \text{Tan}[e + f x]^m \right) / \\ \left( (1+m) \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + \right. \right. \\ \left. \left. 2 \left( b p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] - (a+b) \right. \right. \right.$$

$$\begin{aligned}
 & \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right]\right) \tan[e+fx]^2\right) - \right. \\
 & \left. \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \quad \left. \left. (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx]^2 \tan[e+fx]^m \right) / \right. \\
 & \left. \left( (1+m) \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \right. \right. \\
 & \quad \left. \left. 2 \left( b^p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \right. \right. \right. \\
 & \quad \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right]\right) \tan[e+fx]^2 \right) \right) - \right. \\
 & \left. \left( 2 a (a+b) (3+m) p \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \quad \left. \left. \cos[e+fx] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^p \right. \right. \\
 & \quad \left. \left. \sin[e+fx] \sin[2(e+fx)] \tan[e+fx]^m \right) / \right. \\
 & \left. \left( (1+m) \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \right. \right. \\
 & \quad \left. \left. 2 \left( b^p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \right. \right. \right. \\
 & \quad \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right]\right) \tan[e+fx]^2 \right) \right) + \right. \\
 & \left. \left( 2 (a+b) (3+m) p \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \quad \left. \left. \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \sin[e+fx] \tan[e+fx]^{1+m} \right) / \right. \\
 & \left. \left( (1+m) \left( (a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \right. \right. \\
 & \quad \left. \left. 2 \left( b^p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \right. \right. \right. \\
 & \quad \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right]\right) \tan[e+fx]^2 \right) \right) + \right. \\
 & \left. \left( (a+b) (3+m) \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \right. \right. \\
 & \quad \left. \left. \sin[e+fx] \tan[e+fx]^m \right. \right. \\
 & \quad \left. \left. \left( \frac{1}{(a+b) (3+m)} 2 b (1+m) p \text{AppellF1}\left[1+\frac{1+m}{2}, 1-p, 1, 1+\frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] - \frac{1}{3+m} 2 (1+m) \text{AppellF1}\left[1+\frac{1+m}{2}, -p, \right. \right. \right. \\
 & \quad \left. \left. \left. 2, 1+\frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \right. \\
 & \left. \right) / \left. \right)
 \end{aligned}$$



$$\begin{aligned}
 & \left( (1+m) \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - (a+b) \right. \right. \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) - \\
 & \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right. \\
 & \quad \left. \operatorname{Cos}[e+fx] (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[e+fx] \operatorname{Tan}[e+fx]^m \right. \\
 & \quad \left. \left( 4 \left( b p \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \right. \\
 & \quad \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \right. \\
 & \quad \left. \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + (a+b) (3+m) \left( \left( 2b(1+m) p \operatorname{AppellF1} \left[ 1+\frac{1+m}{2}, 1-p, \right. \right. \right. \right. \\
 & \quad \quad \left. \left. \left. 1, 1+\frac{3+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) / \\
 & \quad \left( (a+b) (3+m) - \frac{1}{3+m} 2(1+m) \operatorname{AppellF1} \left[ 1+\frac{1+m}{2}, -p, 2, 1+\frac{3+m}{2}, \right. \right. \\
 & \quad \quad \left. \left. -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) + 2 \operatorname{Tan}[e+fx]^2 \\
 & \quad \left( b p \left( -\frac{1}{5+m} 2(3+m) \operatorname{AppellF1} \left[ 1+\frac{3+m}{2}, 1-p, 2, 1+\frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, \right. \right. \right. \\
 & \quad \quad \left. \left. -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \left( 2b(3+m)(1-p) \operatorname{AppellF1} \left[ \right. \right. \right. \\
 & \quad \quad \left. \left. \left. 1+\frac{3+m}{2}, 2-p, 1, 1+\frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \right. \right. \\
 & \quad \quad \left. \left. \operatorname{Tan}[e+fx] \right) \right) / ((a+b)(5+m)) - (a+b) \left( \left( 2b(3+m) p \operatorname{AppellF1} \left[ \right. \right. \right. \\
 & \quad \quad \left. \left. \left. 1+\frac{3+m}{2}, 1-p, 2, 1+\frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \right. \right. \\
 & \quad \quad \left. \left. \operatorname{Tan}[e+fx] \right) \right) / ((a+b)(5+m) - \frac{1}{5+m} 4(3+m) \operatorname{AppellF1} \left[ 1+\frac{3+m}{2}, -p, 3, \right. \\
 & \quad \quad \left. \left. 1+\frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) \right) / \\
 & \left( (1+m) \left( (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - (a+b) \right. \right. \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) \right) \right)
 \end{aligned}$$

**Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cot}[e + f x] (a + b \text{Sec}[e + f x]^2)^p dx$$

Optimal (type 5, 114 leaves, 5 steps):

$$- \left( \left( \text{Hypergeometric2F1}\left[1, 1 + p, 2 + p, \frac{a + b \text{Sec}[e + f x]^2}{a + b}\right] (a + b \text{Sec}[e + f x]^2)^{1+p} \right) / \right. \\ \left. (2 (a + b) f (1 + p)) \right) + \frac{1}{2 a f (1 + p)} \\ \text{Hypergeometric2F1}\left[1, 1 + p, 2 + p, 1 + \frac{b \text{Sec}[e + f x]^2}{a}\right] (a + b \text{Sec}[e + f x]^2)^{1+p}$$

Result (type 6, 2055 leaves):

$$\left( (a + 2 b + a \text{Cos}[2 (e + f x)])^p \text{Cot}[e + f x] (\text{Sec}[e + f x]^2)^p (a + b \text{Sec}[e + f x]^2)^p \right. \\ \left( \frac{1}{p} \left( 1 + \frac{(a + b) \text{Cot}[e + f x]^2}{b} \right)^{-p} \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] - \right. \\ \left. \left( 2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] \text{Sin}[e + f x]^2 \right) / \right. \\ \left. \left( 2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] + \right. \right. \\ \left. \left( b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] - \right. \right. \\ \left. \left. (a + b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) / \\ \left( 2 f \left( -a p (a + 2 b + a \text{Cos}[2 (e + f x)])^{-1+p} (\text{Sec}[e + f x]^2)^p \text{Sin}[2 (e + f x)] \right. \right. \\ \left. \left( \frac{1}{p} \left( 1 + \frac{(a + b) \text{Cot}[e + f x]^2}{b} \right)^{-p} \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] - \right. \right. \\ \left. \left( 2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] \text{Sin}[e + f x]^2 \right) / \right. \\ \left. \left( 2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] + \right. \right. \\ \left. \left( b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] - \right. \right. \\ \left. \left. (a + b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) + \\ p (a + 2 b + a \text{Cos}[2 (e + f x)])^p (\text{Sec}[e + f x]^2)^p \text{Tan}[e + f x] \\ \left. \left( \frac{1}{p} \left( 1 + \frac{(a + b) \text{Cot}[e + f x]^2}{b} \right)^{-p} \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] - \right. \right.$$

$$\begin{aligned}
 & \left( 2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sin[e+fx]^2 \right) / \\
 & \left( 2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \left. \left( b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) + \\
 & \frac{1}{2} (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \left( \frac{1}{b} 2(a+b) \cot[e+fx] \right. \\
 & \quad \left. \left( 1 + \frac{(a+b) \cot[e+fx]^2}{b} \right)^{-1-p} \csc[e+fx]^2 \right. \\
 & \quad \operatorname{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + \\
 & \quad 2 \left( 1 + \frac{(a+b) \cot[e+fx]^2}{b} \right)^{-p} \csc[e+fx] \left( \left( 1 + \frac{(a+b) \cot[e+fx]^2}{b} \right)^p - \right. \\
 & \quad \left. \operatorname{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \right) \sec[e+fx] - \\
 & \left( 4 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx] \right. \\
 & \quad \left. \sin[e+fx] \right) / \left( 2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left( b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 2 (a+b) \sin[e+fx]^2 \left( \frac{1}{a+b} b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right], \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2 \right) \sec[e+fx]^2 \tan[e+fx] - \operatorname{AppellF1}\left[2, -p, 2, 3, \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) / \\
 & \left( 2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left( b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) + \\
 & \left( 2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sin[e+fx]^2 \right. \\
 & \quad \left. \left( 2 \left( b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \\
 & \operatorname{Sec}[e+fx]^2 \tan[e+fx] + 2(a+b) \left( \frac{1}{a+b} b p \operatorname{AppellF1}\left[2, 1-p, 1, 3, \right. \right. \\
 & \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \operatorname{AppellF1}\left[2, \right. \right. \\
 & \quad \left. \left. -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \\
 & \tan[e+fx]^2 \left( b p \left( -\frac{4}{3} \operatorname{AppellF1}\left[3, 1-p, 2, 4, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \quad \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{3(a+b)} 4 b (1-p) \operatorname{AppellF1}\left[3, 2-p, 1, \right. \right. \right. \\
 & \quad \left. \left. \left. 4, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) - \right. \\
 & \quad \left. (a+b) \left( \frac{1}{3(a+b)} 4 b p \operatorname{AppellF1}\left[3, 1-p, 2, 4, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{8}{3} \operatorname{AppellF1}\left[3, -p, 3, 4, \right. \right. \right. \\
 & \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \right) / \\
 & \left( 2(a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left( b p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \tan[e+fx]^2 \right) \right) \right)
 \end{aligned}$$

**Problem 447: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 5, 157 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{\cot[e+fx]^2 (a+b \operatorname{Sec}[e+fx]^2)^{1+p}}{2(a+b) f} + \\
 & \left( (a+b-b p) \operatorname{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{a+b \operatorname{Sec}[e+fx]^2}{a+b}\right] (a+b \operatorname{Sec}[e+fx]^2)^{1+p} \right) / \\
 & \left( 2(a+b)^2 f (1+p) \right) - \frac{1}{2 a f (1+p)} \\
 & \operatorname{Hypergeometric2F1}\left[1, 1+p, 2+p, 1+\frac{b \operatorname{Sec}[e+fx]^2}{a}\right] (a+b \operatorname{Sec}[e+fx]^2)^{1+p}
 \end{aligned}$$

Result (type 6, 2951 leaves):

$$\begin{aligned}
 & \left( 2^{-1+p} \cot [e+f x]^3 (a+b \sec [e+f x]^2)^p (1+\tan [e+f x]^2)^p \left( \frac{a+b+b \tan [e+f x]^2}{1+\tan [e+f x]^2} \right)^p \right. \\
 & \left( - \left( \left( 2(a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \tan [e+f x]^2 \right) / \right. \right. \\
 & \left( (1+\tan [e+f x]^2) \left( -2(a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] + \right. \right. \\
 & \left. \left( -b p \operatorname{AppellF1} \left[ 2, 1-p, 1, 3, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] + (a+b) \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ 2, -p, 2, 3, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \right) \tan [e+f x]^2 \right) \right) \left. + \right. \\
 & \frac{1}{(-1+p) p} \cot [e+f x]^2 \left( 1 + \frac{(a+b) \cot [e+f x]^2}{b} \right)^{-p} \\
 & \left( p \operatorname{Hypergeometric2F1} \left[ 1-p, -p, 2-p, -\frac{(a+b) \cot [e+f x]^2}{b} \right] - \right. \\
 & \left. (-1+p) \operatorname{Hypergeometric2F1} \left[ -p, -p, 1-p, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \tan [e+f x]^2 \right) \left. \right) \left. \right) / \\
 & \left( f \left( 2^p p \sec [e+f x]^2 \tan [e+f x] (1+\tan [e+f x]^2)^{-1+p} \left( \frac{a+b+b \tan [e+f x]^2}{1+\tan [e+f x]^2} \right)^p \right. \right. \\
 & \left( - \left( \left( 2(a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \tan [e+f x]^2 \right) / \right. \right. \\
 & \left( (1+\tan [e+f x]^2) \right. \\
 & \left( -2(a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] + \right. \\
 & \left( -b p \operatorname{AppellF1} \left[ 2, 1-p, 1, 3, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] + (a+b) \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ 2, -p, 2, 3, -\frac{b \tan [e+f x]^2}{a+b}, -\tan [e+f x]^2 \right] \right) \tan [e+f x]^2 \right) \right) \left. + \right. \\
 & \frac{1}{(-1+p) p} \cot [e+f x]^2 \left( 1 + \frac{(a+b) \cot [e+f x]^2}{b} \right)^{-p} \\
 & \left( p \operatorname{Hypergeometric2F1} \left[ 1-p, -p, 2-p, -\frac{(a+b) \cot [e+f x]^2}{b} \right] - \right. \\
 & \left. (-1+p) \operatorname{Hypergeometric2F1} \left[ -p, -p, 1-p, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \tan [e+f x]^2 \right) \left. \right) \left. \right) + \\
 & 2^{-1+p} p (1+\tan [e+f x]^2)^p \left( \frac{a+b+b \tan [e+f x]^2}{1+\tan [e+f x]^2} \right)^{-1+p} \\
 & \left( \frac{2 b \sec [e+f x]^2 \tan [e+f x]}{1+\tan [e+f x]^2} - \frac{2 \sec [e+f x]^2 \tan [e+f x] (a+b+b \tan [e+f x]^2)}{(1+\tan [e+f x]^2)^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( - \left( \left( 2 (a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \right) / \right. \right. \\
 & \quad \left( (1+\operatorname{Tan}[e+f x]^2) \right. \\
 & \quad \left. \left( -2 (a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
 & \quad \left. \left( -b p \operatorname{AppellF1} \left[ 2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ 2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) \Bigg) + \\
 & \frac{1}{(-1+p) p} \operatorname{Cot}[e+f x]^2 \left( 1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-p} \\
 & \left( p \operatorname{Hypergeometric2F1} \left[ 1-p, -p, 2-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] - \right. \\
 & \quad \left. (-1+p) \operatorname{Hypergeometric2F1} \left[ -p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
 & 2^{-1+p} (1+\operatorname{Tan}[e+f x]^2)^p \left( \frac{a+b+b \operatorname{Tan}[e+f x]^2}{1+\operatorname{Tan}[e+f x]^2} \right)^p \\
 & \left( \frac{1}{(-1+p) p} \operatorname{Cot}[e+f x]^2 \left( 1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-p} \right. \\
 & \quad \left( -2 (1-p) p \operatorname{Csc}[e+f x] \left( \left( 1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^p - \operatorname{Hypergeometric2F1} \left[ \right. \right. \right. \\
 & \quad \left. \left. \left. 1-p, -p, 2-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \right) \operatorname{Sec}[e+f x] - \right. \\
 & \quad \left. 2 (-1+p) p \left( \left( 1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^p - \operatorname{Hypergeometric2F1} \left[ -p, -p, \right. \right. \right. \\
 & \quad \left. \left. \left. 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \\
 & \quad \left. 2 (-1+p) \operatorname{Hypergeometric2F1} \left[ -p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \right. \\
 & \quad \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Bigg) + \\
 & \left( 4 (a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \\
 & \quad \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]^3 \right) / \left( (1+\operatorname{Tan}[e+f x]^2)^2 \right. \\
 & \quad \left( -2 (a+b) \operatorname{AppellF1} \left[ 1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \\
 & \quad \left. \left( -b p \operatorname{AppellF1} \left[ 2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + (a+b) \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2\right) \right) - \right. \\
 & \left. \left( 4 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right. \right. \\
 & \left. \left. \text{Sec}[e + f x]^2 \tan[e + f x] \right) \right) / \left( (1 + \tan[e + f x]^2) \right. \\
 & \left. \left( -2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + \right. \right. \\
 & \left. \left( -b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + (a + b) \right. \right. \\
 & \left. \left. \left. \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2\right) \right) \right) - \\
 & \left( 2 (a + b) \tan[e + f x]^2 \left( \frac{1}{a + b} b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right. \right. \\
 & \left. \left. \text{Sec}[e + f x]^2 \tan[e + f x] - \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right. \right. \\
 & \left. \left. \left. \text{Sec}[e + f x]^2 \tan[e + f x] \right) \right) \right) / \left( (1 + \tan[e + f x]^2) \right. \\
 & \left. \left( -2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + \right. \right. \\
 & \left. \left( -b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + (a + b) \right. \right. \\
 & \left. \left. \left. \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2\right) \right) \right) + \\
 & \frac{1}{b (-1 + p)} 2 (a + b) \cot[e + f x]^3 \left( 1 + \frac{(a + b) \cot[e + f x]^2}{b} \right)^{-1-p} \csc[e + f x]^2 \\
 & \left( p \text{Hypergeometric2F1}\left[1 - p, -p, 2 - p, -\frac{(a + b) \cot[e + f x]^2}{b}\right] - \right. \\
 & \left. (-1 + p) \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \cot[e + f x]^2}{b}\right] \tan[e + f x]^2 \right) - \\
 & \frac{1}{(-1 + p) p} 2 \cot[e + f x] \left( 1 + \frac{(a + b) \cot[e + f x]^2}{b} \right)^{-p} \csc[e + f x]^2 \\
 & \left( p \text{Hypergeometric2F1}\left[1 - p, -p, 2 - p, -\frac{(a + b) \cot[e + f x]^2}{b}\right] - \right. \\
 & \left. (-1 + p) \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \cot[e + f x]^2}{b}\right] \tan[e + f x]^2 \right) + \\
 & \left( 2 (a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2 \right. \\
 & \left. \left( 2 \left( -b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \\
 & \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - 2 (a+b) \left( \frac{1}{a+b} b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \operatorname{AppellF1}\left[2, \right. \right. \\
 & \quad \left. \left. -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
 & \operatorname{Tan}[e+f x]^2 \left( -b^p \left( -\frac{4}{3} \operatorname{AppellF1}\left[3, 1-p, 2, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{3(a+b)} 4 b (1-p) \operatorname{AppellF1}\left[3, \right. \right. \\
 & \quad \left. \left. 2-p, 1, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
 & (a+b) \left( \frac{1}{3(a+b)} 4 b^p \operatorname{AppellF1}\left[3, 1-p, 2, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{8}{3} \operatorname{AppellF1}\left[3, -p, 3, 4, \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \Bigg) / \\
 & \left( (1+\operatorname{Tan}[e+f x]^2) \left( -2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+f x]^2\right] + \left( -b^p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+f x]^2\right] + (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, \right. \right. \\
 & \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) /
 \end{aligned}$$

**Problem 448: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x]^4 dx$$

Optimal (type 6, 88 leaves, 4 steps):

$$\frac{1}{5 f} \operatorname{AppellF1}\left[\frac{5}{2}, 1, -p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Tan}[e+f x]^5 (a+b+b \operatorname{Tan}[e+f x]^2)^p \left(1+\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p}$$

Result (type 6, 2777 leaves):

$$\left( (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x]^5 \right. \\
 \left. \left( \left( 9 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Cos}[e+f x]^2 \right) / \right. \right.$$



$$\begin{aligned}
 & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] + \right. \\
 & 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - \right. \\
 & \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \left. \right) + \\
 & \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p} \left( -3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] \operatorname{Tan}[e+fx]^2 \right) \left. \right) \Big/ \\
 & \left( 3 f \left( \frac{1}{3} (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{1+p} \right. \right. \\
 & \quad \left. \left( \left( 9 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Cos}[e+fx]^2 \right) \Big/ \right. \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - \right. \\
 & \quad \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \left. \right) + \\
 & \quad \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p} \left( -3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] \operatorname{Tan}[e+fx]^2 \right) \left. \right) - \\
 & \frac{2}{3} a p (a+2b+a \operatorname{Cos}[2(e+fx)])^{-1+p} (\operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[2(e+fx)] \operatorname{Tan}[e+fx] \\
 & \quad \left( \left( 9 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Cos}[e+fx]^2 \right) \Big/ \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - \right. \\
 & \quad \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \left. \right) + \\
 & \quad \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p} \left( -3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] + \right. \\
 & \quad \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] \operatorname{Tan}[e+fx]^2 \right) \left. \right) + \\
 & \frac{2}{3} p (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p \operatorname{Tan}[e+fx]^2 \\
 & \quad \left( \left( 9 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Cos}[e+fx]^2 \right) \Big/ \right.
 \end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] + \right. \\
& 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - \right. \\
& \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \Big) + \\
& \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p} \left( -3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] + \right. \\
& \quad \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] \operatorname{Tan}[e+fx]^2 \right) \Big) + \\
& \frac{1}{3} (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p \operatorname{Tan}[e+fx] \\
& \left( - \left( \left( 18 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Cos}[e+fx] \operatorname{Sin}[ \right. \right. \\
& \quad \left. \left. e+fx \right] \right) / \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - (a+b) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \right) \right) \Big) + \\
& \left( 9 (a+b) \operatorname{Cos}[e+fx]^2 \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \right. \right. \\
& \quad \left. \left. \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) \Big) / \\
& \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] + \right. \\
& 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - \right. \\
& \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \Big) - \\
& \frac{1}{a+b} 2 b p \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-1-p} \\
& \left( -3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] + \right. \\
& \quad \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}\right] \operatorname{Tan}[e+fx]^2 \right) - \\
& \left( 9 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Cos}[e+fx]^2 \right. \\
& \quad \left. \left( 4 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2\right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \\
 & \operatorname{Sec}[e+fx]^2 \tan[e+fx] + 3(a+b) \left( \frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{2}{3} \operatorname{AppellF1}\left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \\
 & 2 \tan[e+fx]^2 \left( bp \left( -\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \quad \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2-p, 1, \right. \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) - \\
 & (a+b) \left( \frac{1}{5(a+b)} 6bp \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \\
 & \quad \left. \left. -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, -p, 3, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Big) \Big) \Big) \Big) \Big) / \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \quad \left. 2 \left( bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right)^2 + \\
 & \left( 1 + \frac{b \tan[e+fx]^2}{a+b} \right)^{-p} \left( 2 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
 & \quad \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - 3 \operatorname{Csc}[e+fx] \operatorname{Sec}[e+fx] \left( -\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \right. \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left( 1 + \frac{b \tan[e+fx]^2}{a+b} \right)^p \right) + 3 \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right. \\
 & \quad \left. \left( -\operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left( 1 + \frac{b \tan[e+fx]^2}{a+b} \right)^p \right) \right) \Big) \Big) \Big) \Big) \Big)
 \end{aligned}$$

**Problem 449: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^p \tan[e+fx]^2 dx$$

Optimal (type 6, 88 leaves, 4 steps):

$$\frac{1}{3f} \text{AppellF1}\left[\frac{3}{2}, 1, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Tan}[e+fx]^3 (a+b+b \text{Tan}[e+fx]^2)^p \left(1 + \frac{b \text{Tan}[e+fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 2465 leaves):

$$\left( (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^p (a+b \text{Sec}[e+fx]^2)^p \text{Tan}[e+fx]^3 \right. \\ \left. \left( \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \left(1 + \frac{b \text{Tan}[e+fx]^2}{a+b}\right)^{-p} - \right. \right. \\ \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] \text{Cos}[e+fx]^2 \right) / \right. \\ \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] + \right. \right. \\ \left. \left. 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] - \right. \right. \right. \\ \left. \left. \left. (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] \right) \text{Tan}[e+fx]^2 \right) \right) \right) / \\ \left( f \left( (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{1+p} \right. \right. \\ \left. \left( \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \left(1 + \frac{b \text{Tan}[e+fx]^2}{a+b}\right)^{-p} - \right. \right. \\ \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] \text{Cos}[e+fx]^2 \right) / \right. \\ \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] + \right. \right. \\ \left. \left. 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] - (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] \right) \text{Tan}[e+fx]^2 \right) \right) - \\ 2 a p (a+2b+a \text{Cos}[2(e+fx)])^{-1+p} (\text{Sec}[e+fx]^2)^p \text{Sin}[2(e+fx)] \text{Tan}[e+fx] \\ \left( \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \left(1 + \frac{b \text{Tan}[e+fx]^2}{a+b}\right)^{-p} - \right. \\ \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] \text{Cos}[e+fx]^2 \right) / \right. \\ \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] + \right. \right. \\ \left. \left. 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] - (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2\right] \right) \text{Tan}[e+fx]^2 \right) \right) \right) + \\ 2 p (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^p \text{Tan}[e+fx]^2$$

$$\begin{aligned}
 & \left( \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}\right] \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^{-p} \right. \\
 & \left. \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx]^2 \right) \right) / \\
 & \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \left. 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) \Bigg) + \\
 & (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p \tan[e+fx] \\
 & \left( -\frac{1}{a+b} 2 b p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\
 & \left. \sec[e+fx]^2 \tan[e+fx] \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^{-1-p} + \right. \\
 & \left. \left( 6(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx] \right. \right. \\
 & \left. \left. \sin[e+fx] \right) \right) / \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \left. 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
 & \left. \left. (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) - \\
 & \left( 3(a+b) \cos[e+fx]^2 \left( \frac{1}{3(a+b)} 2 b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
 & \left. \left. - \tan[e+fx]^2 \right) \sec[e+fx]^2 \tan[e+fx] - \frac{2}{3} \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] \right) \Bigg) / \\
 & \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
 & \left. 2 \left( b p \text{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
 & \left. \left. (a+b) \text{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) + \\
 & \text{Csc}[e+fx] \sec[e+fx] \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^{-p} \\
 & \left( -\text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^p \right) + \\
 & \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx]^2 \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( 4 \left( b^p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \right. \\
 & \quad \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b^p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \right. \right. \\
 & \quad \left. \left. \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{2}{3} \operatorname{AppellF1} \left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) + \\
 & \quad 2 \operatorname{Tan}[e+fx]^2 \left( b^p \left( -\frac{6}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right. \right. \\
 & \quad \left. \left. \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 2-p, 1, \right. \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) - \\
 & \quad (a+b) \left( \frac{1}{5(a+b)} 6 b^p \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{12}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, -p, 3, \right. \right. \\
 & \quad \left. \left. \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) \Big/ \\
 & \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \\
 & \quad 2 \left( b^p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - (a+b) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right)^2 \Big) \Big)
 \end{aligned}$$

**Problem 450: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1} \left[ \frac{1}{2}, 1, -p, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Tan}[e+fx] (a+b + b \operatorname{Tan}[e+fx]^2)^p \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p}$$

Result (type 6, 2137 leaves):

$$\left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Cos}[e+fx] \right.$$

$$\begin{aligned}
 & \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p (a + b \sec[e + fx]^2)^p \sin[e + fx] \right) / \\
 & \left( f \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) \\
 & \left( \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \right. \\
 & \quad \left. \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^{-1+p} \right) / \right. \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) - \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \\
 & \quad \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p \sin[e + fx]^2 \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) + \\
 & \left( 6(a + b) p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right. \\
 & \quad \left. (a + 2b + a \cos[2(e + fx)])^p (\sec[e + fx]^2)^p \sin[e + fx]^2 \right) / \\
 & \left( 3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] + \right. \\
 & \quad 2 \left( b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] - \right. \\
 & \quad \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) - \\
 & \left( 6a(a + b) p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2\right] \cos[e + fx] \right. \\
 & \quad \left. (a + 2b + a \cos[2(e + fx)])^{-1+p} (\sec[e + fx]^2)^p \sin[e + fx] \sin[2(e + fx)] \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \\
& 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \quad \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) + \\
& \left( 3 (a+b) \operatorname{Cos}[e+f x] (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \right. \\
& \quad \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \\
& \quad \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{2}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, \right. \right. \\
& \quad \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Big) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \\
& 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \quad \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) - \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x] \right. \\
& \quad (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \\
& \quad \left( 4 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \right. \\
& \quad \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + 3 (a+b) \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, \right. \right. \right. \\
& \quad \quad \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{2}{3} \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \right. \\
& \quad \quad \left. \left. -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Big) + \\
& 2 \operatorname{Tan}[e+f x]^2 \left( b p \left( -\frac{6}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[ \frac{5}{2}, 2-p, 1, \right. \right. \right. \\
& \quad \quad \left. \left. \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Big) - \\
& (a+b) \left( \frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[ \frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right.
\end{aligned}$$



$$\begin{aligned}
 & -\tan [e+f x]^2 \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2},-p, 3,\right. \\
 & \left.\frac{7}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2 \operatorname{Sec}[e+f x]^2 \tan [e+f x]\right)\right)\right)\right)\right) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right. \\
 & \left.2\left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]- \right. \right. \\
 & \left. \left.(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]\right) \tan [e+f x]^2\right)^2\right)\right)
 \end{aligned}$$

**Problem 451: Result more than twice size of optimal antiderivative.**

$$\int \cot [e+f x]^2 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{1}{f} \operatorname{AppellF1}\left[-\frac{1}{2}, 1,-p, \frac{1}{2},-\tan [e+f x]^2,-\frac{b \tan [e+f x]^2}{a+b}\right] \\
 & \cot [e+f x] (a+b+b \tan [e+f x]^2)^p \left(1+\frac{b \tan [e+f x]^2}{a+b}\right)^{-p}
 \end{aligned}$$

Result (type 6, 2469 leaves):

$$\begin{aligned}
 & \left((a+2 b+a \cos [2(e+f x)])^p \cot [e+f x]^3 (\operatorname{Sec}[e+f x]^2)^p (a+b \operatorname{Sec}[e+f x]^2)^p \right. \\
 & \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2},-p, \frac{1}{2},-\frac{b \tan [e+f x]^2}{a+b}\right] \left(1+\frac{b \tan [e+f x]^2}{a+b}\right)^{-p}- \right. \\
 & \left.3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \sin [e+f x]^2\right) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right. \\
 & \left.2\left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]- \right. \right. \\
 & \left. \left.(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]\right) \tan [e+f x]^2\right)\right)\right) / \\
 & \left(f\left(2^p(a+2 b+a \cos [2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \right. \right. \\
 & \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2},-p, \frac{1}{2},-\frac{b \tan [e+f x]^2}{a+b}\right] \left(1+\frac{b \tan [e+f x]^2}{a+b}\right)^{-p}- \right. \\
 & \left.3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \sin [e+f x]^2\right) / \\
 & \left.3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[ \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) - \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)])^p \operatorname{Csc}[e+f x]^2 (\operatorname{Sec}[e+f x]^2)^p \\
 & \left( -\operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \left( 1+\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} - \right. \\
 & \left. \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sin}[e+f x]^2 \right) \right) / \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \\
 & \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[ \right. \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) \Big) - \\
 & 2 a p (a+2 b+a \operatorname{Cos}[2(e+f x)])^{-1+p} \operatorname{Cot}[e+f x] (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[2(e+f x)] \\
 & \left( -\operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \left( 1+\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} - \right. \\
 & \left. \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sin}[e+f x]^2 \right) \right) / \\
 & \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \\
 & \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[ \right. \right. \right. \\
 & \quad \left. \left. \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) \Big) + \\
 & (a+2 b+a \operatorname{Cos}[2(e+f x)])^p \operatorname{Cot}[e+f x] (\operatorname{Sec}[e+f x]^2)^p \\
 & \left( \frac{1}{a+b} 2 b p \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right. \\
 & \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \left( 1+\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-1-p} - \right. \\
 & \left. \left( 6(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x] \right. \right. \\
 & \left. \left. \operatorname{Sin}[e+f x] \right) \right) / \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \right. \\
 & \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
 & \quad \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
 & \left( 3(a+b) \operatorname{Sin}[e+f x]^2 \left( \frac{1}{3(a+b)} 2 b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -\tan [e+f x]^2] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2},-p, 2,\right. \\
 & \left.\frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right) \Bigg) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right. \\
 & 2\left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]- \right. \\
 & \left.(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]\right) \tan [e+f x]^2)- \\
 & \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x]\left(1+\frac{b \tan [e+f x]^2}{a+b}\right)^{-p}\left(\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \right. \right. \\
 & \left. \left.-p, \frac{1}{2},-\frac{b \tan [e+f x]^2}{a+b}\right]-\left(1+\frac{b \tan [e+f x]^2}{a+b}\right)^p\right)+ \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \sin [e+f x]^2 \right. \\
 & \left(4\left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]- \right. \right. \\
 & \left.(a+b) \operatorname{AppellF1}\left[\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]\right) \\
 & \operatorname{Sec}[e+f x]^2 \tan [e+f x]+3(a+b)\left(\frac{1}{3(a+b)} 2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1,\right. \right. \\
 & \left.\frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{2}{3} \operatorname{AppellF1}\left[\right. \\
 & \left.\frac{3}{2},-p, 2, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right)+ \\
 & 2 \tan [e+f x]^2\left(b p\left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \right. \right. \\
 & \left.\operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{1}{5(a+b)} 6 b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2-p, 1,\right. \right. \\
 & \left.\frac{7}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right)- \\
 & (a+b)\left(\frac{1}{5(a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2},-\frac{b \tan [e+f x]^2}{a+b}, \right. \right. \\
 & \left. \left.-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]-\frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2},-p, 3,\right. \right. \\
 & \left.\frac{7}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan [e+f x]\left.\right)\left.\right)\left.\right) \Bigg) / \\
 & \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2},-p, 1, \frac{3}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]+ \right. \\
 & 2\left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2},-\frac{b \tan [e+f x]^2}{a+b},-\tan [e+f x]^2\right]-\left.(a+b)\right)
 \end{aligned}$$



$$\begin{aligned}
 & \left( \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sin}[e+fx]^2 \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}[e+fx]^2 \right) / \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) - \right. \\
 & \quad \left. \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p} \left( \operatorname{Hypergeometric2F1} \left[ -\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \quad \left. \left. 3 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Tan}[e+fx]^2 \right) \right) - \\
 & \frac{2}{3} a p (a+2b+a \operatorname{Cos}[2(e+fx)])^{-1+p} \operatorname{Cot}[e+fx]^3 (\operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[2(e+fx)] \\
 & \left( \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sin}[e+fx]^2 \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}[e+fx]^2 \right) / \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) - \right. \\
 & \quad \left. \left( 1 + \frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right)^{-p} \left( \operatorname{Hypergeometric2F1} \left[ -\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] - \right. \right. \\
 & \quad \left. \left. 3 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b} \right] \operatorname{Tan}[e+fx]^2 \right) \right) + \\
 & \frac{1}{3} (a+2b+a \operatorname{Cos}[2(e+fx)])^p \operatorname{Cot}[e+fx]^3 (\operatorname{Sec}[e+fx]^2)^p \\
 & \left( \left( 18 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sin}[e+fx]^2 \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}[e+fx] \right) / \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) + \right. \\
 & \quad \left. \left( 18 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Tan}[e+fx]^3 \right) / \right. \\
 & \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + \right. \\
 & \quad \left. \left. 2 \left( b p \operatorname{AppellF1} \left[ \frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \tan[e+fx]^2 + \\
& \left(9(a+b) \sin[e+fx]^2 \tan[e+fx]^2 \left(\frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] - \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx]\right)\right) / \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \\
& \quad \left. 2\left(bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right]\right) \tan[e+fx]^2 + \right. \\
& \quad \left. \frac{1}{a+b} 2bp \sec[e+fx]^2 \tan[e+fx] \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^{-1-p} \right. \\
& \quad \left. \left(\operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\
& \quad \left. \left. 3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2\right) - \right. \\
& \quad \left. \left(9(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sin[e+fx]^2 \right. \right. \\
& \quad \left. \left. \tan[e+fx]^2 \left(4\left(bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right]\right)\right) \right. \right. \\
& \quad \left. \left. \sec[e+fx]^2 \tan[e+fx] + 3(a+b) \left(\frac{1}{3(a+b)} 2bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] - \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx]\right)\right) + \right. \\
& \quad \left. 2 \tan[e+fx]^2 \left(bp \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \right. \\
& \quad \left. \left. \sec[e+fx]^2 \tan[e+fx] - \frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2-p, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx]\right)\right) - \right. \\
& \quad \left. (a+b) \left(\frac{1}{5(a+b)} 6bp \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \right. \\
& \quad \left. \left. -\tan[e+fx]^2\right) \sec[e+fx]^2 \tan[e+fx] - \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, -p, 3, \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \left. \left. \left. \left. \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) \right) \right) / \\ & \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \right. \\ & \quad 2 \left( b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - (a+b) \right. \\ & \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right)^2 - \\ & \left( 1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} \left( -6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right. \\ & \quad \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - 3 \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \left( \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \right. \right. \\ & \quad \left. \left. -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] - \left( 1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^p \right) - 3 \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x] \\ & \quad \left. \left. \left. \left. \left. -\operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \left( 1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^p \right) \right) \right) \right) \right) \right) \end{aligned}$$

**Problem 458: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Tan}[e+f x]^5}{a+b \operatorname{Sec}[e+f x]^3} dx$$

Optimal (type 3, 219 leaves, 11 steps):

$$\begin{aligned} & -\frac{(a^{2/3} + 2 b^{2/3}) \operatorname{ArcTan}\left[\frac{b^{1/3} - 2 a^{1/3} \operatorname{Cos}[e+f x]}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} a^{1/3} b^{4/3} f} - \frac{(a^{2/3} - 2 b^{2/3}) \operatorname{Log}\left[b^{1/3} + a^{1/3} \operatorname{Cos}[e+f x]\right]}{3 a^{1/3} b^{4/3} f} + \\ & \frac{(a^{2/3} - 2 b^{2/3}) \operatorname{Log}\left[b^{2/3} - a^{1/3} b^{1/3} \operatorname{Cos}[e+f x] + a^{2/3} \operatorname{Cos}[e+f x]^2\right]}{6 a^{1/3} b^{4/3} f} - \\ & \frac{\operatorname{Log}\left[b + a \operatorname{Cos}[e+f x]^3\right]}{3 a f} + \frac{\operatorname{Sec}[e+f x]}{b f} \end{aligned}$$

Result (type 7, 251 leaves):

$$\begin{aligned} & \frac{1}{3 a b f} \left( 3 b \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right] - \operatorname{RootSum}\left[-8 a + 12 a \#1 - 6 a \#1^2 + a \#1^3 - b \#1^3 \&, \right. \right. \\ & \quad \left. \left. \left( -4 a^2 \operatorname{Log}\left[1 - \#1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + 4 a b \operatorname{Log}\left[1 - \#1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right. \\ & \quad \left. \left. 2 a^2 \operatorname{Log}\left[1 - \#1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1 - 8 a b \operatorname{Log}\left[1 - \#1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1 + \right. \right. \\ & \quad \left. \left. a b \operatorname{Log}\left[1 - \#1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1^2 - b^2 \operatorname{Log}\left[1 - \#1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1^2 \right) \right) / \\ & \quad \left. \left( 4 a - 4 a \#1 + a \#1^2 - b \#1^2 \right) \& \right] + 3 a \operatorname{Sec}[e+f x] \end{aligned}$$

### Problem 459: Result is not expressed in closed-form.

$$\int \frac{\tan[e + f x]^3}{a + b \sec[e + f x]^3} dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{b^{1/3}-2a^{1/3}\cos[e+fx]}{\sqrt{3}b^{1/3}}\right]}{\sqrt{3}a^{1/3}b^{2/3}f} - \frac{\operatorname{Log}\left[b^{1/3}+a^{1/3}\cos[e+fx]\right]}{3a^{1/3}b^{2/3}f} +$$

$$\frac{\operatorname{Log}\left[b^{2/3}-a^{1/3}b^{1/3}\cos[e+fx]+a^{2/3}\cos[e+fx]^2\right]}{6a^{1/3}b^{2/3}f} + \frac{\operatorname{Log}\left[b+a\cos[e+fx]^3\right]}{3af}$$

Result (type 7, 242 leaves):

$$\frac{1}{3af} \left( -3 \operatorname{Log}\left[\sec\left[\frac{1}{2}(e+fx)\right]^2\right] + \operatorname{RootSum}\left[-a-b+3a\#1-3b\#1-3a\#1^2-3b\#1^2+a\#1^3-b\#1^3 \&, \right.\right.$$

$$\left. \left(-a \operatorname{Log}\left[-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - b \operatorname{Log}\left[-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right.\right.$$

$$\left. 4a \operatorname{Log}\left[-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \#1 - 2b \operatorname{Log}\left[-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \#1 + \right.$$

$$\left. a \operatorname{Log}\left[-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \#1^2 - b \operatorname{Log}\left[-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \#1^2\right) /$$

$$\left. \left(a-b-2a\#1-2b\#1+a\#1^2-b\#1^2\right) \&\right)$$

### Problem 461: Result is not expressed in closed-form.

$$\int \frac{\cot[e + f x]}{a + b \sec[e + f x]^3} dx$$

Optimal (type 3, 295 leaves, 11 steps):

$$-\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3}-2a^{1/3}\cos[e+fx]}{\sqrt{3}b^{1/3}}\right]}{\sqrt{3}a^{1/3}(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3})f} + \frac{\operatorname{Log}\left[1-\cos[e+fx]\right]}{2(a+b)f} +$$

$$\frac{\operatorname{Log}\left[1+\cos[e+fx]\right]}{2(a-b)f} - \frac{(a^{2/3}+b^{2/3})b^{2/3} \operatorname{Log}\left[b^{1/3}+a^{1/3}\cos[e+fx]\right]}{3a^{1/3}(a^2-b^2)f} +$$

$$\frac{(a^{2/3}+b^{2/3})b^{2/3} \operatorname{Log}\left[b^{2/3}-a^{1/3}b^{1/3}\cos[e+fx]+a^{2/3}\cos[e+fx]^2\right]}{6a^{1/3}(a^2-b^2)f} - \frac{b^2 \operatorname{Log}\left[b+a\cos[e+fx]^3\right]}{3a(a^2-b^2)f}$$

Result (type 7, 290 leaves):



$$\frac{1}{3 a (a-b) (a+b) f} \left( 3 \left( a (a+b) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (e+f x) \right] \right] + b^2 \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \right] + a (a-b) \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2} (e+f x) \right] \right] \right) - b \operatorname{RootSum} \left[ -8 a + 12 a \#1 - 6 a \#1^2 + a \#1^3 - b \#1^3 \&, \left( -4 a^2 \operatorname{Log} \left[ 1 - \#1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 4 a b \operatorname{Log} \left[ 1 - \#1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 2 a^2 \operatorname{Log} \left[ 1 - \#1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \#1 - 2 a b \operatorname{Log} \left[ 1 - \#1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \#1 + a b \operatorname{Log} \left[ 1 - \#1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \#1^2 - b^2 \operatorname{Log} \left[ 1 - \#1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \#1^2 \right) / \left( 4 a - 4 a \#1 + a \#1^2 - b \#1^2 \& \right) \right)$$

**Problem 462: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cot}[e+f x]^3}{a+b \operatorname{Sec}[e+f x]^3} dx$$

Optimal (type 3, 393 leaves, 11 steps):

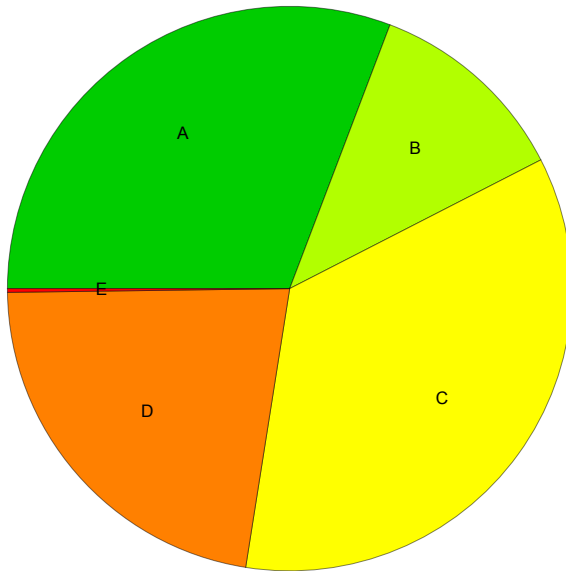
$$\frac{b^{4/3} (a^2 - 3 a^{2/3} b^{4/3} + 2 b^2) \operatorname{ArcTan} \left[ \frac{b^{1/3} - 2 a^{1/3} \operatorname{Cos}[e+f x]}{\sqrt{3} b^{1/3}} \right]}{\sqrt{3} a^{1/3} (a^2 - b^2)^2 f} - \frac{1}{4 (a+b) f (1 - \operatorname{Cos}[e+f x])} - \frac{1}{4 (a-b) f (1 + \operatorname{Cos}[e+f x])} - \frac{(2 a + 5 b) \operatorname{Log}[1 - \operatorname{Cos}[e+f x]]}{4 (a+b)^2 f} - \frac{(2 a - 5 b) \operatorname{Log}[1 + \operatorname{Cos}[e+f x]]}{4 (a-b)^2 f} - \frac{b^{4/3} (a^2 + 3 a^{2/3} b^{4/3} + 2 b^2) \operatorname{Log}[b^{1/3} + a^{1/3} \operatorname{Cos}[e+f x]]}{3 a^{1/3} (a^2 - b^2)^2 f} + \frac{b^{4/3} (a^2 + 3 a^{2/3} b^{4/3} + 2 b^2) \operatorname{Log}[b^{2/3} - a^{1/3} b^{1/3} \operatorname{Cos}[e+f x] + a^{2/3} \operatorname{Cos}[e+f x]^2]}{6 a^{1/3} (a^2 - b^2)^2 f} - \frac{b^2 (2 a^2 + b^2) \operatorname{Log}[b + a \operatorname{Cos}[e+f x]^3]}{3 a (a^2 - b^2)^2 f}$$

Result (type 7, 336 leaves):

$$\frac{1}{24 f} \left( -\frac{3 \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{a+b} + \frac{12(-2 a+5 b) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]\right]}{(a-b)^2} - \frac{12(2 a+5 b) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{(a+b)^2} + \frac{1}{a\left(a^2-b^2\right)^2} 8 b^2 \left( 3\left(2 a^2+b^2\right) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right] + (-a+b) \operatorname{RootSum}\left[-8 a+12 a \#1-6 a \#1^2+a \#1^3-b \#1^3 \&, \left(8 a^2 \operatorname{Log}\left[1-\#1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right]-4 a b \operatorname{Log}\left[1-\#1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right]-6 a^2 \operatorname{Log}\left[1-\#1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1+2 a^2 \operatorname{Log}\left[1-\#1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1^2+b^2 \operatorname{Log}\left[1-\#1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \#1^2}\right) / \left(4 a-4 a \#1+a \#1^2-b \#1^2\right) \& \right) - \frac{3 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{a-b} \right)$$

## Summary of Integration Test Results

471 integration problems



A - 145 optimal antiderivatives

B - 55 more than twice size of optimal antiderivatives

C - 165 unnecessarily complex antiderivatives

D - 105 unable to integrate problems

E - 1 integration timeouts